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A laboratory study of the effects of porosity and bed tilting on the discharge rate of groundwater

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Abstract : The study of movement of underground water helps to predict the extent of flow in as much as we can determine the volume rate among other things. Sand samples from river bed were used as porous media, a laboratory experiment was set up to look at longitudinal dispersion in term of volume of liquid flowing across a unit cross sectional area per unit time in these materials. Water was made to flow through a cylindrical pipe drilled sideways at intervals. Values of pressure were taken at regular interval and using appropriate basic equations, the volume flux rate was determined at various angles of tilt, from which volume rate of flow was also determined. A graph of volume rate of flow against angle of tilt gives the value of volume rate of flow as 1.00 ´ 10^{-10} m^3/s irrespective of porosity and permeability of the medium provided that the angle of tilt is 1.42°.

Keywords : Volume flux, hydraulic conductivity, permeability, hydrostatic angle, porosity.

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1. Introduction

When considering porous media, it is not new to think that more fluid (volume wise) will flow through a porous material, which is more porous per second than that which is less porous. In the same vein, it is expected that volume flux rate should increase with increasing porosity and as the flow is being tilted [1]. It is then evidently clear that there exists a strong connection between porosity, volume flux and the angle of tilt for flow, for a liquid flowing through a porous material. This work is aimed at examining the variation in the volume flux in materials of different porosities with angle of tilt, θ of flow between $0° \le \theta \le 25°$ [1].

2. Theory

Consider a flow through a porous material of permeability κ, Darcy law [2] is written as :

$$V_1 \frac{\kappa}{\mu} \nabla(p - g\rho z), \tag{1}$$

which can be re-expressed as :

$$V_1 - \frac{\kappa}{\mu}\left(\frac{dp}{dl} - g\rho\frac{dz}{dl}\right), \tag{2}$$

where

l = distance in the direction of flow, always positive,

V_l = volume flux across a unit area of the porous medium in unit time along flow path l,

Z = vertical coordinate, considered positive downward,

ρ = density of the liquid,

g = acceleration of gravity,

$\frac{dp}{dl}$ = pressure gradient along l at the point to which V_l refers,

$\frac{dz}{dl}$ = $\sin\theta$, where θ is the angle between l and the horizontal.

$$V_l = Q/A, \tag{3}$$

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where Q [3] is the volume rate of flow or simply, discharge rate and A is the average cross sectional area perpendicular to the lines of flow.

From eq. (2),

$$\frac{dp}{dl} = g\rho\sin\theta - V\frac{\mu}{\kappa}. \tag{4}$$

If a sample is completely saturated with an incompressible fluid [4] and is horizontal, then $\frac{dz}{dl} = 0$; that is $\theta = 0$.

Eq. (4) reduces to

$$\frac{dp}{dl} = -V\frac{\mu}{\kappa}. \tag{5}$$

The minus sign indicates that pressure decreases with distance down the direction of flow. To determine the permeability κ in eqs. (4) and (5), a separate experiment was performed using the same sand samples. Darcy equation [5] of the form :

$$K = Q\frac{s}{A(h+s)t} \tag{6}$$

was employed,

where Q = volume rate of flow through sand filter (m^3),

t = time (sec),

s = length of the sand filter (m),

A = cross-sectional area of the sand filter (m^2) and

K = proportionality constant (ms^{-1}), it is called the hydraulic conductivity.

The permeability κ is related to the hydraulic conductivity K of a porous media by Hubert King relation [4] :

$$\kappa = K\frac{\mu_w}{\rho_w g}, \tag{7}$$

where μ_w =viscosity of water (Pa sec),

ρ_w =density of water (kgm^{-3}) and

g = acceleration due to gravity (ms^{-2}).

3. Materials and methods

River bed sand samples were collected at different rivers in and around University of Ibadan. These were dried in an oven and the unwanted grains and organic particles sieved off. The porosity, ϕ of each sample was determined by volumetric approach [2]. In an attempt to determine the hydraulic conductivity of the samples, the set up in Figure 1 was used. A transparent turfnol material of radius 9.4×10^{-2} m was used with s = 0.11 m and h = 0.04 m and the volume rate of flow Q, drained in the beaker per minute was noted. The hydraulic conductivity of each sample was then determined using eq. (6). Thus these values were used to determine the permeability of the samples using eq. (7).

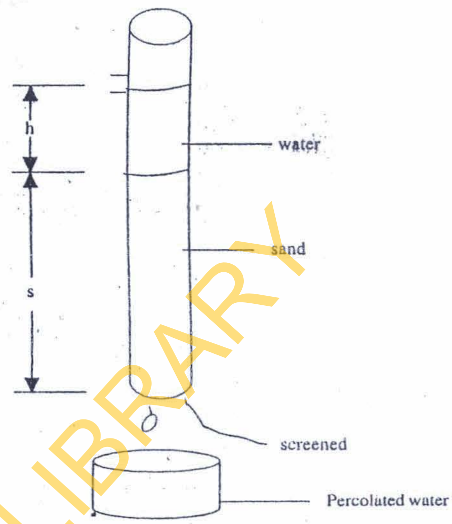

Figure 1. A set up for determination of hydraulic conductivity.

In another experiment involving the same samples, a cylindrical pipe was used and to allow for uniform compaction, the pipe was half- filled with water and the sand were dropped to fill it. The pvc pipe used was drilled at interval of 0.20 m and has a diameter of 3.45×10^{-2} m was fixed in a horizontal position [6] with the aid of a plumb. The pipe was later made to stand at angles θ = 5°,10°,15°,20° and 25° as shown in Figure 2.

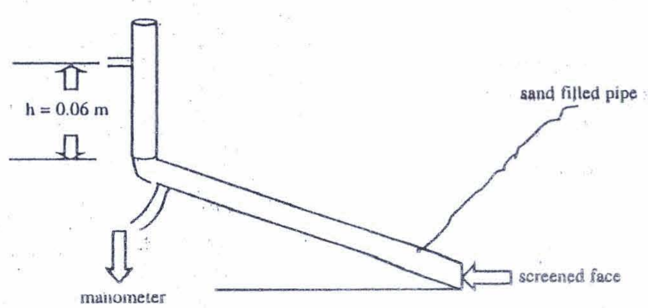

Figure 2. A set up showing water flowing through sand filled pipe.

An elbow joint made room for an L-shaped structure, which stored water at a constant height of 0.06 m. This height of water was maintained by drilling a hole at the said point and excess water was drained off through the connected rubber tubing. This constant pressure start for all samples at all angles.

A manometer [6] of tube diameter 0.01 m connected to a rubber tubing which ended with a tapered metal brass and plugged into the 4mm holes drilled in the pvc pipe, measured the pressure [3] along the pipe. Measurements were made at every angle of tilt *i.e.* θ = 5°, 10°, 15°, 20° and 25° for each sample.

4. Results and Analysis

For each sample A-E, values of pressure at each point at interval of 0.20 m were recorded using a manometer. Horizontal flow was first considered, that is, flow at θ = 0°. Using eq. (4), the value of the volume flux rate was obtained from the slope of graph of pressure against distance of flow. This graph indicates that pressure decreases with the increase in the distance of flow [1,2] and using eq (4) the volume rate of flow Q, was determined knowing that $Q = VA$, where $A = \pi\left(\frac{d}{2}\right)^2$ and d is the diameter of the pipe used given as 3.45×10^{-2} m.

Using eq (4), the slope of the graph equals $g\rho\sin\theta - V\frac{\mu}{\kappa}$. Hence, volume flux rate for each sample can be calculated *vis-à-vis* volume rate of flow (discharge rate) using eq (3). Figure 3 shows the fits for the graph of pressure against distances of flow for horizontal set up for all the samples (A-E); here pressure decreases along the distance of flow. Moreover, a graph of pressure against distance of flow for tilted flow indicates that pressure increases with increasing distance of flow and also increases as the angle of tilt has been increased from θ = 5° to θ = 25° (Figures 4–8). Table 1 shows calculated values of hydraulic conductivity and permeability used for calculating various volume flux rate that was later used to determine the volume rate of flow (discharge rate). Table 2 shows the calculated values of volume flux while Table 3 shows the calculated values of volume rate of flow from the slopes of fits from Figures (3–8) using eq. (5) and eq. (4) for θ = 0° and 5° ≤ θ ≤ 25°, respectively. The Tables 2 and 3 clearly show that both the volume flux rate and the volume rate of flow increase as the angle of flow increases.

A graph of volume rate of flow against angles of flow (Figure 9) reveals that the curves intersect at a

Table 1. Values of porosity, hydraulic conductivity and permeability (as determined experimentally) for various samples

Sample	Porosity	Hydraulic conductivity (m/s)	Permeability (m²)
A	0.361 ± 0.001	1.100 E–4	1.210 E–11
B	0.375 ± 0.001	1.430 E–4	1.457 E–11
C	0.417 ± 0.006	2.024 E–4	2.062 E–11
D	0.448 ± 0.02	2.510 E–4	2.558 E–11
E	0.467 ± 0.01	3.433 E–4	3.498 E–11

$\rho = \rho_w = 1000$ kgm^{-3}, $\mu = \mu_w = 10^{-3}$ Pa.s, $g = 9.80665$ ms^{-2}.

Table 2. Experimentally determined values of volume flux for samples at various angles

Angle (degree)	Vol. flux rate (m/s)*E–5;(A)	Vol. flux rate (m/s)*E–5;(B)	Vol. flux rate (m/s)*E–5;(C)	Vol. flux rate (m/s)*E–5;(D)	Vol. flux rate (m/s)*E–5;(E)
0	–0.08 ± 0.18	–0.11 ± 0.14	–0.17 ± 0.15	–0.24 ± 0.13	–0.33 ± 0.25
5	0.58 ± 0.18	0.68 ± 0.17	0.92 ± 0.13	1.08 ± 0.07	1.45 ± 0.13
10	1.26 ± 0.07	1.48 ± 0.06	1.97 ± 0.05	2.43 ± 0.08	3.25 ± 0.06
15	2.06 ± 0.08	2.42 ± 0.06	3.38 ± 0.06	4.10 ± 0.10	5.45 ± 0.09
20	2.65 ± 0.20	3.16 ± 0.01	4.46 ± 0.07	5.38 ± 0.04	7.29 ± 0.07
25	3.29 ± 0.08	3.92 ± 0.11	5.34 ± 0.02	6.59 ± 0.14	8.97 ± 0.14

Values of volume flux rate vis-a-vis the corresponding angles (in degrees).

Table 3. Experimentally determined values of volume rate of flow for samples at various angles

Angle (degree)	Vol. Rate of flow (m/s)*E–8;(A)	Vol. Rate of flow (m/s)*E–8;(B)	Vol. Rate of flow (m/s)*E–8;(C)	Vol. Rate of flow (m/s)*E–8;(D)	Vol. Rate of flow (m/s)*E–8;(E)
0	–0.83±0.18	–1.10±0.14	–0.16±0.15	–0.22±0.13	–0.31±0.25
5	0.55±0.18	0.65±0.17	0.87±0.13	1.01±0.07	1.35±0.13
10	1.18±0.07	1.39±0.06	1.85±0.05	2.28±0.08	3.04±0.06
15	1.93±0.08	2.27±0.06	3.16±0.06	3.84±0.10	5.09±0.09
20	2.49±0.02	2.951±0.01	4.18±0.07	5.03±0.04	6.81±0.07
25	3.10±0.08	3.66±0.11	4.99±0.02	6.17±0.14	8.39±0.14

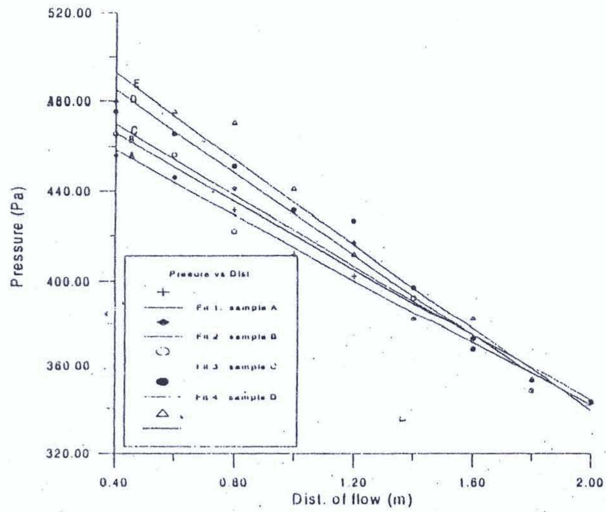

Figure 3. Plot of pressure *against* distance of flow for samples with different porosity.

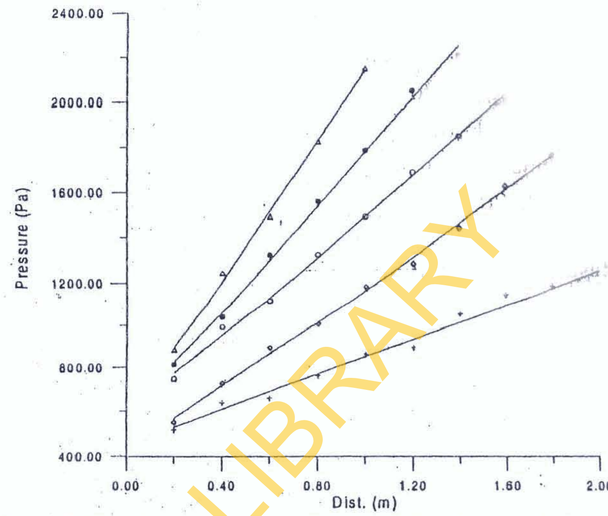

Figure 6. Plot of pressure *against* distance of flow for sample C at various angles of tilt.

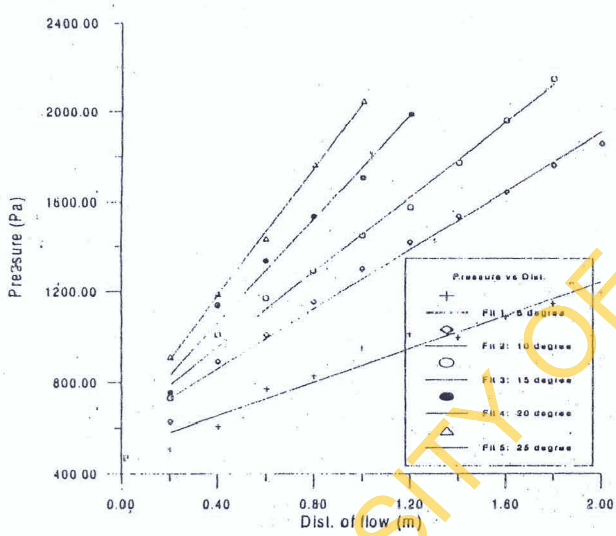

Figure 4. Plot of pressure *against* distance of flow for sample A at various angles of tilt.

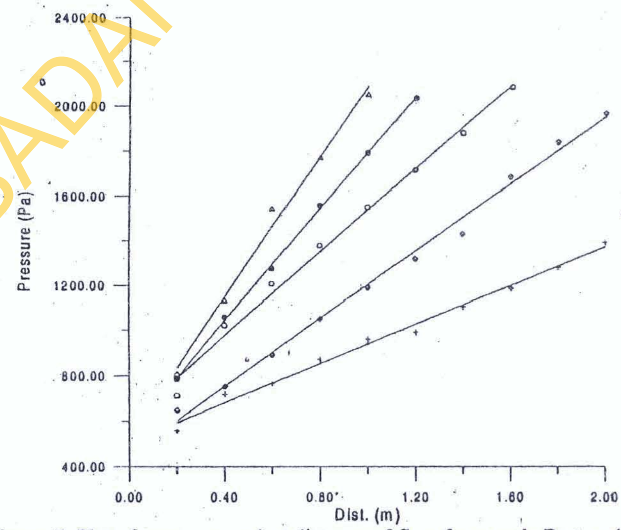

Figure 7. Plot of pressure *against* distance of flow for sample D at various angles of tilt.

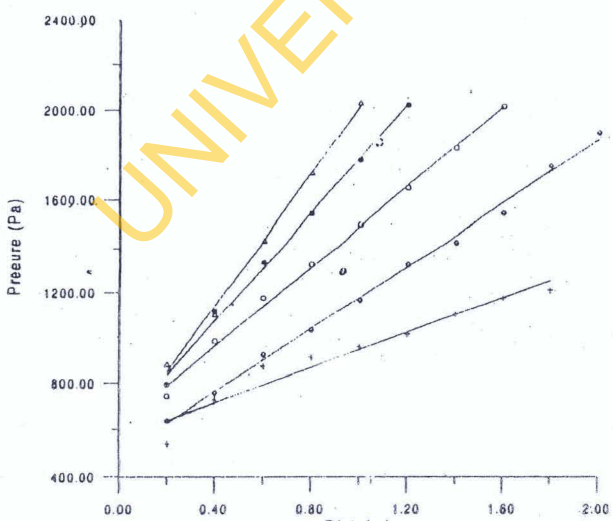

Figure 5. Plot of pressure *against* distance of flow for sample B at various angles of tilt.

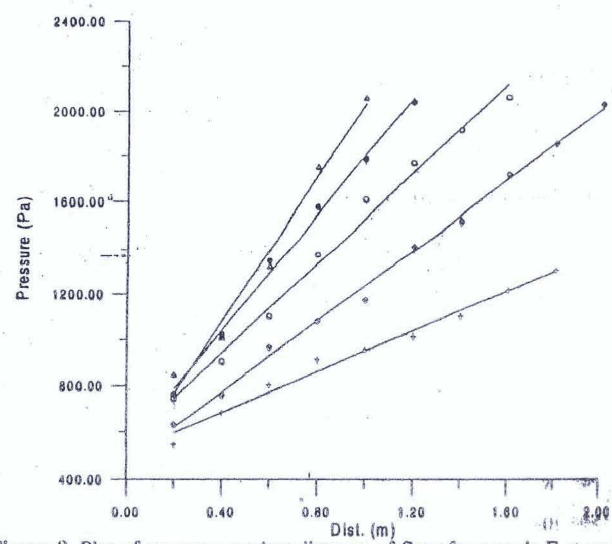

Figure 8. Plot of pressure *against* distance of flow for sample E at various angles of tilt.

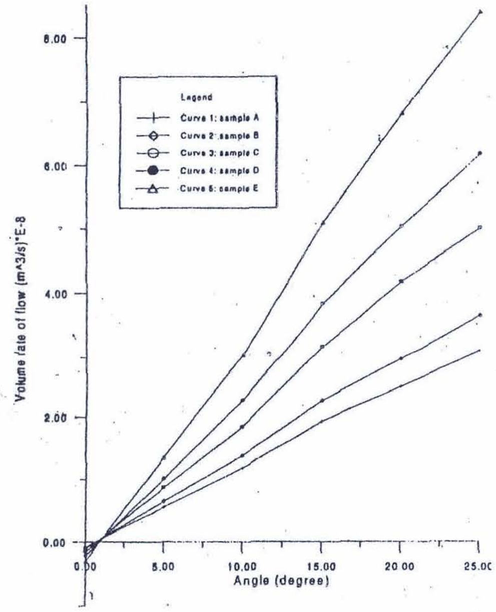

Figure 9. Plot of discharge rate (volume rate of flow) *against* angle of flow for all samples.

point where volume rate of flow equals 1.00×10^{-10} m^3/s and the angle θ at this point is 1.42°.

5. Conclusion

As revealed from the discussion there exists an angle of tilt 1.42° in this case where the volume rate of flow remains constant. One interesting fact is that this value is independent of porosity and permeability. Obviously, the result cannot be used for an impervious material but it holds for porous and permeable materials, only. A cursory look at Figure 9 clearly shows that all the samples converges at this point.

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