FUNDAMENTALS

OF

ELECTROMECHANICAL

SYSTEMS

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PREFACE

Electromechanical Systems has evolved from just a subject of study in Electrical Engineering to one with a wide range of applications in many diverse fields. This owns to the fact that most electrical or electronic devices or equipments and machineries are made up of one or more of these systems.

The basic concepts of Electromechanical Systems such as Magnetic circuits, Transformer, DC Generators and DC Motors were explained in this book in a concise and easy-to-read manner. The book is focused on the key ideas of electromechanical system and is theoretically and mathematically self-contained. It contains many solved problems and is suitable for Higher National Diploma Students and University undergraduate level students in Electrical and Electronics Engineering, Computer Engineering and Applied Physics.

The book is organized into five chapters. Chapter one introduces Magnetic Circuit and the underlying principles of Magnetic Circuits. It further compared Electric Circuits and Magnetic Circuits. This is followed by Chapter two which gives a concise description of the construction, working principles and the various equivalent circuits of a Transformer. Chapters three introduce the generalized theory of direct current machines and also serve as a preparation for discussions on dc generators and motors in the subsequent chapters. Chapter four described the dc Generator and its various variations. The chapter described aptly the generator's field system, armature core, armature winding, commutator and brushes. Chapter five discussed explicitly with relevant diagrams, the dc motor, types of dc motors and the characteristics of the various types of dc motors.

Errors might have crept in despite utmost care to avoid them and authors shall be grateful if these are pointed out along with suggestions for the improvement of the book.

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CHAPTER ONE

MAGNETIC CIRCUITS

1.1 Introduction

A magnetic circuit is made up of one or more closed loop path containing a magnetic flux. The flux is generated by permanent magnets or electromagnets. Magnetic circuits are employed to efficiently channel magnetic flux in many devices such as generators, motors, transformers, relays, e.t.c.

1.2 Definitions of Terms

i. **Magnetic circuit-** may be defined as the close route/path which is followed by magnetic flux (In SI system of units, a unit N-pole is supposed to radiate out flux of one Weber. Its symbol is Φ. Therefore, the magnetic flux coming out of a N-pole of mWeber (Wb) is given by; Φ = mWb)

ii. **Magnetic Field-** The space or region around a magnet which is permeated by the lines of force within which conductors carrying electric current are perceptibly influenced is called magnetic field of force or simply a magnetic field.

iii. **Flux Density-** Flux density at a point is given by the magnetic flux passing per unit cross-section at that point. It is usually designated by the letter β and is measured in Weber/meters sq or Tesla.

If Φ is the total flux passing through an area of Am^2, then

$$\beta = \frac{\Phi}{A} \, Wb/m^2 \text{ or Tesla } (T)$$

Iv. **Magnetomotive Force (MMF)**- It tends to drive flux through a magnetic circuit and corresponds to electromotive force (e.m.f). It is equal to the work done in Joules in carrying a unit magnetic pole once through the entire magnetic circuit. It is measured in ampere-turns.

As Potential Difference (Pd) between any two points is measured by the work done in carrying a unit charge from one point to another, similarly, m.m.f between two points is measured by the work done in Joules in carrying a unit magnetic pole from one point to another.

v. **Ampere-Turns (AT)**:- It is the unit of (m.m.f) and is given by the product of number of turns of a magnetic circuit and the current in ampere in those turns. The laws of magnetism are quite similar to (but not the same as) those of electric circuit.

Magnetic Field strength $H = \frac{NI}{L}$ 1.1

I = current, L = length,

N = number of turns

$\beta = \mu_0 \mu_r H$ 1.2

β = flux density, μ_0, μ_r = absolute & relative permeability respectively.

Total flux Φ = β x A 1.3

Φ = total flux, A = cross sectional area

From Eqn 1.1 sub $H = \frac{NI}{l}$ into Eqn 1.2

$$B = \frac{\mu_o \mu_r NI}{l} \qquad 1.4$$

Note: $\mu_r = 1$ for non magnetic materials

$\mu_r > 1$ for paramagnetic materials

$\mu_r < 1$ for diamagnetic materials

Sub Eqn 1.4 into Eqn 1.3

$$\Phi = \frac{\mu_0 \mu_r NI A}{l}$$

$$\Phi = \frac{NI}{l / \mu_0 \mu_r A} \qquad 1.5$$

The numerator of Equation 1.5, NI, which produces magnetization in the magnetic circuit, is known as magnetmotive force (m.m.f). Its unit is in ampere-turn. It is analogous to e.m.f in an electric circuit.

The denominator $\frac{l}{\mu_0 \mu_r A}$ is called the reluctance of the circuit and is analogous to resistance in an electric circuit.

$$flux = \frac{m.m.f}{reluctance} \quad \text{or} \quad \Phi = \frac{F}{S} \qquad 1.6$$

vi. **Reluctance-** It is the name given to that property of a matter which opposes the creation of magnetic flux in it. It measures the opposition offered to the passage of magnetic flux through a material and is analogous to resistance in an electric circuit. Its unit is A/Wb

$$S = \frac{l}{\mu A} = \frac{l}{\mu_0 \mu_r A} \qquad 1.7$$

Reluctance of a magnetic circuit is the number of amp-turn required per meter of magnetic flux in the circuit.

vii. **Permeance-** It is the reciprocal of reluctance and implies the ease or readiness with which magnetic flux is developed. It is analogue to conductance in electric circuits. It is measured in Wb/AT or Henry.

1.3 Composite Magnetic Circuits

Figure 1.1 shows a composite magnetic circuit consisting of three different magnetic materials of different permeabilities and lengths and are air gap ($\mu_r = 1$) Each path will have its own reluctance. The total reluctance is the sum of individual reluctances as they are joined in series.

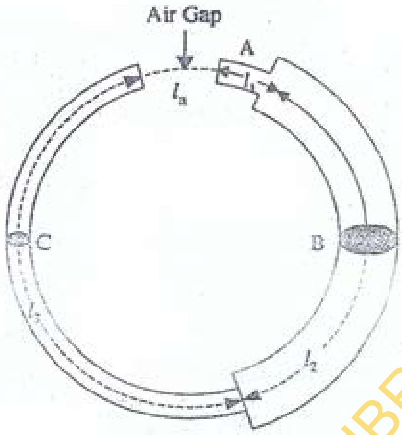


Figure 1.1: A Composite Magnetic Circuit

$$\therefore \text{Total reluctance} = \Sigma \frac{l}{\mu_o \mu_r A}$$

$$= \frac{l_1}{\mu_0 \mu_{r1} A_1} + \frac{l_2}{\mu_0 \mu_{r2}\ A_2} + \frac{l_3}{\mu_0 \mu_{r3}\ A_3} + \frac{l_a}{\mu_0\ A_g}$$

$$\text{Flux}\ \phi = \frac{m.m.f}{\Sigma \frac{l}{\mu_1 \mu_r\ A}} \qquad 1.8$$

1.3.1 Calculating Ampere-turn for Composite Magnetic Circuits

a) Find H for each portion of the composite circuit. For air,

$H = B/\mu_0$

otherwise $H = \beta/\mu_0\mu_r$

b) Find ampere-turns for each path separately by using the relation $AT = H \times L$

c) Add up these ampere turns to get the total ampere turns for the entire circuit.

1.4 Comparison between Electric & Magnetic Circuits

1.4.1 Similarities

	Magnetic Circuit	Electric Circuit
(1)	$\text{Flux} = \frac{m.m.f}{reluctance}$	$\text{Current} = \frac{e.m.f}{reluctance}$
(2)	m.m.f (ampere-turn)	e.m.f (in volt)
(3)	Flux (in Webers)	current (in amperes)
(4)	Flux density β (in Wb/m^2)	current density (A/m^2)
(5)	Reluctance $S = \frac{l}{\mu A} = \frac{l}{\mu_0 \mu_r A}$	Resistance $R = \rho \frac{1}{A} = \frac{l}{\rho A}$
(6)	$\text{Permeance} = \frac{1}{reluctance}$	$\text{conductance} = \frac{1}{resistance}$
(7)	Reluctivity	Resistivity
(8)	$\text{Permeability} = \frac{1}{reluctivity}$	$\text{Conductivity} = \frac{1}{resistivity}$

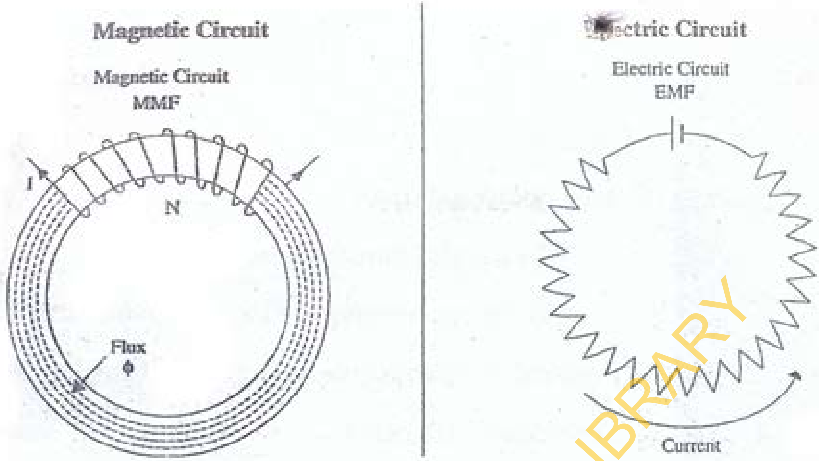


Figure 1.2: Comparison between Electric & Magnetic Circuits

1.4.2 Differences

i) Flux does not actually 'flow' in the sense in which an electric current flows.

ii) If temperature is kept constant, then conductivity (or resistively) of an electric circuit is constant and is independent of current strength but the magnetic conductivity i.e. permeability depends on the total flux or flux density established through the material.

iii) Flow of current in an electric circuit involves continuous expender of energy but in a magnetic circuit, energy needed only for creating the flux initially but not for maintaining it

Example 1.1

A mild steel ring having a cross-sectional area of $5cm^2$ and a mean circumference of 40cm has a coil of 200 turns wound uniformly around it. Calculate:

(i) The reluctance of the ring

(ii) The current required to produce a flux of 800μWb in the ring.

Assume relative permeability of mild steel to be 380 at the flux density developed in the core.

$$S = \frac{l}{\mu_0 \mu_r A}$$

$$= \frac{0.4}{4\pi \times 10^{-7} \times 380 \times 5 \times 10^{-4}}$$

$$= 1.675 \times 10^{6} \text{ AT/Wb}$$

$$\Phi = \frac{F}{S} = \frac{NI}{S}$$

$$800 \times 10^{-6} = \frac{200 \times I}{1.676 \times 10^{6}} = 6.7A$$

Example 1.2

An iron ring of mean length 50cm has an air-gap of 1mm and a winding of 200 turns. If the permeability of the iron is 300 when a current of 1.5A flows through the coil, find the flux density.

Solution

Let A be the area of cross-section of the iron path as well as that of the air gap

$$\Phi = \frac{F}{S} = \frac{F}{S_1 + S_2}$$

$$\beta A = \frac{F}{\dfrac{l_1}{\mu_0 \mu_r A} + \dfrac{l_2}{\mu_0 A}} = \mu_0 \mu_r A \times \frac{F}{l_1 + \mu_r l_2}$$

$$\beta = \mu_0 \mu_r \times \frac{F}{l_1 + \mu_r l_2} = 4\pi \times 10^{-7} \times 300 \times \frac{200 \times 1.5}{0.5 + 300 \times 1 \times 10^{-3}}$$

$$\beta = 0.14 Wb/m^2$$

Example 1.3

A magnetic circuit has a mean core length of 100cm and a uniform cross-section of $5cm^2$. It has an air gap of 0.8mm and is wound with a coil of 1200 turns. Determine the total reluctance of the circuit if the core material has a relative permeability of 1000

Solution

$$S_1 = \frac{l_1}{\mu_0 \mu_r A} = \frac{1}{4\pi \times 10^{-7} \times 1000 \times 5 \times 10^{-4}} = \frac{1}{62.84 \times 10^{-8}} = 1.59 \times 10^6 \ AT/wb$$

$$S_2 = \frac{l_2}{\mu_0 \mu_r A} = \frac{0.8 \times 10^{-3}}{4\pi \times 10^{-7} \times 1 \times 5 \times 10^{-4}} = \frac{8 \times 10^7}{20 \times 3.142} = 1.27 \times 10^6 \ AT/wb$$

$$S = S_1 + S_2$$

$$= 1.59 \times 10^6 + 1.27 \times 10^6 = (1.59 + 1.27) \times 10^6$$

$$= 2.86 \times 10^6 AT/wb$$

Example 1.4

A coil of 200 turns is wound uniformly over a wooden ring having a mean circumference of 600mm and a uniform cross-sectional area of $500mm^2$ if the current through the coil is 4A, calculate;

a) The magnetic field strength

b) The flux density

c) The total flux (Take $\mu_0 = 4\pi \times 10^{-7}$ H/m)

Solution

a) mean circumference = 600mm = 0.6m

$$H = 4 \times \frac{200}{0.6} = 1333 A/m$$

b) flux density $\beta = \mu_0 H$

$= 4\pi \times 10^{-7} \times 1333$

$= 0.001675T$

$= 1675\mu T$

c) Cross sectional area $= 500mm^2 = 500 \times 10^{-6} m^2$

Total flux $= 1675 [\mu T] \times (500 \times 10^{-6}) m^2$

$= 0.8375\ \mu Wb$

Example 1.5

Calculate the m.m.f require to produce a flux of 0.015wb across an air gap 2.5mm long, having an effective area of $200cm^2$.

Area of air gap $= 200 \times 10^{-4}$

$= 0.02m^2$

$$B(\text{flux density}) = \frac{0.015Wb}{0.02m^2} = 0.75T$$

Magnetic field strength for gap: $H = \frac{\beta}{\mu_0}$

$$= \frac{0.75}{4\pi \times 10^{-7}} = 597{,}000\ A/m$$

Length of gap = 2.5mm

= 0.0025m

∴ m.m.f required to send flux across gap

$$= 597{,}000\,[A/m] \times 0.0025[m] = 1492.5A$$

CHAPTER TWO

TRANSFORMERS

2.1 Introduction

A transformer is a static piece of apparatus by means of which electric power in one circuit is transformed to electric power of the same frequency in another circuit. It can raise or lower the voltage in a circuit but with a corresponding decrease or increase in current. The physical basis of a transformer is mutual inductance between two circuits linked by a common magnetic flux through a path of low reluctances. The two coils possess high mutual inductance. If one coil is connected to a source of alternating voltage, an alternating flux is set up in the laminated core; most of which is linked up with the other coil via magnetic circuit. The alternating voltage in one coil produces mutually induced e.m.f according to Faraday's laws of electromagnetic induction i.e.

$$e = M\frac{di}{dt}$$ where e = induced e.m.f

M = mutual inductance

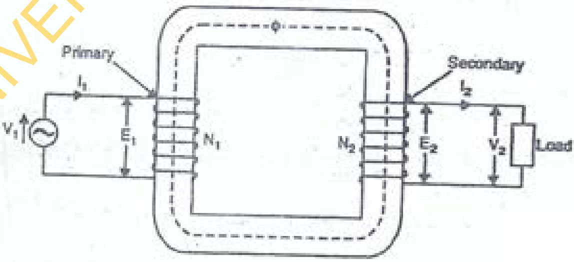


Figure 2.1: A Typical Transformer

In summary, a transformer is a device that:

i) Transfers electric power from one circuit to another without change of frequency.

Ii) Accomplishes this by electromagnetic induction[1]

2.2 Transformer Construction

The simple elements of a transformer consist of 2 coils having mutual inductance and a laminated steel core. The two coils are insulated from each other and the steel core. Other necessary parts are; some suitable containers for the assembled core and windings, suitable medium for insulating the core and its winding from its container. In all types of transformers, the core is constructed of transformer. sheet steel laminations assembled to provide a continuous magnetic path with minimum of air-gap included. The steel used is of high silicon content and sometimes heat treated to produce a high permeability and a low hysteresis[2] loss at the usual operating flux density.

Constructionally, transformers are of two general types, distinguished from each other merely by the manner in which the primary and secondary coils are placed around the laminated core.

[1]Magnetic induction- magnet can impact magnetism to a magnetic substance without actually coming in physical contact with it. If an unmagnetic piece of soft iron be placed in the field of a magnet it also becomes magnetized. The iron piece is under induction, the magnet is the inducing body and this phenomenon is known as magnetic induction.

[2](Hysteresis may be defined as the lagging of magnetization or induction flux density β behind the magnetizing force H)

The two types are:

(i) core type

(ii) Shell type

In the core type transformer, the winding surrounds a considerable part of the core whereas in shell type, transformer, the core surrounds a considerable portion of the windings as shown in figure 2.2.

In the simplified diagram of the core type transformer (figure 2.2b), the primary and secondary winding are shown located on the opposite legs of the core, but in actual construction, these are always inter-leaved in order to reduce the leakage flux.

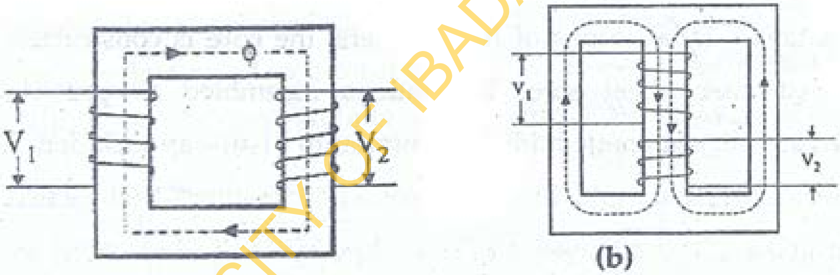


Figure 2.2: Transformer Types: (a) Core Type (b) Shell Type

2.3 Principle of Operation

The general arrangement of a transformer is as shown in figure 2.1. A steel core C consist of laminated sheets of about 0.35mm thick, insulated from one another by thin layers of paper or vanish or by spraying the laminations with a mixture of flour, chalk and water which when dried adheres to the metal. The purpose of laminating The core is to reduce the loss due to eddy current induced in by the alternating magnetic flux. The vertical portions of the core are referred to as limbs and the top and bottom portions are the yokes.

Coils P and S are wound on the limbs. Coils P is connected to the supply and therefore termed the primary while coil S is termed secondary and is connected to the load. An alternating voltage applied to P circulates an alternating current through P and this current produces an alternating flux Φ in the steel core. If the whole of the flux produce by P passes through S, the e.m.f induced in each turn is the same for P and S.

Let N_1 and N_2 be the number of turns on P & S respectively, then

$$\frac{Total\ e.m.f\ induced\ in\ S}{total\ emf\ induced\ in\ P} = \frac{N_2 \times e.m.f\ \ per\ turn}{N_1 \times e.m.f\ \ per\ turn} = \frac{N_2}{N_1} \qquad 2.1$$

When the secondary is open circuit, its terminal voltage is the same as the induced e.m.f. the primary current is then very small, so that the applied voltage V_1 is practically equal and opposite to the e.m.f induced in P.

Hence

$$\frac{V_2}{V_1} \approx \frac{N_2}{N_1} \qquad 2.2$$

2.4 Ideal Transformer

An ideal transformer is one which has no losses i.e. which windings has no ohmic resistance so that there is no I^2R loss and no core loss and in which there is no magnetic leakage. In other words, an ideal transformer consists of true purely inductive coils wound on a loss free core.

In summary, an ideal transformer is one that has;

i) No winding resistance.

ii) No leakage flux i.e. the same flux links both the windings.

Iii) No iron losses (i.e. eddy current and hysteresis losses) in the core.

Although, ideal transformer cannot be physically realized, yet its study provides a very powerful tool in the analysis of a practical transformer. In actual fact, practical transformers have properties that approach very close to an ideal transformer.

Let us consider an ideal transformer whose secondary is open and whose primary is connected to a sinusoidal alternating voltage V_1. The Pd causes the flow of an alternating current in the primary winding.

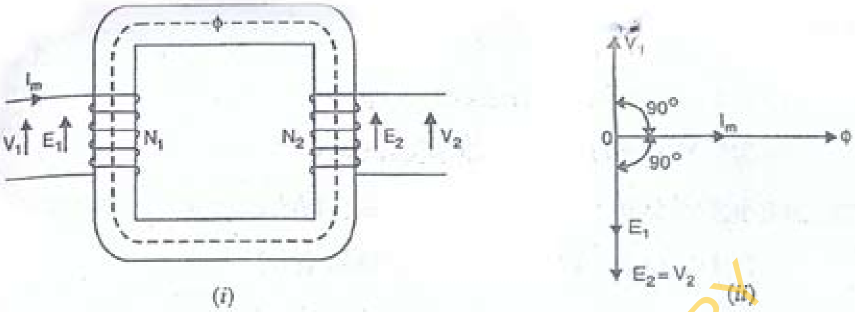


Figure 2.3: Ideal Transformer on No-Load

Since the primary coil is purely inductive and there is no output (secondary being open), the primary draws magnetizing current I_m only. The function of this current is merely to magnetize the core; it is small in magnitude and lags V_1 by 90^0. This alternating current I_m produces an alternating flux ϕ which is, at all times, proportional to the current (assuming permeability of the magnetic circuit to be constant) and hence is in phase with it. This changing flux is linked with both the primary and the secondary windings. Therefore, it produces self-induced e.m.f in the primary. This self induced e.m.f E_1 is, at every time, equal to and in opposition to V_1. It is also known as counter e.m.f or back e.m.f of the primary.

Similarly, there is produced in the secondary an induced e.m.f E_2 which is known as mutually induced e.m.f. This e.m.f is anti-phase with V_1 and its magnitude is proportional to the rate of change of flux and the number of secondary turns.

2.5 E.M.F Equation of a Transformer

Let N_1 = Number of turns in primary

N_2 = Number of turns in secondary

ϕ_m = maximum flux in the core in Webers = $\beta_m \times A$

f = frequency of a.c. input in Hertz (Hz)

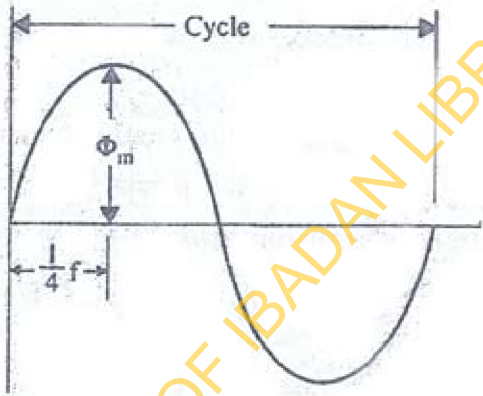


Figure 2.4: Rate of change of flux

As shown in the figure 2.4, the core flux increases from its zero value to maximum value ϕ_m in one quarter of the cycle i.e. in ¼f second.

:. Average rate of change of flux $= \frac{\Phi_m}{\frac{1}{4} \times f} = 4f\Phi_m Wb/s$ 2.3

Now, rate of change of flux per turn means induced e.m.f in volts

:. Average e.m.f induced/turn = $4f\phi_m$ volt. 2.4

If flux ϕ varies sinusoidally, then r.m.s value of induced e.m.f is obtained by multiplying the average value with form factor.

$$\text{Form factor} = \frac{r.m.s\ value}{average\ value} = 1.11 \qquad 2.5$$

$\therefore$ r.m.s value of e.m.f/turn $= 1.11 \times 4f\phi_m$

$$= 4.44f\phi_m \text{ volt} \qquad 2.6$$

Now r.m.s value of induced e.m.f in the whole of primary winding

= (induced e.m.f/turn) x No. of primary turns

$$\therefore \quad E_1 = 4.44fN_1\phi_m = 4.44N_1\beta_m A$$

Similarly, r.m.s value of induced e.m.f in secondary is

$$E_2 = 4.44fN_2\phi_m = 4.44N_2\beta_m A \qquad 2.8$$

In an ideal transformer on no load,

$$V_1 = E_1 \text{ and} \qquad 2.9$$

$$E_2 = V_2 \text{ where } V_2 \text{ is the terminal voltage} \qquad 2.10$$

2.6 Voltage Transformation Ratio (K)

By finding the ratios of equations 2.7 and 2.8; 2.9 and 2.10, and combining the two results gives the transformation ratio, K.

$$\frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{N_2}{N_1} = K \qquad 2.11$$

This constant K is known as voltage transformation ratio.

i) If $N_2 > N_1$ i.e. $K > 1$, then the transformer is called step-up transformer.

ii) If $N_2 < N_1$ i.e. $K < 1$, then the transformer is known as step-down transformer.

Again, for an ideal transformer,

Input VA = output VA

$$V_1 I_1 = V_2 I_2 \qquad 2.12$$

$$\frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{1}{K} \qquad 2.13$$

Hence, currents are in the inverse ratio of the (voltage) transformation ratio.

Example 2.1

A 200KVA, 3300/240V, 50Hz single phase transformer has 80 turns on the secondary winding. Assuming an ideal transformer, calculate:

i) Primary & secondary currents on full-load.

ii) The maximum value of flux.

iii) The number of primary turns.

Solution

A 3300/240V transformer is one whose normal primary secondary voltages are 3300V and 240V respectively

(I) $$\frac{N_2}{N_1} = \frac{V_2}{V_1}$$

$$\frac{80}{N_1} = \frac{12}{165}$$

$$N_1 = \frac{80 \times 165}{121} = 1100$$

(ii) $P = I_2 V_2$

$200{,}000 = I_2 (240)$

$I_2 = 833A$

(iii) $$K = \frac{I_1}{I_2}$$

$$0.073 = \frac{I_1}{833}$$

$I_1 = 0.073 \times 833 = 60.6A$

(iv) $E_1 = 4.44 f N_1 \phi_m$

$3300 = 4.44 \times 50 \times 1100 \times \phi_m$

$$\phi_m = \frac{3300}{244200}$$

$\Phi_m = 13.5_m \text{Wb}$

Example 2.2

The no load ratio of a 50113 single phase transformer is 6000/250V: Estimate the number of turns in each winding if the maximum flux is 0.06Wb in the core.

Solution

$E_1 = 4.44fN_1\phi_m$

$6000 = 4.44 \times 50 \times N_1 \times 0.06$

$N_1 = 450$

Similarly,

$250 = 4.44 \times 50 \times N_2 \times 0/06$

$N_2 = 19$

Example 2.3

The number of turns in the low voltage winding of a 200KVA, 50Hz, 2000/250V single phase transform is 25. Calculate:

i. Peak value of the magnetic flux is core
ii. The full load current in the low voltage winding
iii. The number turns in the high voltage winding.

Solution

$P = 200KVA, V_1 = 200V, V_2 = 250V, N_2 = 25$

$E_2 = 4.44fN_2\phi_m$

$250 = 4.44 \times 50 \times 25 \times \phi_m$

$$\phi_m = \frac{250}{4.44 \times 50 \times 25} = \frac{250}{5550} = 45_m Wb$$

$P = IV$

$200 \times 1000 = I \times 250$

$I = 800A$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$\frac{250}{2000} = \frac{25}{N_1}$$

$N_1 = 2000$

Example 2.4

A double-wound single phase transformer is required to step down from 2100V to 230V, 50H_z. It is to have 1.8.V per turn. Calculate the required number of turns on the primary and secondary winding respectively. The peak value of the flux density is required not to be more than 1.4T. Calculate the required cross-sectional area of the steel core.

Solution

$V_1 = 2100V, V_2 = 230V, f = 50Hz, \beta_m = 1.4T$

$$N_1 = \frac{2100}{1.8} = 1167$$

$$N_2 = \frac{230}{1.8} = 128$$

$$E_1 = 4.44fN_1\phi_m$$

$$2100 = 4.44 \times 50 \times 21167 \times \phi_m$$

$$2100 = 259074\phi_m$$

$$\Phi_m = 8.1_m Wb$$

$$\beta_m = \frac{\phi_m}{A}$$

$$A = \frac{\phi_m}{A}$$

$$A = \frac{\phi_m}{\beta} = \frac{8.1 \times 10^{-3}}{1.4} = 5.79 \times 10^{-3} m^2$$

2.7 Practical Transformer

A practical transformer differs from an ideal transformer in many respects. The practical transformer has:

(i) Iron losses

(ii) Winding resistance

(iii) Magnetic leakage, giving rise to leakage reactances

These short comings of the practical transformer are described in the following subsections.

2.7.1 Iron losses

Since the iron core is subjected to alternating flex, there occurs eddy current and hysteresis losses in it. These two losses together are known as iron losses or core losses. The iron losses depend upon the supply frequency, maximum flux density in the core, volume of the core e.t.c. It may be noted that magnitude of iron losses is quite small in a practical transformer.

2.7.2 Winding resistances

Since the windings consist of copper conductors, it immediately follows that both primary and secondary will have winding resistance. The primary resistance R_1 and secondary resistance R_2 acts in series with the respective windings as shown in figure 2.5

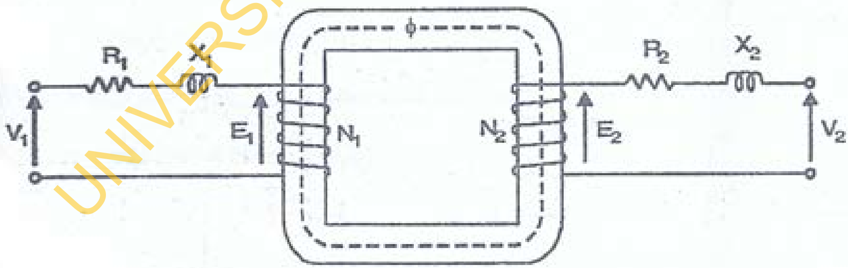


Figure 2.5: Practical Transformer

2.7.3 Leakage reactances

Both primary and secondary currents produce flux. The flux ϕ which links both the windings is the useful flux and is called mutual flux. However, the primary current would produce some flux ϕ which would not link the secondary winding. Similarly, secondary current would produce some flux ϕ_2 that would not link the primary winding. The flux such as ϕ_1 or ϕ_2 which links only one winding is called leakage flux. The leakage flux paths are mainly through the air. The effect of these leakage fluxes would be the same as though inductive reactance were connected in series with each winding of the transformer that had no leakage flux as shown below:

In other words, the effect of primary leakage flux ϕ_1 is to introduce an inductive reactance X_1 in series with the primary winding as shown in the diagram. Si...larly, the secondary leakage flux ϕ_2 introduces an inductive reactance X_2 in series with the secondary winding.

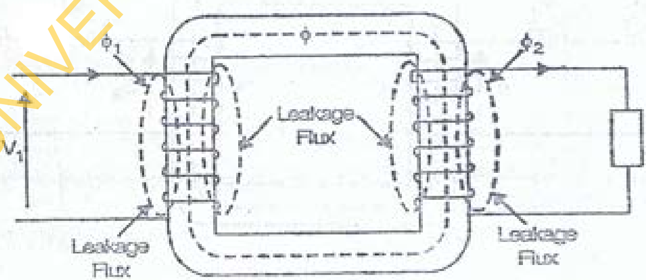


Figure 2.6: Practical Transformer showing Leakage Flux

2.8 Practical Transformer on No Load

Consider a practical transformer on no load i.e. secondary is on open circuit as shown in figures 2.7(i) and figure 2.7(ii). The primary will draw a small current I_o to supply

(i) the iron losses

(ii) a very small amount of copper loss in the primary

Hence the primary no load current I_o is not 90^0 behind the applied voltage V_1 but lags it by an angle $< 90^0$ as shown in the phasor diagram of figure 2.7(ii)

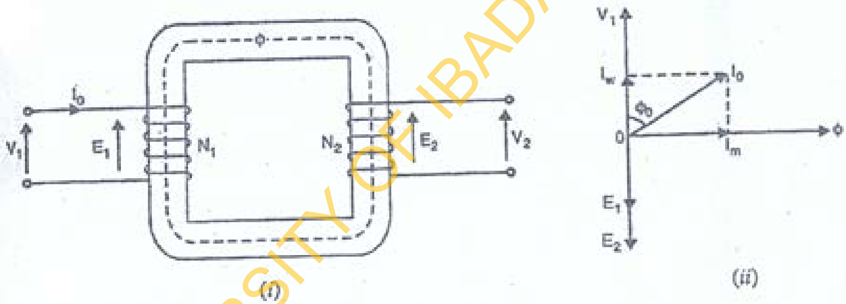


Figure 2.7: Practical Transformer on No Load

No load input power, $W_o = V_1 I_o \cos \phi_o$ 2.14

From the phasor diagram, the no-load primary current I_o can be resolved into two rectangular components viz:

i. The component I_w in phase with the applied voltage V_1. This is known as active (working) or iron loss component and supplies the iron loss and a very small primary copper loss.

$$I_w = I_o \cos \phi_o \quad 2.15$$

ii. The component I_m lagging behind V_1 by 90^0 and is known as magnetizing component. It is this component which produces the mutual flux ϕ in the core.

$$I_m = I_o \sin \phi_o \quad 2.16$$

Clearly, I_o is phase sum of I^2_m and I^2_w

$$\therefore I_o = \sqrt{I_m^2 + I_w^2} \quad 2.17$$

No load power factor $\cos \phi_o = \dfrac{I_w}{I_0}$ 2.18

It is emphasized here that no load primary copper loss (i.e. R_1) is very small and may be neglected. Therefore, the no load primary input power is practically equal to the iron loss in the transformer i.e. No load input power, W_o = Iron loss.

2.9 Ideal Transformer on Load

Let us connect a load Z_L across the secondary of a ideal transformer as shown in figure 2.8(i)

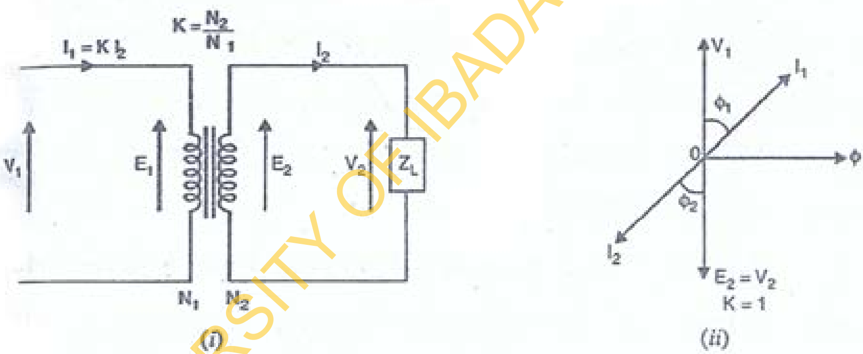


Figure 2.8: Ideal Transformer on Load

The secondary e.m.f E_2 will cause a current I_2 to flow through the load.

$I_2 = \frac{E_2}{Z_l} = \frac{V_2}{Z_l}$ (Since there is no voltage drop in an ideal transformer $E_2 = V_2$) 2.19

The angle at which I_2 leads or lags V_2 (or E_2) depend upon the resistance and reactance of the load. In the present case, we have considered inductive load so that current I_2 lags behind V_2 (or E_2) by ϕ_2.

The secondary current I_2 sets up an m.m.f N_2I_2 which produce a flux in the opposite direction to the flux ϕ originally set up in the primary by the magnetizing current. This will change the flux in the core from the original value. However, the flux in the core should not change from the original value. In order to fulfill this condition, the primary must develop an m.m.f which exactly counterbalances the secondary m.m.f $N_2 I_2$. Hence a primary current I_1 most flow such that:

$N_1 I_1 = N_2 I_2$

or $\quad I_1 = \dfrac{N_2}{N_1} I_2 = KI_2$

This when a transformer is loaded and carries a secondary current I_2, then a current $I_1 = KI_2$ must flow in the primary to maintain the m.m.f balance. In other words, the primary must draw enough current to neutralize the demagnetizing effect of secondary current so that mutual flux ϕ remains constant. Thus as the secondary current increase, the primary current $I_1 = KI_2$ increases in unison and keeps the mutual flux ϕ constant. The power input, therefore, automatically increases with the output.

Phasor Diagram

The phasor diagram (figure 2.8(ii)) shows an ideal transformer on load. From the diagram, the value of K has been assumed unity so that primary phasors are equal to secondary phasors. The secondary current I_2 lags behind V_2 (or E_2) by ϕ_2. It causes a primary current $I_1 = KI_2 = 1 \times I_2 = I_2$ which is in antiphase with it.

(i) $\Phi_1 = \phi_2$

$$\cos\phi_1 = \cos\phi_2 \qquad 2.21$$

This power factor p.f on the primary side is equal to the power factor on the secondary side.

(ii) Since there are no losses in ideal transformer, input primary power is equal to the secondary output power i.e.

$$V_1 I_1 \cos\phi_1 = V_2 I_2 \cos\phi_2 \qquad 2.22$$

2.10 Practical Transformer on Load

We shall consider 2 cases:

(i) When such transformer is assumed to have no winding resistance and leakage flux.

(ii) When the transformer has winding resistance and leakage flux.

2.10.1 No winding resistance and leakage flux

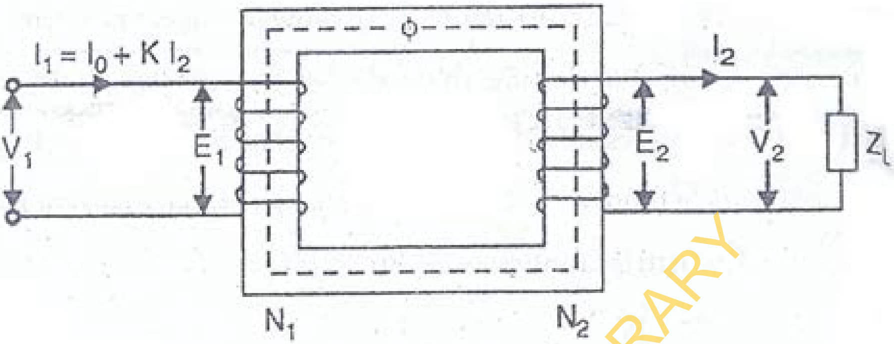


Figure 2.9: Practical Transformer on Load

Figure 2.9 shows a practical transformer with the assumption that resistances and leakage reactances of the windings are negligible With this assumption, $V_2 = E_2$ and $V_1 = E_1$ (however, V_1 and E_1 are 180° out of phase). Let us consider the usual case of inductive load which causes the secondary current I_2 to lag the secondary voltage V_2 by ϕ_2. The total primary current I_1 must most two requirements viz.

(a) It must supply the no-load current I_o to meet the iron losses in the transformer and to provide flux in the core.

(b) It must supply a current I?$_2$ to counteract the demagnetizing effect of secondary current I_2. The magnitude of I?$_2$ will be such that:

$N_1 I_2' = N_2 I_2$

Or $I_2' = \frac{N_2}{N_1} I_2 = K I_2$

The total primary current I_1 is the phase sum of $I?_2$ and I_o i.e. $I_1 = I?_2 + I_o$

Where $I_2 = -K I_2$

Note that $I?_2$ is 180^0 out of the phase with I_2.

Phasor Diagram

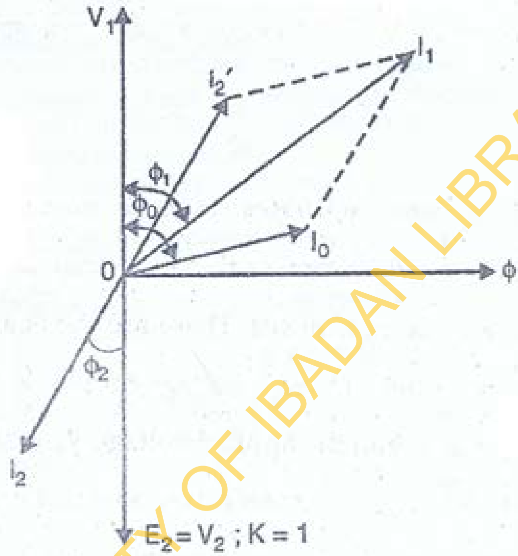


Figure 2.10: Phasor diagram of Practical Transformer on Load

Figure 2.10 shows the phasor diagram for the usual case of inductive load. Both E_1 and E_2 lag behind the mutual flux ϕ by 90^0. The current $I?_2$ represents the primary current to neutralize the demagnetizing effect of secondary current I_2. Now $I'_2 = KI_2$ and is antiphase with I_2. I_o is the no-load current of the transformer. The phasor sum of I'_2 and I_0 gives the total primary current I_1.

Primary p.f = cos ϕ_1; secondary p.f = cos ϕ_2 2.26

Primary input power = $V_1 I_1 \cos \phi_1$; secondary output power = $V_2 I_2$ Cos ϕ_2 2.27

2.10.2 **Transformer with Resistance & Leakage Reactance**

Figure 2.11 shows a practical transformer having winding resistances and leakage reactances. These are the actual conditions that exist in a transformer. There is voltage drop in R_1 and X_1 so that primary e.m.f E_1 is less than the applied voltage V_1. Similarly, there is voltage drop in R_2 and X_2 so that secondary terminal voltage V_2 is less than the secondary e.m.f E_2 Considering the usual case of inductive load which cause the secondary current I_2 to lag behind the secondary voltage V_2 by ϕ_2;

The total primary current I_1 must meet two requirements viz:

(a) It must supply the no-load current I_o to meet the iron losses in the transformer and to provide flux in the core.

(b) It must supply a current I?$_2$ to counteract the demagnetizing effect of secondary current I_2.

nitude of I'_2 will be such that:

$$N_1 I'_2 = N_2 I_2$$

$$\text{or } I'_2 = \frac{N_2}{N_1} I_2 = K I_2 \qquad 2.28$$

The total primary current I_1 will be the phasor sum of I'_2 and I_0 i.e.

$$I_1 = I'_2 + I_0 \text{ where } I_2' = -KI_2$$

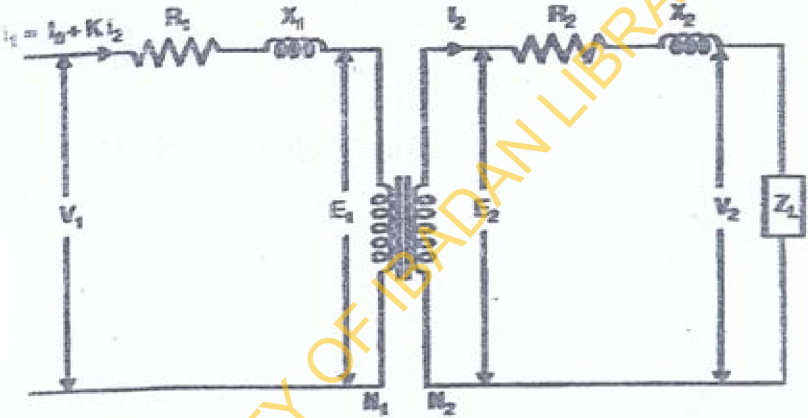


Figure 2.11: Practical Transformer with Resistance & Leakage Reactance

$$V_1 = -E_1 + I_1(R_1 + jx_1) \text{ where } I_1 = I_0 + (-KI_2) = -E_1 + I_1 Z_1 \qquad 2.29$$

$$V_2 = E_2 - I_2(R_2 + jx2) = E_2 - I_2 Z_2 \qquad 2.30$$

2.11 Impedance Ratio

Consider a transformer having impendence Z_2 in the

Secondary as shown in the figure below.

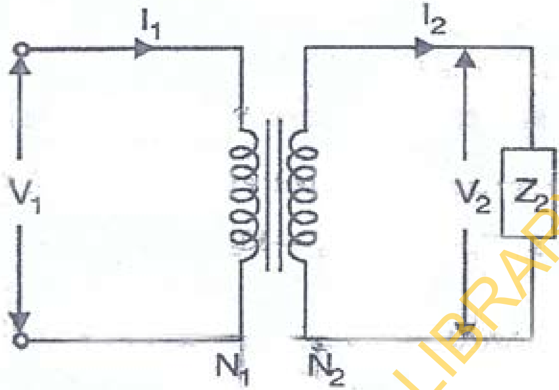


Figure 2.12: Transformer impedance Ratio

$$Z_2 = \frac{V_2}{I_2} \qquad 2.31$$

$$Z_1 = \frac{V_1}{I_1} \qquad 2.32$$

$$\frac{Z_2}{Z_1} = \frac{V_2}{V_1} \times \frac{I_1}{I_2}$$

$$\therefore \frac{Z_2}{Z_1} = K^2 \qquad 2.33$$

I.e. impedance ratio (Z_2/Z_1) is equal to the square of voltage transformation ratio. In other words, an impedance Z_2 in secondary become $\frac{Z_2}{K^2}$ when transferred to primary. Likewise, an impedance Z_1 in the primary becomes K^2Z_1 when transferred to the secondary.

Similarly, $\frac{R_2}{R_1} = K^2 and \frac{X_2}{X_1} = K^2$

Note the importance of the above relations. The parameters can be transferred from one winding to the other thus:

(i) A resistance R_1 in the primary becomes K^2R_1 when transferred to the secondary.

(ii) A resistance R_2 in the secondary becomes R_2/K^2 when transferred to the primary.

(iii) A reactance X_1 in the primary becomes K^2X_1 when transferred to the secondary.

(Iv) A reactance X_2 in the secondary becomes X_2/K^2 when transferred to the primary.

Example 2.5

A 2,200/200V transformer draws a no-load primary current of 0.6A and absorbs 400 Watts. Find the magnetizing and iron-loss current.

Solution

$$\text{Iron loss current} = \frac{\textit{no load in watt}}{\textit{primary voltage}}$$

$$= \frac{400}{2200} = 0.182A$$

$I_o^2 = I_w^2 + I_m^2$

$I_m = \sqrt{(0.6^2\ 0.18^2)}$

$= 0.572A$

Example 2.6

A 2,200/250V transformer takes 0.5A at a power factor of 0.3 on open circuit. Find the magnetizing and working components of no-load primary current.

Solution

$I_o = 0.5A, \cos\phi_o = 0.3, I_w = I_o \cos\phi_o$

$= 0.5 \times 0.3$

$= 0.15A$

$$I_m = \sqrt{(0.5^2\ 0.15^2)}$$

$$= 0.477A$$

Example 2.7

The no-load current of a transformer is 5A at 0.25 power factor when supplied at 235V, 50Hz. The number of turns on the primary winding is 200.Calculate:

a) The maximum valve of flux in the core

b) The core loss

c) The magnetizing component

Solution

(a) $E_1 = 4.44 f N \phi_m$

$E_1 = 235V, f = 50Hz, N_1 = 200$

$235 = 4.44 \times 50 \times 200 \times \phi_m$

$$\phi_m = \frac{235}{4.44 \times 50 \times 200}$$

$\phi_m = 5.29 mWb$

(b) Primary no-load input represents the core loss

$\therefore$ Core loss $= V_1 I_o \cos \phi_o$

$= 235 \times 5 \times 0.25$

$= 294$ W

(c) Magnetizing component is

$I_m = I_o \sin \phi_o$

$\sin \phi_o = \sqrt{1 - \cos \phi_o}$

$$= \sqrt{\frac{15}{16}} = 0.9682$$

$I_m = 5 \times 0.9682$

$= 4.82A$

2.12 Approximate Equivalent Circuit of a Transformer

The no-load current I_o in a transformer is only 1-3% of the rated primary current and may be neglected without any serious error.

The transformer can then be shown as in the figure 2.13;

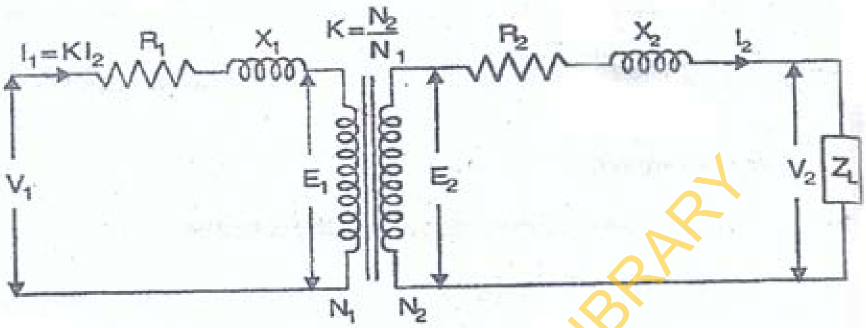


Figure 2.13: Approximate Equivalent Circuit of a Transformer

In figure 2.13, if we refer all the quantities to one side (primary or secondary), the ideal transformer stands removed and we get the equivalent circuit.

2.12.1 Equivalent circuit of transformer referred to primary

If all the secondary quantities are referred to the primary, we get the equivalent circuit of the transformer referred to the primary as shown in the figure 2.14. Note that when secondary quantities are referred to primary, resistances / reactances are divided by K^2, voltages are divided by K and current are multiplied by K.

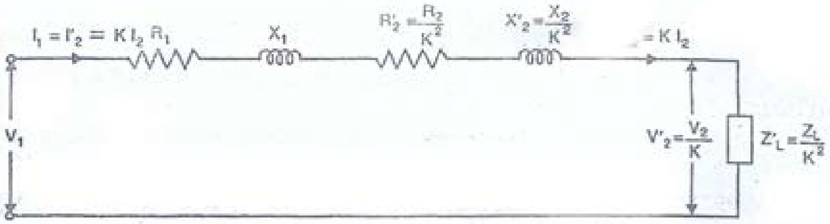


Figure 2.14: Equivalent Circuit of Transformer Referred To Primary

The equivalent circuit shown in Figure 2.14 is an electrical circuit and can be solved for various currents and voltages. Thus if we find and, then actual secondary values can be determined as follows:

Actual secondary voltage, $V_2 = K V'_2$ 2.35

Actual secondary current, $I_2 = I'_2 / K$ 2.36

2.12.2 Equivalent circuit of transformer referred to secondary

If all the primary quantities are referred to the secondary, we obtain the equivalent circuit of the transformer referred to secondary as shown in figure 2.15.

Note that when primary quantities are referred to secondary, resistance / reactances are multiplied by K^2, voltages are multiplied by K and currents are divided by K.

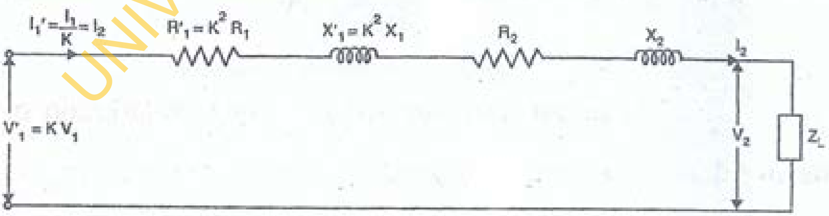


Figure 2.15: Equivalent Circuit of Transformer Referred To Secondary

The equivalent circuit shown in figure 2.15 is in electric circuit and can be solved for various voltages and currents. Thus we can solve for V'_1 and I'_1, then actual primary values can be determined as under:

Actual primary voltage, $V_1 = V'_1/K$ 2.37

Actual primary current $I_1 = KI'_1$ 2.38

Note that the same final answers will be obtained whether we use the equivalent circuit referred to primary or secondary. The use of a particular equivalent circuit would depend upon the conditions of the problem.

2.13 Voltage Regulation

The voltage regulation of a transformer is the arithmetic difference (not phasor difference) between the no-load secondary voltage ($_oV_2$) and the secondary voltage V_2 on load expressed as percentage of no-load voltage i.e.

$$\% \text{ voltage regulation} = \frac{_oV_2 - V_2}{_oV_2} \times 100 \qquad 2.39$$

It may be noted that percentage voltage regulation of the transformer will be the same whether primary or secondary side is considered.

2.14 Transformer Test

The circuit constants, efficiency and voltage regulation of a transformer can be determined by two simple tests.

(i) open circuit test

(Ii) short circuit test

These tests are very convenient as they provide the required information without actually loading the transformer. Further, the power required to carry out these is very small as compared with full load output of the transformer.

2.14.1 Open circuit or No-load test

In this test, the rated voltage is applied to the primary (usually low voltage side) while the secondary is left open-circuited. The applied primary voltage V_1 is measured by the voltmeter, the no-load current I_o by ammeter and no-load input power W_o by wattmeter as shown in figure 2.16.

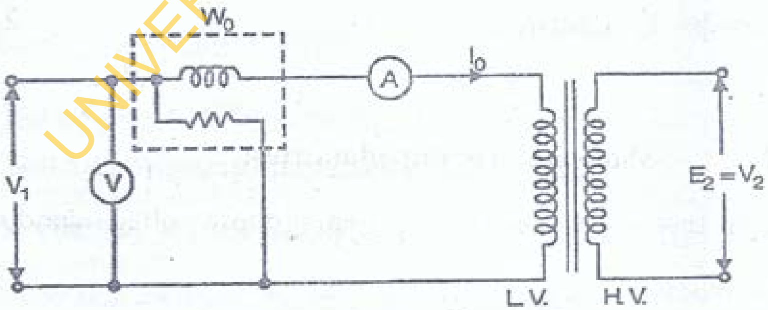


Figure 2.16: Open Circuit Test

As the normal rated voltage is applied to the primary, therefore, normal iron losses will occur in the transformer core. Hence wattmeter will record the iron losses and small copper loss in the primary. Since no-load current I_o is very small (usually 2 -10% of rated current), copper losses in the primary under no load condition are negligible as compared with iron losses. Hence, the wattmeter reading practically gives the iron losses in the transformer. It is reminded that iron losses are the same at all loads.

Iron losses P_i = Wattmeter reading = W_o

No load current = Ammeter reading = I_o

Applied voltage = Voltmeter reading = V_1

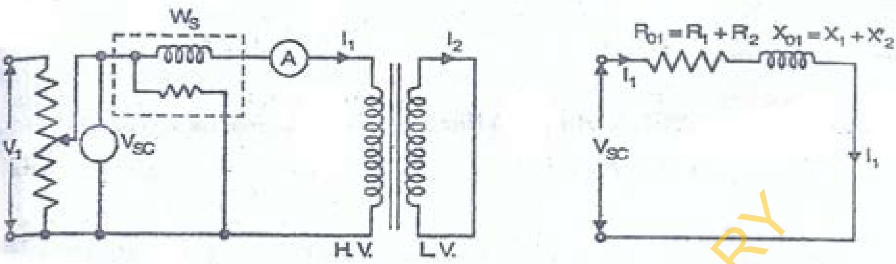
Input power $W_o = V_1 I_o \cos\phi_o$

$$\therefore \text{No-load p.f, } \cos\phi_o = \frac{W_o}{V_1 I_o} \qquad 2.40$$

$$I_w = I_o \cos\phi_o;\ I_m = I_o \sin\phi_o \qquad 2.41$$

2.14.2 Short circuit or impedance test

In this test, the secondary (usually low voltage winding) is short-circuited by a thick conductor and variable low voltage is applied to the primary as shown below.



(*i*) (*ii*)

Figure 2.17: Short Circuit Test

The low input voltage is gradually raised till at voltage V_{sc}, full-load current I_1 flows in the primary. Then I_2 in the secondary also has full-load value since .

Under such conditions the copper loss in the windings is the same as that on full load.

There is no output from the transformer under short circuit conditions. Therefore, input power is all loss and this loss is almost entirely copper loss. It is because iron loss is negligibly small since the voltage V_{sc} is very small.

Hence, the wattmeter will practically register the full-load copper losses in the transformer winding Figure 2.17 (ii) shows the equivalent circuit of a transformer on short circuit as referred to primary; the no-load current being neglected due to it smallness.

Full load Cu loss, P_c = Wattmeter reading = W_s

Applied voltage = Voltmeter reading = V_{sc}

F.L. Primary Current = Ammeter reading = I_1

$P_c = I_1^2 R_1 + I_1^2 R_2' = I_1^2 R_1$ 2.42

$R_0 = P_c / I_1^2$ 2.43

Where R_{01} is total resistance of transformer referred to primary.

Total impedance referred to primary, $Z_{01} = \dfrac{V_{sc}}{I_1}$ 2.44

Total leakage reactance referred to primary $X_{01} = \sqrt{Z_{01}^2 + R_{01}^2}$ 2.45

Short circuit p.f $\cos\phi_1 = \dfrac{P_c}{V_{sc} I_1}$ 2.46

Thus short-circuit test gives full-load Copper (Cu) loss, R_{01} and X_{01}

2.15 Losses in a Transformer

The power losses in a transformer are of two types, namely;

1. Core of iron losses
2. Copper losses

These losses appear in the form of heat and produce:

(i) An increase in temperature and

(ii) A drop in efficiency

2.15.1 Core or iron losses

These consist of hysteresis and eddy current losses and occur in the transformer core due to the alternating flux. These can be determined by open circuit test.

Hysteresis loss $= K_h f \beta m^{1.6}$ Watt/m^3 2.47

Eddy current loss $= K_e f^2 \beta_m^2 t^2$ Watt/m^3 2.48

Both hysteresis and eddy current losses depend upon:

(i) Maximum flux density β_m in the core

(ii) Supply frequency f.

Since transformers are connected to constant frequency, constant voltage supply, both f and β_m are constant. Hence core or iron losses are practically the same of all loads.

Iron or core losses, P_i = Hysteresis loss + Eddy current loss

= constant losses

The hysteresis loss can be minimized by using steel of high silicon content whereas eddy current loss can be reduced by using core of thin laminations.

2.15.2 Copper Losses

These losses occur in both the primary and secondary windings due to their Ohmic resistance. These can be determined by short-circuit test

Total Cu losses, $P_c = I_1^2 R_1 + I_2^2 R_2$

$$= I_1{}^2 R_{01} \text{ or } I_2{}^2 R_{02} \qquad 2.49$$

It is clear that copper losses vary as the square of load current. Thus if copper losses are 400W at a load current of 10A, then they will be $(½)^2 \times 400 = 100W$ at a load current of 5A.

Total losses in a transformer = $P_i + P_c$ = constant losses + variable losses

It may be noted that in a transformer, copper losses account for about 90% of the total losses.

2.16 Efficiency of a Transformer

Like any other electrical machine, the efficiency of a transformer is defined as the ratio of output power in (Watts /KW) to input power (Watts/KW) i.e.

$$\text{Efficiency } \eta = \frac{oupt\ power}{input\ power} \qquad 2.50$$

It may appear that efficiency can be determined by directly loading the transformer and measuring the input power and output power. However, this method has the following drawbacks.

(i) Since the efficiency of a transformer is very high, even 1% error in each wattmeter (output and input) may give ridiculous results. This test, for instance, may give efficiency higher than 100%.

(ii) Since the test is performed with transformer in load, considerable amount of power is wasted.

(iii) The test gives no information about the proportion of various losses.

(iv) It is generally difficult to have a device that is capable of absorbing all the output power.

Due to these drawbacks, direct loading method is seldom used to determine the efficiency of a transformer. In practice open-circuit and short-circuit tests are carried out to find the efficiency

$$Efficiency = \frac{output}{input} = \frac{output}{output + losses} \qquad 2.51$$

2.16.1 Efficiency from Transformer Tests

Full Load (F.L) Iron loss = P_i

F.L Cu loss = P_c

Total F.L. losses = $P_i + P_c$

$$\text{F.L efficiency } \eta_{FL} = \frac{Full-Load VA \times P.f}{(Full-Load VA \times P.F) + P_i + P_c} \qquad 2.52$$

Also for any load equal to x × full load

Corresponding total loss = $P_i + x^2 P_c$

Corresponding η_x = $$\frac{(x \times full - load\ kVA \times p.f.)}{(x \times full - load\ kVA \times p.f.) + P_i + x^2 P_c} \qquad 2.53$$

Note that iron loss remains the same at all loads.

Example 2.8

A transformer takes a current of 0.6A and absorbs 64W when the primary is connected to its normal supply of 200V, 50Hz, the secondary being an open circuit. Calculate:

(a) The Iron loss current

(b) The Magnetizing current

(Hint: Practical transformer on no-load)

Solution

a) No load primary /input power, $W_o = V_1 I_o \cos\phi_o$

But $I_w = I_o \cos\phi_o$ (Iron loss component)

$\therefore W_o = V_1 I_w$

$$I_w = \frac{W_o}{V_1} = \frac{64}{200} = 0.32A$$

b) Magnetizing current, $I_m = \sqrt{I_0^2 - I_w^2}$

$$= \sqrt{(0.6)^2 - (0.32)^2}$$

$$= 0.507A$$

Example 2.9

An iron cored transformer working at a maximum flux density of $0.8Wb/m^2$ is replaced by silicon steel working at a maximum flux density of $1.2Wb/m^2$. If the total flux is to remain unchanged, what is the reduction in volume expressed as a percentage of the original volume? The frequency and voltages per turn are the same.

Solution

Voltage per turn = $4.44f\phi_m = 4.44f\beta_m A$

Since voltage per turn and frequency are the same, $\beta_m A$ = constant

$$A \alpha \frac{1}{\beta_m}; A_1 = \frac{1}{0.8} = 1.25 \ and \ A_2 \alpha \frac{1}{1.2} = 0.83$$

:. % reduction is core volume = reduction in core area

$$= \frac{1.25 - 0.83}{1.25} \times 100 = 33\%$$

[3] If rated VA(i.e. full load VA) of a transformer is 1000, then half-load VA = 500. Obviously, the value of x = 1/2.

Example 2.10

A low-voltage outdoor lighting system uses a transformer that steps 120V down to 24V for safety. The equivalent resistance of all low-voltage laps is 9.6Ω. What is the current in the secondary coil? Assume the transformer is ideal and there are no losses in the line. Also, calculate the current in the primary coil and the power used.

Solution

Secondary current $I_2 = \frac{V_2}{R}$

$$= \frac{24}{9.6} = 2.5A$$

Primary current $I_1 = \frac{V_2 I_2}{I_1} = \frac{24 \times 2.5}{120} = 0.5A$

For lamp load, power factor is unity

Therefore $P = V_2 I_2 \cos\phi_2$

$$= 24 \times 2.5 \times 1 = 60W$$

Example 2.11

A 230/2300V transformer takes no-load current of 6.5A and absorbs 187W. If the resistance of the primary is 0.06Ω. Calculate:

(a) Core loss.

(b) The no-load power factor.

(c) The active component of current.

(d) The magnetizing current.

Solution

(a) No load loss $W_o = 187W$

Primary Cu loss = $I_o^2 R_1 = (6.5)^2 \times 0.06 = 2.5W$

W_o = Iron loss + Primary Cu loss

Iron loss = W_o Primary Cu loss

$= 187 \ 2.5 = 184.5W$

(b) $W_o = V_1 I_o \cos\phi_o$

$$\cos\phi_o = \frac{W_o}{V_1 I_o} = \frac{187}{230 \times 6.5} = 0.125 \quad \text{lag}$$

(c) Active component $I_w = \frac{W_o}{V_1} = \frac{187}{230} = 0.81A$

(d) $I_o^2 = I_m^2 + I_w^2$

$I_m^2 = I_o^2 - I_w^2$

$I_m = \sqrt{I_o^2 \ I_w^2}$

$= \sqrt{(6.5)^2 \ (0.81)^2}$

$= 6.4A$

Example 2.12

A 10KVA, 2000/400V transformer (single phase) has a primary resistance and inductive reactance of 5Ω and 12Ω respectively. The secondary values are 0.2Ω and 0.48Ω respectively. Calculate:

(i) The equivalent impedance of the transformer referred to the primary side

(ii) The impedance of the transformer referred to secondary side

Solution

(i) $K = \frac{400}{2000}$ i.e. $K = \frac{V_2}{V_1}$

$= 1/5$

$R_{01} = R_1 + \frac{R_2}{K_2} = 5 + \frac{0.2}{(1/5)^2}$

$= 10\Omega$

$X_{01} = X_1 + \frac{X_2}{K_2} = 12 + \frac{0.48}{(1/5)^2} = 24\Omega$

$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$

$\therefore Z_{01} = \text{v}\ 10^2 + 24^2 = 26\ \Omega$

(ii) $R_{02} = R_2 + K^2 R_1$

$= 0.2 + (1/5)^2 \times 5 = 0.4\Omega$

$X_{02} = X_2 + K^2 X_1 = 0.48 + (1/5)^2 \times 12 = 0.96\Omega$

$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2}$

$= v(0.4)^2 + (0.96)^2 = 1.04\Omega$

2.17 Shifting Impedances in a Transformer

The figure 2.18 shows a transformer where resistances and reactances are shown external to the windings. The resistance and reactance of one winding can be transferred to the other by appropriately using the factor K^2. This makes the analysis of the transformer a simple are because we have to work in one winding only.

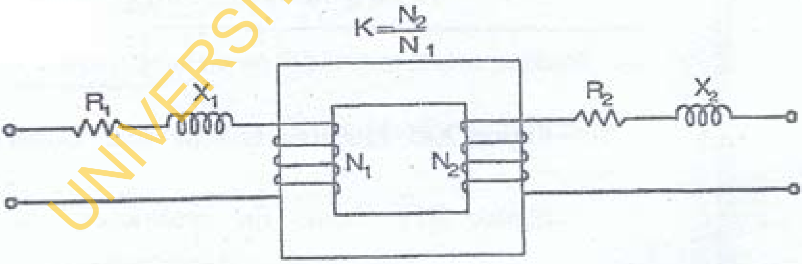


Figure 2.18: Transformer with Resistance & Reactance External to Winding

I. **Referred to primary** When secondary resistance or reactance is transferred to the primary, it is divided by K^2. It is then called equivalent secondary resistance or reactance transferred to primary and is denoted by or equivalent resistance of transformer referred to primary.

$$R_{01} = R_1 + R'_2 = R_1 + \frac{R_2}{K_2} \qquad 2.54$$

Equivalent reactance of transformer referred to primary

$$X_{01} = X_1 + X'_2 = X_1 + \frac{X_2}{K_2} \qquad 2.55$$

Equivalent impedance of transformer referred to primary

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$$

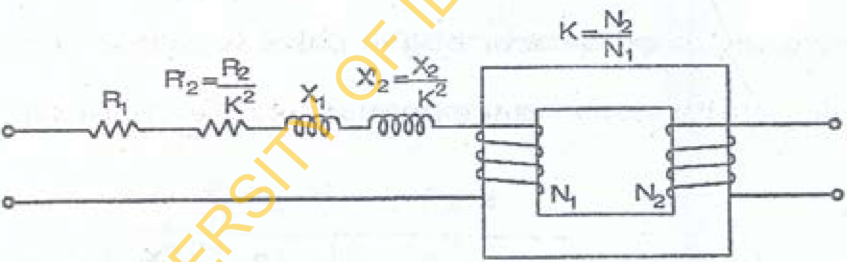


Figure 2.19: Resistance/ Reactance Referred to Primary

Figure 2.19 shows the resistance and reactance of the secondary referred to the primary. Note that secondary now has no resistance or reactance.

II. **Referred to secondary** When primary resistance or reactance is transferred to the secondary, it is multiplier by K^2. It is then called equivalent primary resistance or reactance referred to the secondary and is denoted by R'_1 or X'_1

Equivalent resistance of transformer referred to secondary

$$R_{01} = R_2 + R_1' = R_2 + K^2 R_1 \qquad 2.57$$

Equivalent reactance of transformer referred to secondary

$$X_{02} = X_2 + X_1' = X_2 + K^2 X_1 \qquad 2.58$$

Equivalent impedance of transformer referred to secondary

$$Z_{01} = \sqrt{R_{02}^2 + X_{02}^2}$$

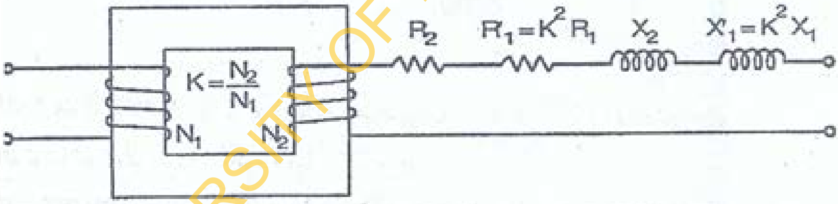


Figure 2.20: Resistance/ Reactance Referred to Secondary

Figure 2.20 shows the resistance and reactance of primary referred to the secondary. Not that primary now has no resistance or reactance.

INTRODUCTION TO THE GENERALIZED THEORY OF ELECTRICAL MACHINES

3.1 Introduction

Electrical machines are, in general, used to convert mechanical energy into electrical energy, as in electric generators, or electrical energy into mechanical energy as in electric motors. Most electrical machines consist of an outer stationary member and an inner rotating member. The stationary and rotating members consist of steel cores, separated by airgap and form a magnetic circuit in which magnetic flux is produced by current flowing through the windings situated on the two members.

3.2 Winding

One member of an electrical machine has a winding, referred to as field winding, the function of which is to produce a magnetic flux. The other member has a winding or group of coils, termed an armature winding, in which an e.m.f. is generated by the movement of the winding relative to the magnetic flux produced by the field winding.

In d.c machines the field winding is stationary and the armature winding is located on a rotating steel core. In a.c. synchronous generator and motors, the field winding is usually on the rotor and the armature winding is stationary.

The types of windings used on electrical machines can be grouped thus:

1. **Concentrated /coil windings** wound around the salient poles of d.c machine and relatively slow-speed a.c. synchronous generator and motors.

2. **Phase /Distributed Windings** Distributed in slots located on the inner surface of stator and/or in slots located on the outer surface of a cylindrical rotor.

3. **Commutator Windings:** These are windings in which coils are connected to commutator segments. This type of winding must be on the rotating member 1. of a machine on order that the commutator may act as a switch which automatically reverses the direction of the current in a cal as the latter passes brush bearing on the commutator surface.

Commutator windings are used on the armature of

(i) Direct current (dc) machines

(ii) single phase series motors

(iii) 3 Phase variable speed commutator motors.

3.3 Electromechanical Energy Conversion

In the process of converting mechanical energy to electrical energy or vice-versa. some of the input energy is converted in to heat owing to I^2R, core and friction losses in the machine. There is also an increase or a decrease of energy stored in the magnetic field of the machine if the magnetic flux is increased or decreased. If a change of load on an electric motor is accompanied by a change of speed, some of the input energy is converted into kinetic energy during the period that the machine and its load are being accelerated.

Note: Input power= output power + power dissipated in form of heat

3.4 DC Machine as a Generator / Motor

There is no difference of construction between a dc generator and a dc motor. In fact, the only difference is that in a generator, the generated e.m.f is less than the terminal voltage. Consider for instance, a shunt generator D in figure 3.1 to be driven by an engine and connected through a centre-zero ammeters A to a battery B. If the field regulator R is adjusted until the reading on A is zero, the e.m.f E_D, generated in D is then exactly generated in D exceeds that of B and the excess e.m.f is available to circulate a current I_D through the resistance of the armature circuit, the battery and the connecting conductors.

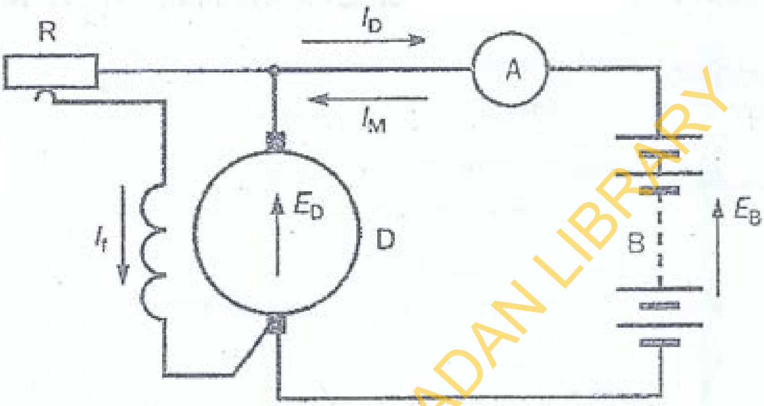


Figure 3.1: A dc Machine

Since I_D is in the same direction as E_D, machine D is generator of electrical energy.

The relationship between the current, the e.m.f e.t.c for machine D may be expressed thus;

If E_b = e.m.f generated in armature,

V = terminal voltage

R_a = resistance of armature circuit Eqn 3.2

and I_a = armature current

then when D is operating as a generator

$E_b = V - I_a R_a$ Eqn 3.1

when the machine is operating as a motor the e.m.f, is less than the applied voltage V, and the direction of the current I_a is the reverse of that when the machine is acting as a generator, hence

$E_b = V \ I_a R_a$

$V = E_b + I_a R_a$

CHAPTER FOUR

DC GENERATOR

4.1 Introduction

An electric generator is a machine that converts mechanical energy into electrical energy. An electric generator is based on the principle that whenever flux is cut by a conductor, an e.m.f. is induced which will cause a current to flow if the conductor circuit is closed. The direction of induced e.m.f (and hence current) is given by Fleming's right hand rule (stretch the thumb, fore-finger and middle finger of your right hand so that they are at right angles to each other. If the fore finger points in the direction of field, thumb in the direction of motion of the conductor, then middle finger will point in the direction of induced e.m.f).

Therefore, the essential components of a generator are;

(i) a magnetic field

(ii) conductor or a group of conductors

(Iii) motion of conductor w.r.t. magnetic field

As energy converter, the d.c generator is not 100% efficient because there are energy losses in the machine.

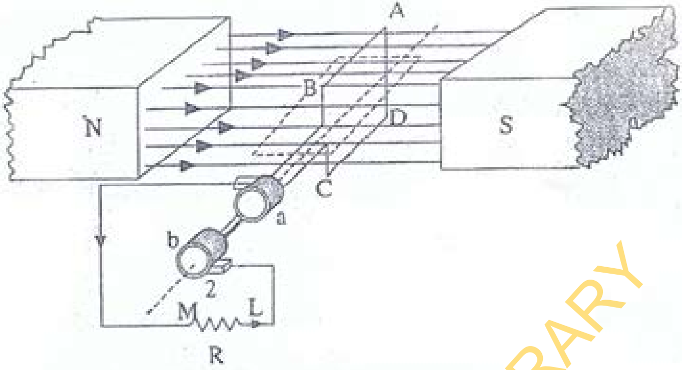


Figure 4.1: Simple dc generator

Figure 4.1 shows a single rectangle copper coil ABCD rotating abou its own axis in a magnetic field provided either by permanen magnets or electromagnets.

The ends of the coil are joined to two slip rings or disc 'a' and 'b' which are insulated from each other from the central shaft. Tw connecting carbon brushes press against the slip rings. There functio is to collect the current induced in the coil and convey it to the extern load resistance R

The rotating coil may be called armature and the magnets a field magnets.

4.2 Working Principles

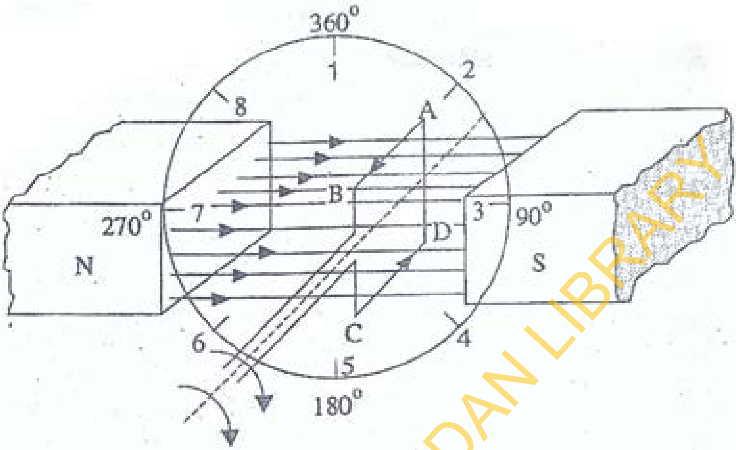


Figure 4.2: Coil & conductor rotating in a magnetic field

Let assume the coil in figure 4.2 to be rotating in clockwise direction. As the coil assumes successive positions in the field, the flux linked with it changes. Hence an e.m.f. is induced in it which is proportional to the rate of change of the flux linkages

$$e = \frac{-Nd\Phi}{dt} \qquad 4.1$$

When the plane of the coil is at right angles to the flux i.e. position 1 (figure 4.2), the flux linked with the coil is maximum but rate of change of flux linkages is minimum.

This is so because in this position, the coil sides AB and CD do not cut or shear the lines of flux; rather they slide along them i.e. they move parallel to them. Hence, there is no induced e.m.f in the coil. Let us consider the no e.m.f. /Vertical position of the coil as the starting position. The angle of rotation or time will be measured from this position.

As the coil continues rotating further, the rate of change of flux linkages increases till position 3 is reached where angle of rotation $\phi = 90^0$. Here, the coil plane is horizontal (i.e. parallel to the lines of flux). The flux linked with the coil is minimum but rate of change of flux linkages or rate of flux cutting is maximum. Hence maximum e.m.f is induced in the coil when in this position.

In the next quarter revolution i.e. 90^0 to 180^0, the flux linked with the coil gradually increase but the rate of change of flux decreases. Hence, the induced e.m.f decreases gradually till position 5 of the coil where it is reduced to zero value.

4.3 Construction of D.C. Generator

The dc generator and dc motor have the same general construction. Any dc generator can be run as a dc motor and vice-versa. All dc machines have five principal components:

(i) field system
(ii) armature core
(iii) armature winding
(iv) commutator
(v) brushes

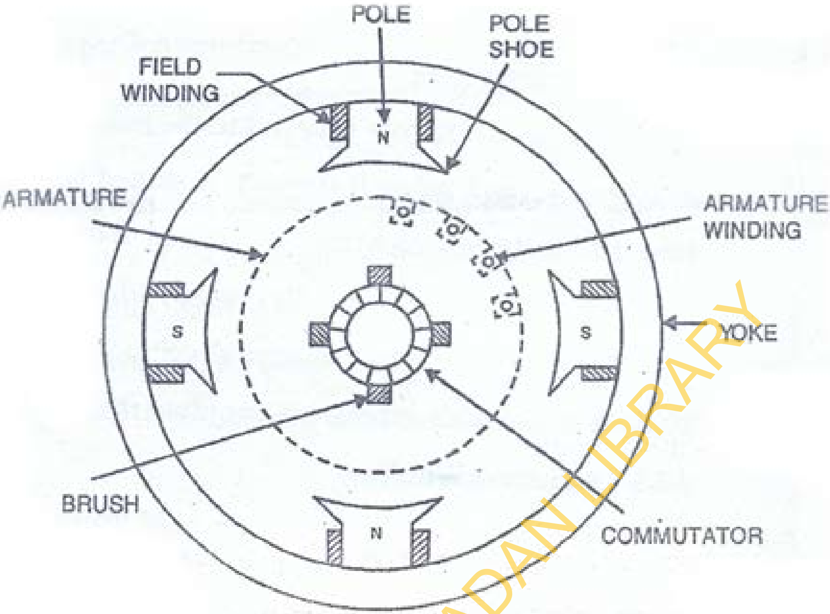


Figure 4.3: Parts of a dc Generator

4.3.1 Field System

The function of the filed system is to produce uniform magnetic field within which the armature rotates. It consists of a number of salient poles (even number) bolted to the inside of circular frame (general called yoke). The yoke is usually made of solid cast steel whereas the pole pieces are composed of stacked lamination. Field coils are mounted on the poles and carry the dc exciting current. The field coils are connected in such a way that adjacent poles have opposite polarity.

4.3.2 Armature Core

The armature core is keyed to the machine shaft and rotates between the field poles. It consists of slotted soft-iron laminations (about 0.4 to 0.6mm thick) that are stacked to form a cylindrical core. The laminations are individually coated with a thin insulating film so that they do not come in electrical contact with each other. The purpose of laminating the core is to reduce the eddy current loss.

4.3.3 Armature Winding

The slots of the armature core hold insulated conductors that are connected in a suitable manner. This is known as armature winding. This is the winding in which "working" e.m.f. is induced. The armature conductors are connected in series-parallel, the conductors being connected in series so as to increase the voltage and in parallel paths so as to increase the current. The armature of a dc machine is a close- circuit winding; the conductors being connected in a systematical manner forming a closed loop or series of closed loops.

4.3.4 Commutator

A commutator is a mechanical voltage rectifier which converts the alternating voltage generated in the armature into direct voltage across the brushes. The commutator is made of copper segments insulated from each other by mica sheets and mounted on the shaft of the machine. The armature conductors are soldered to the commutator segments in a suitable manner to give rise to the armature winding. Depending upon the manners in which the armature conductors are connected to the commutator segments, there are two types of armature winding in a dc machine viz;

(a) Lap winding

(b) Wave winding

4.3.5 Brushes

The purpose of brushes is to ensure electrical connections between the rotating commutator and stationary external load circuit. The brushes are made of carbon and rest n the commutator. The brush pressure is adjusted by means of adjustable springs. If the brush pressure is very large, the friction produces heating effect on the commutator and the brushes. On the other hand, if it is too weak, the imperfect contact with the commutator my produce sparking.

4.4 Types of Armature Windings

There are two types of simple armature windings viz:

(i) Simple wave winding

(ii) Simple lap winding

4.4.1 Simple wave winding

In this arrangement, the armature winding is divided into two parallel paths irrespective of the number of poles of the machine. If there are Z armature conductors, then Z/2 conductors will be in series in each parallel path as shown below. Each parallel path will carry a current Ia/2 where I_a is the total armature current.

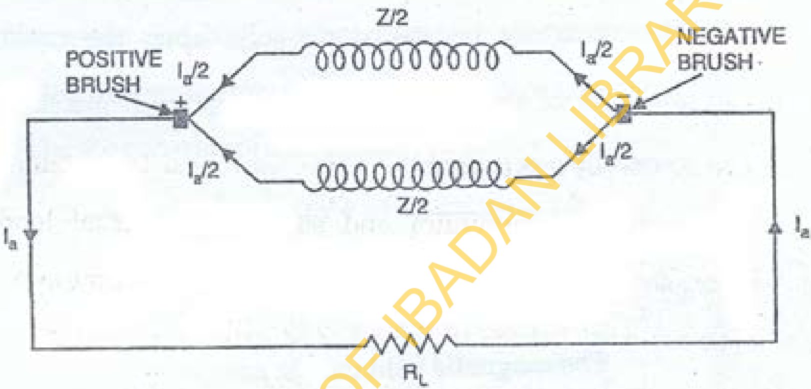


Figure 4.4: Wave Winding

To sum up, in a simple wave winding;

- There are two parallel paths irrespective of number of poles of the machine.
- Each parallel path has Z/2 conductors in series; Z being total number of armature conductors.
- e.m.f generated = e.m.f /parallel path
- total armature current, $I_a = 2 \times$ current / parallel path

4.4.2 Simple lap winding

In this arrangement, the armature coils are connected in series through commutator segments in such a way that the armature winding is divided into as many parallel paths as the number of poles of the machine if there are Z conductors and P poles, then there will be P parallel paths, each containing Z/P conductors in series. Each parallel path will carry a current of Ia/P where I_a is the total armature current. Here it is assumed that P = 4 so that there are 4 parallel palls.

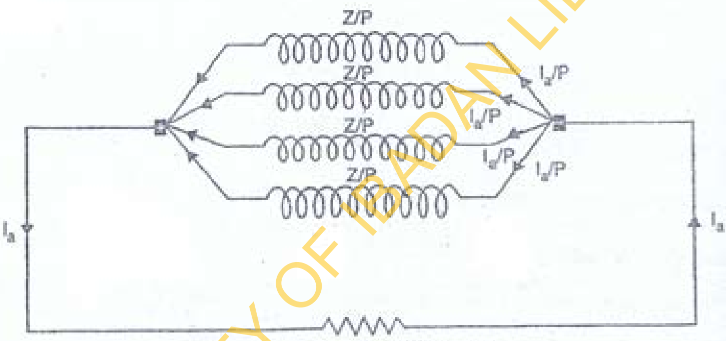


Figure 4.5: Lap Winding

To sum up, in a simple lap winding;

- There are as many parallel paths as the number of poles (P) of the machine.
- Each parallel path has Z/P conductors in series where Z and P are the total number of armature conductors and poles respectively.
- e.m.f. generated = e.m.f. /path (parallel)

total armature current, $I_a = P \times$ current / parallel path

4.5 E.M.F Equation of a DC Generator

Let ϕ = flux /pole in Wb

Z = total number of armature conductors

P = number of poles

A = number of parallel paths

= 2 for wave winding

= P for lap winding

N = speed of armature in r.p.m

E_g = e.m.f. of the generator = e.m.f. /parallel path

Flux cut of one conductor in one revolution of the armature,

$d\phi = P\phi$ Webers 4.2

Time taken to complete one revolution,

$dt = 60/N$ second 4.3

e.m.f generated / conductor = $\frac{d\phi}{dt} = \frac{p\phi}{60/N} = \frac{p\phi N}{60}$ *volts* 4.4

e.m.f of generator, E_g = e.m.f per parallel path

= (e.m.f/conductor) × no. of conductors in seri[es]

per parallel path

$$= \frac{P\phi N}{60} \times \frac{Z}{A}$$

$$E_g = \frac{P\phi ZN}{60A} \qquad 4.5$$

where A = 2 for wave winding

= P for lap winding

4.6 Armature Resistance (R_a)

The resistance offered by the armature circuit is known as armature resistance (R_a) and includes:

(i) resistance of armature winding

(ii) resistance of brushes

The armature resistance depends upon the constructor of the machine of the machine. Except for small machines, its value is generally less than 1Ω.

4.7 Types of DC Generators

The magnetic field in a dc generator is normally produced by electromagnets rather than permanent magnets. Generators are generally classified according to their methods of field excitation. On this basis, dc generators are divided into the following two classes:

(i) separately excited dc generators

(ii) self-excited dc generators

The behaviour of dc generator on load depends upon the method of field excited adopted.

4.7.1 Separately excited generator

A dc generator whose field magnet winding is supplied from an independent external dc source (e.g. a battery) is called a separately generator. Figure 4.6 shows the connections of a separately excited generator. The voltage output depends upon the speed of rotation of armature and the field current (E_g = PφZN/60A). The greater the speed and field current, greater is the generated e.m.f. It may be noted that separately excited d.c generators are rarely used in practice. The d.c generators are normally of self-excited type.

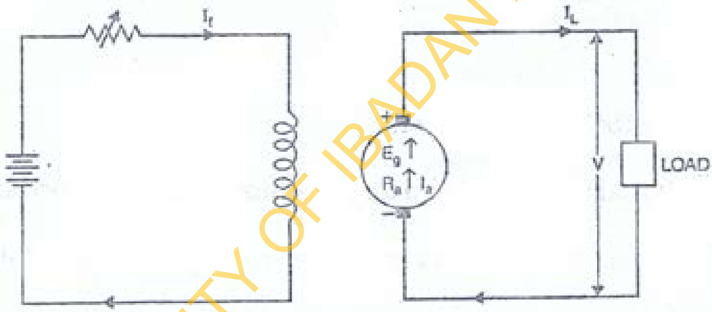


Figure 4.6: Separately Excited Generator

Armature current, $I_a = I_L$ 4.6

Terminal Voltage, $V = E_g - I_a R_a$ 4.7

Electric Power developed $= E_g I_a$ 4.8

Power delivered to load $= E_g I_a \;\; I_a^2 R_a$ 4.9

$= I_a (Eg \;\; I_a R_a)$

$= VI_a$ 4.10

4.7.2 Self excited generators

A dc generator whose field magnet winding is supplied current from the output of the generator itself is called a self excited generator. There are three types of self-excited generators depending upon the manner in which the field winding is connected to the armature namely:

(i) Series generator

(ii) Shunt Generator

(iii) Compound Generator

4.7.2.1 series generator

In the series wound generator, the field winding is connected in series with armature winding so that whole armature current flows through the field winding as well as the load. Figure below shows the connecting of a series-wound generator. Since the field winding carries the whole of load current, it has a few turns of thick wire having law resistance. Series generators are rarely used except for special purposes e.g. as boosters.

Armature current, $I_a = I_{se} = I_L = I$ 4.11

Terminal Voltage, $V = E_g \; I(R_a + R_{se})$ 4.12

Power developed in armature $= E_g I_a$ 4.13

Power delivered to load $= E_g I_a \; I_a^2(R_a + R_{se})$ 4.14

$= I_a[E_g \; I_a(R_a + R_{se})]$

$= VI_a$ or VI_L 4.15

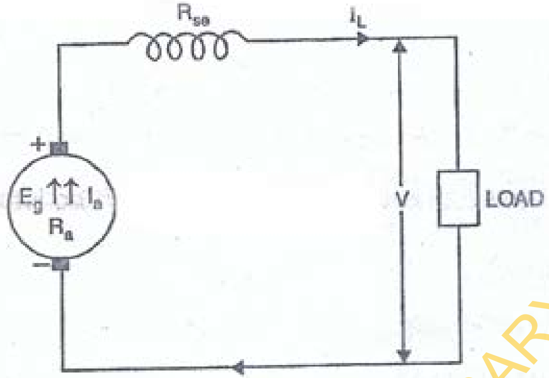


Figure 4.7: Series-wound Generator

4.7.2.2 shunt generator

In a shunt generator, the field winding is connected in parallel with the armature winding so that terminal voltage of the general is applied across it. The shunt winding has many turns of fine wire having high resistance. Therefore, only a part of armature current flows through shunt field winding and the rest flows through the load.

Figure below shows the connections of a short wound generator.

Shunt field current, $I_{sh} = \frac{V}{R_{sh}}$ 4.16

Armature current $I_a = I_L + I_{sh}$ 4.17

Terminal Voltage, $V = E_g - I_a R_a$ 4.18

Power developed in armature $= E_g I_a$ 4.19

Power delivered to load $= VI_L$ 4.20

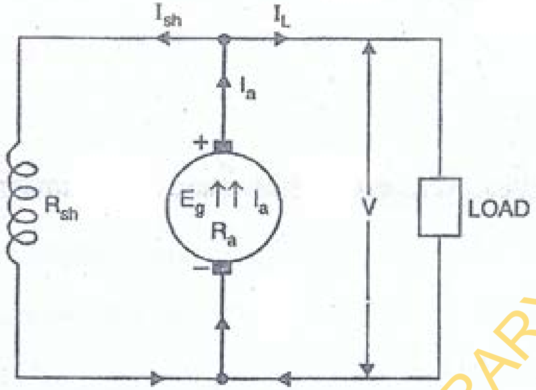
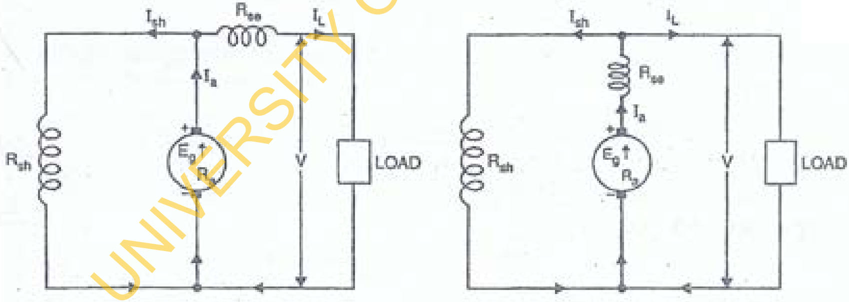


Figure 4.8: Shunt-wound Generator

4.7.2.3 compound generator

In a compoundwound generator, there are two sets of field windings on each pole-one are in series and the other in parallel with the armature. A compound wound generator may be:



(a) Short Shunt (b) Long Shunt

Figure 4.9: Compound Generator

(a) Short shunt in which only shunt field winding is in parallel with the armature winding.

(b) Long shunt in which shunt field winding is a parallel with both series and armature winding.

Short Shunt

Series field current, $I_{se} = I_L$ 4.21

Shunt field current, $I_{sh} = \dfrac{V + I_{se} R_{se}}{R_{sh}}$ 4.22

Terminal Voltage, $V = E_g - I_a R_a - I_{se} R_{se}$ 4.23

Power developed in armature $= E_g I_a$ 4.24

Power delivered to load $= VI_L$ 4.24

Long Shunt

Series field current, $I_{se} = I_a . I_L + I_{sh}$ 4.25

Shunt field current, $I_{sh} = \dfrac{V}{R_{sh}}$ 4.26

Terminal Voltage, $V = E_g - I_a (R_a + R_{se})$ 4.27

Power delivered to load $= VI_L$ 4.28

4.8 Losses in a DC Machine

The losses in a dc machine (generator or motor) may be divided into three classes viz:

(i) Copper losses

(ii) Iron or core losses

(iii) Mechanical losses

All these losses appear as heat and thus raise the temperature of the machine. They also lower the efficiency of the machine

Copper losses – Armature Cu loss, shunt field Cu Loss and series field Cu loss

Losses in a dc machine — Iron losses Hysteresis loss and Eddy current

Mechanical losses Friction and Windage

4.8.1 Copper losses

These losses occur due to currents in the various windings of the machine

(a) Armature copper loss = $I^2_a R_a$

(b) Shunt filed copper loss $I^2_{sh} R_{sh}$

(C) Series field copper loss = $I^2_{se} R_{se}$

4.8.2 Iron or core losses

These losses occur in the armature of a dc machine and are due to the rotation of armature in the magnetic field of the poles. They are of two types viz:

(a) Hysteresis loss The Hysteresis loss occur in the armature of the d.c machine since any given part of the armature is subjected to magnetic reversals as it passes under successive poles. It is given by;

Hysteresis loss, $P_h = \eta \beta_{max}^{1.6} fV$ Watts 4.29

Were β_{max} = maximum flux density in armature

f = frequency of magnetic reversals

= Np/120 where N is r.p.m.

V = volume of armature in m^3

η = Steinmetz hysteresis co-efficient

(b) Eddy current loss when armature rotates in the magnetic field of the poles, an e.m.f. is induced in it which circulates eddy currents in the armature core. The power loss due to these eddy currents is called eddy current loss. In order to reduce this loss, the armature core is build t up of thin laminations insulated from each other by a thin layer of vanish.

Eddy current loss, $P_e = K_e \beta^2_{max} f^2 t^2 V$ Watts 4.30

Where K_e = constant

β_{max} = Maximum flux density in the core

f = frequency of magnetic reversals

t = thickness of lamination

V = volume of core in m^3

4.8.3 Mechanical Losses

These losses are due to friction and windage.

(I) Friction loss e.g. bearing friction, brush friction e.t.c.

(ii) Windage loss i.e. air friction of rotating armature.

NOTE: Iron losses and mechanical losses together are called stray losses.

4.9 Constant and Variable Losses

The losses in a d.c generator (or d.c motor) may be sub-divided into

(i) Constant losses

(ii) Variable losses

4.9.1 Constant losses

These losses in a dc generator which remains constant at all loads are known as constant losses. The constant losses in a dc generator are:

(a) Iron losses

(b) Mechanical losses

(c) Shunt field losses

4.9.2 Variable losses

Those losses in a dc generator which vary with load are known as variable losses. The variable losses in a dc generator are;

(a) Copper loss in armature winding $(I\ r_a)$

(B) Copper loss in series field winding $I\ R_{se})$

Total losses = constant losses + variable losses

4.10 POWER STAGES

The various power stages in a d.c generator are represented diagrammatically as shown in figure 4.10:

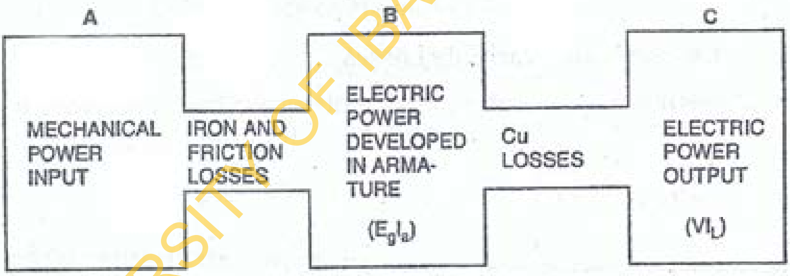


Figure 4.10: Power Stages in a dc Generator

Note: A B = Iron and friction losses

B C = Copper losses

(i) Mechanical Efficiency

$$\eta_m = \frac{\beta}{A} = \frac{E_g I_a}{Mechanical\ Power\ Input}$$

(ii) Electrical Efficiency, $\eta_e = \frac{C}{\beta} = \frac{VI_L}{E_g I_a}$

(iii) Commercial / Overall Efficiency

$$\eta_c = \frac{C}{A} = \frac{VI_L}{Mechanical\ Power\ Input}$$

Clearly, $\eta_c = \eta_m \times \eta_e$

Commercial Efficiency $\eta_c = \frac{C}{A} = \frac{Output}{Input} = \frac{Input - Losses}{Input}$

Example 4.1

A shunt generator delivers 195A at a terminal Pd of 250V. The armature resistance and shunt field resistance are 0.02Ω and 50Ω respectively. Calculate:

(i) The value of generated e.m.f

(ii) The value of copper losses

Solution

(i) $I_{sh} = \frac{250}{50} = 5A$

$\therefore I_a = I_{sh} + I_L = 5 + 195$

$= 200A$

Armature drop $= I_a R_a$

$= 200 \times 0.02 = 4V$

Generated e.m.f $E_g = V + I_a R_a$

$$= 250 + 4 = 254V$$

(ii) Armature Cu loss $= I R_a$

$$= (200)^2 \times 0.02 = 800W$$

Shunt Cu loss $= VI_{sh} = 250 \times 5$

$$= 1250W$$

Total Cu loss $= 1250 + 800$

$$= 2050W$$

Example 4.2

A 30KW, 300V dc shunt generator has armature and field resistance of 0.05Ω and 100Ω respectively. Calculate:

(i) The generated e.m.f

(ii) The total power developed when the generator delivers full output power

Solution

$$I_L = \frac{30 \times 10^3}{300} = 100A$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{300}{100} = 3A$$

$$\therefore I_a = I_L + I_{sh} = 100 + 3$$

$$= 103A$$

Generated e.m.f $E_g = V + I_a R_a$

$$= 300 + 103 \times 0.5$$

$$= 300 + 5.15 = 305.15V$$

(ii) Power developed in armature = E_g Ia

$$= \frac{305.15 \times 103}{1000} = 31.43KW$$

Example 4.3

A 6 pole lap wound dc generator has 6000 conductors on its armature. The flux per pole is 0.02Wb. Calculate the speed at which the generator must run to generate a voltage of 300V. Also, calculate the speed of the generator if it is wave connected.

Solution

(i) $E_g = \frac{\phi ZN}{60} \times \frac{P}{A}$

$E_g = 300, \phi = 0.02Wb, Z = 6000, P = 6, A = 6$

$300 = \frac{0.02 \times 6000 \times N}{60} \times \frac{6}{6}$

N = 50 r.p.m

(ii) $E_g = \frac{\phi ZN}{60} \times \frac{P}{A}$

$E_g = 300, \phi = 0.02Wb, Z = 6000, P = 6, A = 2$

$300 = \frac{0.02 \times 6000 \times N}{60} \times \frac{6}{2}$

N = 50 r.p.m

Example 4.4

The no-load voltage of a generator is 240V and rated load voltage is 220V. Calculate the voltage regulation of the generator.

Solution

$$\text{Voltage regulation} = \frac{240 - 220}{220} \times 100$$

$= 9.1\%$

Example 4.5

The resistance of the field circuit of a short d.c generator is 200Ω. When the output of the generator is 100KW, the terminal voltage is 500V and the generated e.m.f is 325V. Calculate the armature resistance.

Solution

Armature voltage drop $I_a R_a = E_g - V$

$= 525 - 500$

$= 25V$

$$I_{sh} = \frac{V}{R_{sh}}$$

$$= \frac{500}{200} = 2.5A$$

$$\text{Power delivered} = VI_L$$

$$I_L = \frac{100 \times 10^3}{500} = 200\ A$$

Now $I_a = I_L + I_{sh}$

$= 200 + 2.5 = 202.5A$

$$\frac{I_a R_a}{I_a} = \frac{25}{202.5} = 0.123\Omega$$

Example 4.6

A dc shunt generator supplies a load of 7.5KW at 200V. The armature resistance is 0.6Ω and field resistance is 80Ω. Calculate the generated e.m.f.

Solution

$$I_{sh} = \frac{V}{R_{sh}} = \frac{200}{80} = 2.5A$$

$$I_L = \frac{7.5 \times 1000}{200}$$

$$= 37.5A$$

Armature current $I_a = I_L + I_{sh}$

$$= 37.5 + 2.5$$

$$= 40A$$

∴ Generated voltage, $Eg = V + I_a R_a$

$$= 200 + (40 \times 0.6)$$

$$= 224V$$

CHAPTER FIVE

DC MOTORS

5.1 Introduction

DC motors are seldom used in ordinary applications because all electric supply companies furnish alternating current. However, for special applications such as in steel mills, mines and electric trains, it is advantageous to convert alternating current into direct current in order to use d.c motors. The reason is that speed/torque characteristics of d.c motors are much more superior to that of a.c motors. Like generators, d.c motors are also of three types viz: series-wound, shunt wound and compound wound. The use of a particular d.c motor depends upon the type of mechanical load it has to drive.

5.2 Working Principle

The general arrangement of the brush and field connections of a 4 pole d.c motor is a shown in figure 5.1.

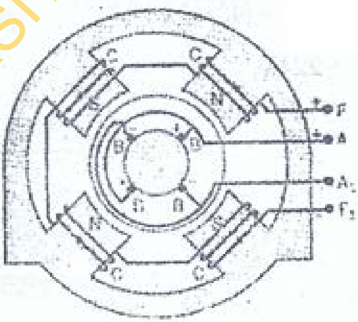


Figure 5.1: A 4-Pole dc Motor

The 4 brushes B make contact with the commutator. The positive brushes are connected to the positive terminal A and the negative brushes to the negative terminal A_1. From the figure above, it will be seen that the brushes are situated approximately in line with the centre of the poles. This position enables then to make contact with conductors in which little or no e.m.f is being generated since these conductors are the moving between the poles.

The 4 exciting or field coils C are usually joined in series and the end are brought out to terminals F and F_1. These coils must be so connected to produce N and S poles alternately.

5.3 Back/Counter E.m.f.

When the armature of a d.c motor rotates under the influence of the driving torque, the armature conductors move through the magnetic field and hence e.m.f is induced in them as in a generator. The induced e.m.f acts in opposite direction to the applied voltage. V and this is known as back or counter e.m.f, E_b. The back e.m.f is always less than the applied voltage V, although this difference is small when the motor is running under normal condition.

5.4 Voltage Equation of DC Motor

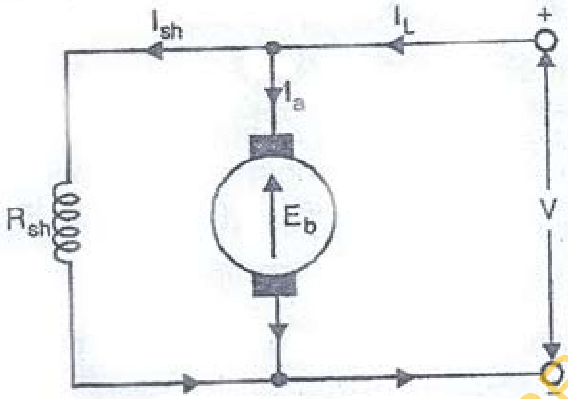


Figure 5.2: A dc Motor

Consider the figure 5.2,

V = applied voltage

E_b = back e.m.f

R_a = armature resistance

Since back e.m.f E_b acts in opposition to the applied voltage V, the net voltage across the armature circuit is V - E_b. The armature current I_a is given by

$$I = \frac{V - E_b}{R_a}$$

$$V = E_b + I_a R_a \qquad 5.2$$

5.5 Power Equation of dc Motor

If equation 5.2 above is multiplied throughout by I_a,

$$VI_a = E_bI_a + IR_a \qquad 5.3$$

This is known as power equation of the dc motor

VI_a = electric power supplied by armature (armature input)

E_bI_a = power developed by armature (armature output)

IR_a = electric power wasted in armature (armature Cu loss)

5.6 Condition for Maximum Power

The mechanical power developed by a dc motor is expressed as

$$P_m = E_bI_a \qquad 5.4$$

Substituting P_m for E_bI_a in equation 5.2

$$VI_a = P_m + Ir_a \qquad 5.5$$

$$P_m = VI_a - IR_a \qquad 5.6$$

Since V and R_a are fixed, power developed by the motor depends upon armature current. For maximum power, dP_m/dI_a should be zero.

$$\frac{dp_m}{dI_a} = V - 2\,I_a\,R_a = 0 \qquad 5.7$$

or $\quad I_aR_a = V/2$

From equation Eqn 5.1 above, $V = E_b + I_aR_a$

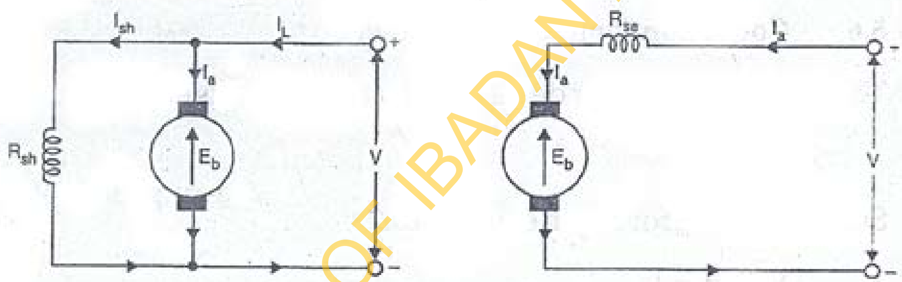
$$= E_b + V/2$$

$$E_b = V/2 \qquad 5.8$$

Hence mechanical power developed by the motor is maximum when back e.m.f is equal to half the applied voltage.

5.7 Types of DC Motors

Like generators, there are three types of d.c motors characterized by the connections of field windings in relation to the armature as shown in figures 5.3 and 5.4;



(a): Shunt Wound Motor (b):Series Wound Motor

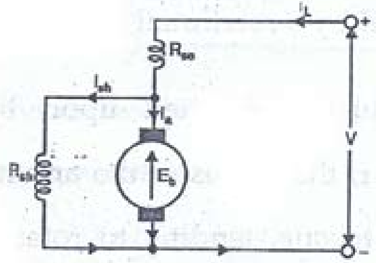
Figure 5.3: dc Motor Type

(i) Shunt wound motor in which the field winding is connected in parallel with the armature (Figure 5.3a)

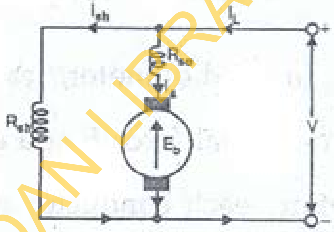
(ii) Series wound motor in which the field winding is connected in series with the armature (Figure 5.3b)

(iii) Compoundwound motor which has two field windings; one connected in parallel with the armature and other in series with it. There are two types of compound motor connections.

When the shunt field winding is directly connected across the armature terminals, it is called short shunt connection.



Short-Shunt Connection



Long-Shunt Connection

Figure 5.4: Types of Compound Wound Motors

When the shunt field winding is so connected that it shunts the series combination of armature and series fields, it is called long shunt connection.

5.8 Armature Torque of DC Motor

Torque is the turning moment of a force about an axis as shown in figure 5.5 and is measured by the product of force (F) and radius (r) at right angle to which the force acts. i.e $T = F \times r$

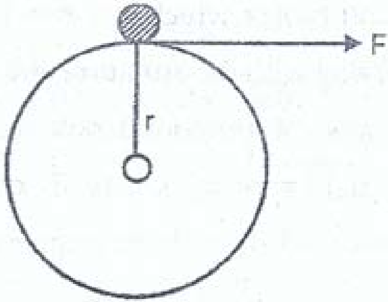


Figure 5.5: Torque produced by a rotational motion

In a d.c motor, each conductor is acted upon by a circumferential force F at a distance r, the radius of the armature. Therefore, each conductor exerts a torque, tending to rotate the armature. The sum of the torques due to all armature conductors is known as gross armature torque (T_a)

Let in a d.c motor,

r = average radius of armature in m

l = effective length of each conductor in m

Z = total number of armature conductors

A = Number of parallel paths

i = current in each conductor = I_a/A

β = average flux density in Wb/m^2

Φ = flux per pole in Wb

P = number of poles

Force on each conductor, $F = Bil$ Newton 5.9

T torque due to one conductor $= F \times r$ Newton meter 5.10

Total armature torque, $T_a = ZFr$ Newton-meter

$$= Z\beta ilr \quad 5.11$$

Now $i = I_a/A$ 5.12

And $\beta = \phi/a$ 5.13

where a is the x-sectional area of flux path/pole at radius r.

Clearly, $a = 2\pi rl/p$ 5.14

$$T_a = Z \times (\phi/a) \times (I_a/A) \times l \times r$$

$$= Z \times \frac{\phi}{2\pi rl/P} \times \frac{I_a}{A} \times l \times r$$

$$= \frac{Z\phi I_a P}{2\pi A} Nm$$

$T_a = 0.159 Z\phi I_a (P/A)$ N-m 5.15

Since Z, P and A are fixed for a given machine,

$$T_a \phi I_a \quad 5.16$$

Hence torque in a d.c motor is directly proportional to flux per poles and armature current.

(I) For a shunt motor, flux ϕ is practically constant

$\therefore T_a \alpha I_a$ 5.17

(ii) For series motor, flux ϕ is directly proportional to armature current I_a provided magnetic saturation does not take place.

$T_a \alpha I_a^2$ (up to magnetic saturation) 5.18

Alternative Expression for T_a,

$$E_b = \frac{P\Phi ZN}{60A} \quad 5.19$$

$$\frac{P\Phi Z}{A} = \frac{60 \times E_b}{N} \quad 5.20$$

From equation Eqn 5.15, we get expression of T_a as:

$$Ta = 0.159 \times \left(\frac{60 \times E_b}{N}\right) \times I_a = 9.55 \times \frac{E_b\ I_a}{N}\ N-m \quad 5.21$$

5.9 Shaft Torque T_{sh}

The torque which is available at the motor for doing useful work is known as shaft torque. It is represented by T_{sh}. Figure below illustrates the concept of shaft torque.

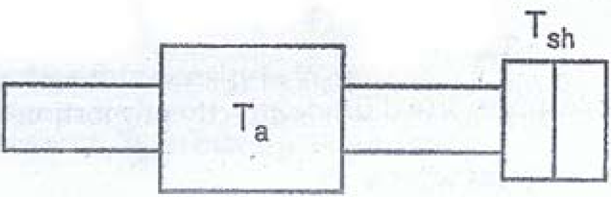


Figure 5.6: Shaft Torque of dc Motor

The total torque T_a developed in armature of a motor is fully not available at the shaft as a part of it is lost in overcoming the iron and frictional losses in the motor. Therefore, shaft torque T_{sh} is somewhat less than the total armature torque, T_a. The difference $T_a - T_{sh}$ is known as lost torque. It follows from equation 5.21 that,

$$T_{sh} = 9.55 \times \frac{output}{N}\ Newton - meter \qquad 5.22$$

$$\text{Lost torque} = 9.55 \times \frac{iron\ and\ friction\ losses}{N}\ Newton - metre \qquad 5.23$$

5.10 Speed of a dc Motor

$$E_b = V\ I_a R_a \qquad 5.24$$

$$\text{But, } E_b = \frac{P\Phi ZN}{60A} \qquad 5.25$$

$$\therefore \quad \frac{P\Phi ZN}{60A} = V - I_a R_a \qquad 5.26$$

$$\text{Or } N = \frac{V - I_a R_a}{\Phi} \times \frac{60A}{PZ} \qquad 5.27$$

$$\text{Or } N = \frac{K(V - I_a R_a\ )}{\Phi}\ where\ K = 60A / PZ$$

But $V - I_a R_a = E_b$

$$N = \frac{KE_b}{\phi} \quad 5.28$$

$$\text{or } N \alpha \frac{E_b}{\phi} \quad 5.29$$

Therefore, in a d.c motor, speed is directly proportional to back e.m.f E_b and inversely proportional to flux per pole ϕ.

5.11 Speed Relations

If a dc motor has initial values of speed, flux per pole and back e.m.f as N_1, ϕ and E_{b1} respectively and the corresponding final values are N_2, ϕ_2 and E_{b2}, then,

$$N_1 \alpha \frac{E_{b1}}{\phi_1} \text{ and } N_2 \alpha \frac{E_{b2}}{\phi_2} \quad 5.30$$

$$\therefore \frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \times \frac{\phi_2}{\phi_1} \quad 5.31$$

(i) For shunt motors flux practically remains constant so that $\phi_1 = \phi$

$$\therefore \frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \quad 5.32$$

(Ii) For a series motor, $\phi \alpha I_a$ prior to saturation

$$\therefore \frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \times \frac{I_{a2}}{I_{a1}} \qquad 5.33$$

Where I_{a1} = initial armature current

I_{a2} = final armature current

5.12 Speed Regulation

The speed regulation of a motor is the change in speed from full-load (F.L) to no load (N.L) and is expressed as a percentage of the speed at full-load i.e.

$$\%\,\text{Speed Regulation} = \frac{N.L\ speed - F.L\ Speed}{F.L.\ speed} \times 100 \qquad 5.34$$

$$= \frac{N_0 - N}{N} \times 100 \qquad 5.35$$

Where N_0 = No load speed

N = full load speed

5.13 Losses in a dc Motor

The losses occurring in a dc motor are the same as in a dc generator. There are:

(i) Copper losses

(ii) Mechanical losses

(iii) Iron losses

As in a generator, these losses cause

(a) an increase of temperature

(b) reduction in efficiency of the d.c motor

5.14 Efficiency of a dc Motor

Like a d.c generator, the efficiency of a dc motor is the ratio of input power to the input power i.e.

$$\text{Efficiency, } \eta = \frac{output}{input} \times 100 = \frac{output}{output + losses} \times 100 \qquad 5.36$$

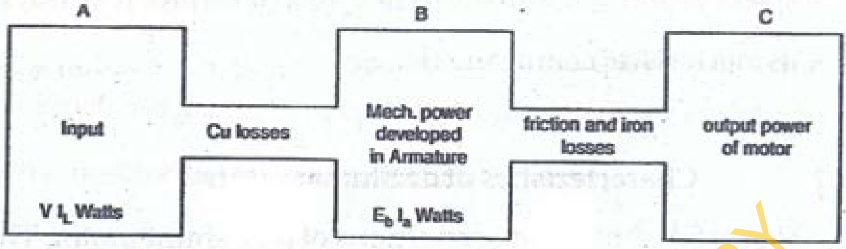
As for a generator, the efficiency of a d.c motor will be maximum when:

Variable losses = constant losses.

5.15 Power Stages

The power stages in a d.c motor are represented diagrammatically as shown in figure 5.7

A B = copper losses



B – C = Iron and friction losses

Figure 5.7: Power Stages in a d.c Motor

Overall efficiency, η_c = C/A

Electrical efficiency η_e = B/A

Mechanical efficiency η_m = C/B

5.16 DC Motor characteristics

The performance of a dc motor can be judged from its characteristic curves known as motor characteristics. Following are the three important characteristics of a dc motor:

I. Torque and Armature current characteristic (T_a/I_a): It is the curve between armature torque T_a and armature current I_a of a dc motor. It is also known as electrical characteristic of the motor.

II. Speed and armature current characteristic (N/I_a): It is the curve between speed N and armature current I_a of a dc motor. It is very important characteristic as it is often the deciding factor in the selection of the motor for a particular application.

III. Speed and torque characteristic (N/T_a): It is the curve between speed N and armature torque T_a of a dc motor. It is also known as mechanical characteristic.

5.16.1 Characteristics of dc Shunt motors

Figure 5.8 shows the connections of a dc shunt motor. The field current I_{sh} is constant since the field winding is directly connected to the supply voltage V which is assumed to be constant. Hence, the flux in a shunt motor is approximately constant.

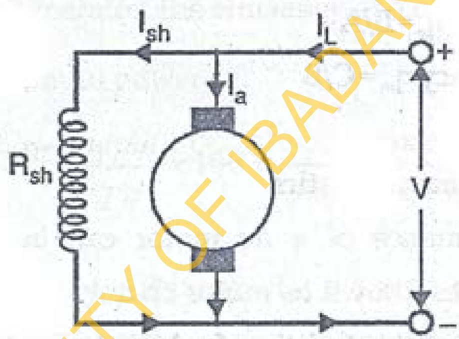


Figure 5.8: connections of a dc shunt motor

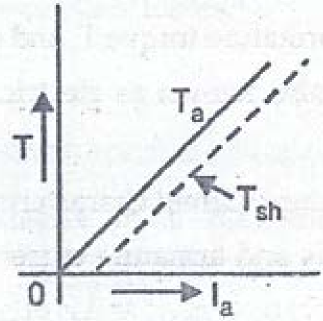


Figure 5.9: T_a/I_a Characteristic of a dc shunt motor

I. T_a/I_a **Characteristic:** In a dc motor,

$$T_a \propto \Phi I_a$$

Since the motor is operating from a constant supply voltage, flux Φ is constant (neglecting armature reaction).

$$T_a \propto I_a$$

Hence T_a/I_a characteristic is a straight-line passing through the origin as shown in Figure 5.9. The shaft torque (T_{sh}) is less than T_a and is shown by a dotted line. It's clear from the curve that a very large current is required to start a heavy load. Therefore, a shunt motor should not be started on heavy load.

II. N/I_a **Characteristic:** The speed N of a motor is given by:

$$N \propto \frac{E_b}{\Phi}$$

The flux Φ and back e.m.f E_b in a shunt motor are almost constant under normal conditions. Therefore, speed of a shunt motor will remain constant as the armature current varies (dotted line AB in Figure 5.10). When load is increase, E_b ($= V - I_a R_a$) and Φ decrease due to the armature resistance drop and armature reaction respectively. However E_b decreases slightly more than Φ so that the speed of the motor decreases slightly with load (line AC)

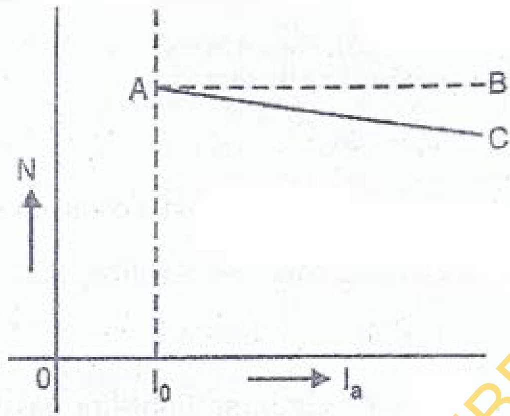


Figure 5.10: N /I_a Characteristic of a dc shunt motor

III. **N/T_a Characteristic:** This curve is obtained by plotting the values of N and T_a for various armature currents (Figure 5.11). It may be seen that speed falls somewhat as the load torque increases.

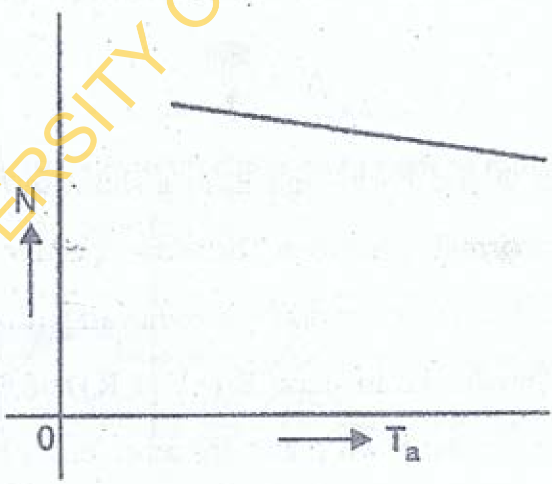


Figure 5.11: N /T_a Characteristic of a dc shunt motor

Conclusion: The following two important conclusions are drawn from the above characteristic:

a) There is slight change in the speed of a shunt motor from no load to full-load. Hence, it is essentially a constant-speed motor.

(ii) The starting torque is not high because $T_a \propto I_a$

5.16.2 Characteristics of dc Series Motors

Figure 5.12 shows the connections of a dc series motor. Note that current passing through the field winding is the same as that in the armature. If the mechanical load on the motor increases, the armature current also increases. Hence, the flux in a series motor increases with the increase in armature current and vice versa.

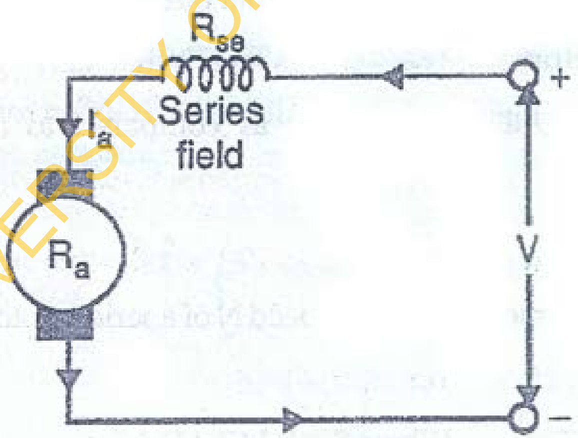


Figure 5.12: Connections of a dc Series Motor

I. T_a/I_a **Characteristic:** We know that:

$$T_a \propto \Phi I_a$$

Up to magnetic saturation, $\Phi \propto \Phi I_a$ so that $T_a \propto I^2_a$

After magnetic saturation, Φ is constant so that $T_a \propto I_a$

Thus up to magnetic saturation, the armature torque is directly proportional to the square of armature current. If I_a is doubled, T_a is almost quadrupled.

Therefore, T_a/I_a curve up to magnetic saturation is a parabola (portion OA of the curve in Figure 5.13). However, after magnetic saturation, torque is directly proportional to the armature current. Therefore, T_a/I_a curve after magnetic saturation is a straight line (portion AB of the curve).

It may be seen that in the initial portion of the curve (i.e. up to magnetic saturation), $T_a \propto I^2_a$ This means that starting torque of a dc series motor will be very high as compared to a shunt motor (where $T_a \propto I_a$)

II. N/I_a **Characteristic:** The speed N of a series motor is give by;

$$N \propto \frac{E_b}{\Phi} \qquad \text{Where } E_b = V I_a(R_a + R_{se})$$

When the armature current increases, the back e.m.f. E_b decreases due to $I_a(R_a+R_{se})$ drop while the flux increases. However, $I_a(R_a+R_{se})$ drop is quite small under normal conditions and may be neglected.

Therefore, $N \propto \frac{1}{\Phi}$

$$\propto \frac{1}{I_a} \quad \text{up to magnetic saturation}$$

Thus up to magnetic saturation, the N / I_a curve follows the hyperbolic path as shown in Figure 5.14. After saturation, the flux becomes constant and so does the speed.

III. **N/T_a Characteristic:** The N / T_a characteristic of a series motor is shown in Figure 5.14. It is clear that series motor develops high torque at low speed and vice-versa. It is because an increase in torque requires an increase in armature current, which is also the field current. The result is that flux is strengthened and hence the speed drops. Reverse happens should the torque be low.

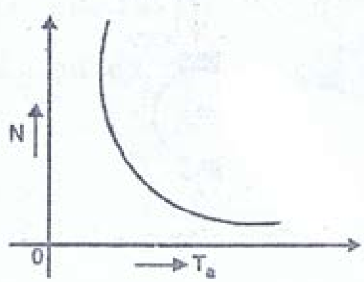


Figure 5.14: N /T_a Characteristic of a dc Series Motor

5.17 Characteristics of Compound Motors

A compound motor has both series and shunt field. The shunt field is always stronger than the series. Compound motors are of two types:

i. Cumulative-compound motors in which series field aids the shunt field.

ii. Differential compound motors in which series field opposes the shunt field.

Differential-compound motors are rarely used due to their poor torque characteristics at heavy loads.

5.18 Characteristics of Cumulative Compound Motors

Figure 5.15 shows the connections of a cumulative-compound motor. Each pole carries a series as well as shunt field winding; the series field aiding the shunt field.

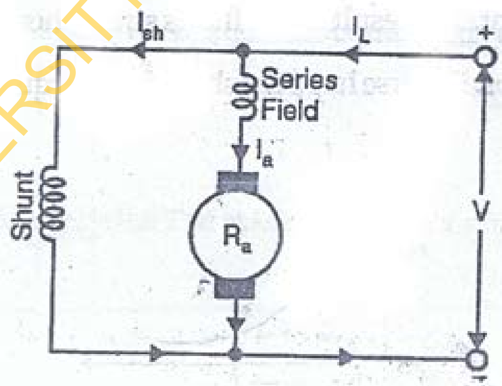


Figure 5.15 Connections of a Cumulative-Compound Motor

I. T/I_a Characteristic: As the load increases, the series field increases but shunt field strength remains constant. Consequently, total flux is increased and hence the armature torque. It may be noted that torque of a cumulative-compound motor is greater than that of shunt motor for a given armature current due to series field (Figure 5.16)

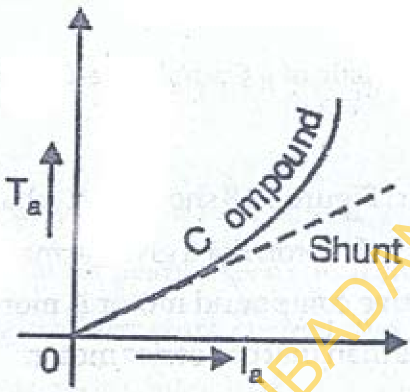


Figure 5.16: T/I_a Characteristic of a Cumulative-Compound Motor

II. N / I_a Characteristic: As earlier discussed, as the load increases, the flux per pole also increases. Consequently, the speed (N 1/Φ) of the motor falls as the load increases (Figure 5.17). It may be noted that as the load in added, the increased amount of flux causes the speed to decrease more than does the speed of the shunt motor. Thus the speed regulation of a cumulative compound motor is poorer than that of a shunt motor.

Note: Due to shunt field, the motor has a definite no load speed and can be operated safely at no load.

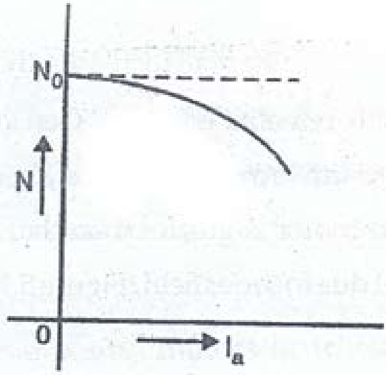


Figure 5.17: T/I$_a$ Characteristic of a Cumulative-Compound Motor

III. **N / T$_a$ Characteristic:** Figure 5.18 shows N/T$_a$ characteristics of a cumulative compound motor. For a given armature current, the torque of a cumulative compound motor is more than that of a shunt motor but less than that of a series motor.

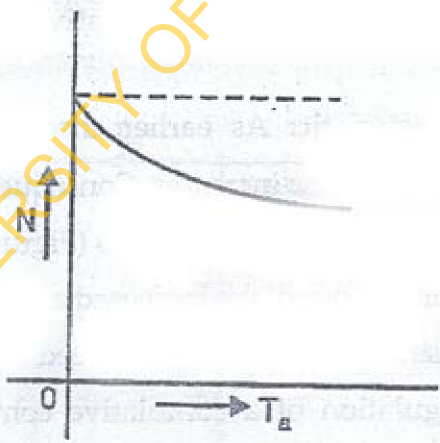


Figure 5.17: N/I$_a$ Characteristic of a Cumulative-Compound Motor

Conclusion: A cumulative compound motor has characteristics intermediate between series and shunt motors.

a) Due to the presence of shunt field, the motor is prevented from running away at no load.

b) Due to the presence of series field, the starting torque is increased.

5.19 Comparison of the Three Types of Motors

I. The speed regulation of a shunt motor is better than that of a series motor. However, speed regulation of a compound wound motor lies between shunt and series motor.

II. For a given armature current, the starting torque of a series motor is more than that of a shunt motor. However, the starting torque of a cumulative compound motor lies between series and shunt motor.

III. Both shunt and compound motors have definite no load speed. However, a series motor has dangerously high speed at no load.

5.20 Applications of DC Motors

Shunt Motors

Shunt Motors are used:

A) Where the speed is required to remain almost constant from no load to full load.

b) Where the load has to be driven at a number of speeds and one of which is required to remain nearly constant.

Industrial use: Lathes, drills, boring mills, sharpers, spinning and weaving machine

Services Motors

Series motors are used:

a) Where large starting torque is required.

b) Where the load is subjected to heavy fluctuations and the speed is automatically required to reduce at high torque and vice-versa.

Industrial use: Electric traction, cranes, elevators, air compressors, vacuum cleaners etc

Compound Motors

Compound motors are used where a fairly constant speed is required with irregular loads or a suddenly applied load e.g. presses, shares, reciprocating machines etc.

Example 5.1

A 220V d.c shunt motor takes a total current of 80A and runs at 800 r.p.m. Resistance of shunt field are 50Ω and that of armature 0.1Ω. The iron and friction losses amount to 1600W. Calculate:

i. The driving power of the motor.

ii. The lost torque.

Solution

i. $N=800$ r.p.m, $R_{sh}=50$?, $R_a=0.1$?

$I_{sh}=V/R_{sh}=220/50=4.4A$

$I_a=80\ 4.4$

$=75.6A$

$E_b=V-I_aR_a$

$=220\ 75.6\times0.1=212.44A$

Driving power$=E_bI_a=212.44\times75.6\ ^{-}16050W$

ii. Lost Torque $=9.55\times\frac{iron\ and\ \ friction\ losses}{N}$

$=\ 9.55\times\frac{1600}{800}=19\,Nm$

Example 5.2

A 220V series motor takes a current of 35A and runs at 500r.p.m. armature resistance is 0.25Ω and series field resistance is 0.3Ω. If iron and friction losses amount to 600W, calculate:

i. The armature torque.

ii. The shaft torque.

Solution

i. $R_m=R_a+R_{se}$

$R_m=0.25+0.3=0.55\Omega$

$E_b=V-I_aR_m$

$=200-35\times0.55=200.75V$

$$T_a = 9.55 \times \frac{E_b I_a}{N}$$

$$= 9.55 \times \frac{200.75 \times 35}{500} = 134 Nm$$

Ii. Lost Torque $= 9.55 \times \frac{\text{iron and friction losses}}{N}$

$$= 9.55 \times \frac{600}{500} = 11.5 Nm$$

Shaft Torque $T_{sh} = T_a$ - Lost Torque

$$= 134 \quad 11.5 = 122.5 Nm$$

Example 5.3

A 4-pole, 250V series motor has a wave connected armature with 1254 conductors. The flux per pole is 22mWb when the motor is taking 50A. Iron and friction losses amount to 1000W. Armature resistance=0.2Ω series field resistance=0.2Ω. What is the speed of the motor?

Solution

$$E_b = V - I_a(R_a + R_{se})$$

$$= 250 \quad 50(0.2 + 0.2) = 230V$$

$$E_b = \frac{\phi Z N}{60} \times \frac{P}{A}$$

$$230 = \frac{22 \times 10^{-3} \times 1254 \times N \times 4}{60 \times 2}$$

Example 5.4

A 220V d.c machine has an armature resistance of 1Ω. If the full load current is 20A, what is the difference in the induced voltage when the machine is running as a motor and as a generator?

Solution

As a motor: $E_1 = V - IR_a = 220 - 20\times1 = 220V$

As a generator: $E_2 = V + IR_a = 220 + 20\times1 = 240V$

Difference: $E_2 \; E_1 = 240 - 220 = 40V$

Example 5.5

A 4-pole 500V shunt motor has 720 wave connected conductors on its armature. The full load armature current is 60A and the flux per pole is 0.03Wb. The armature resistance is 0.2Ω and the contact drop per brush is 1V. Calculate:

i. The back e.m.f

Ii. The full load speed of the motor.

Solution

i. $E_b = V - IR_a$ brush drop

$E_b = 500 \; (60\times0.2) \; 2 = 486V$

ii. $E_b = \frac{\phi Z N}{60} \times \frac{P}{A}$

$$486 = \frac{0.03 \times 720 \times N}{60} \times \frac{4}{2}$$

$$N = 675 \text{ r.p.m}$$

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