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An Assignment Problem for n Machines and m Operators.

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Abstract

The assignment problem is a special type of linear programming problem which is concerned with allocating m operators (men) to n machines (tasks) given that each operator is qualified for certain of the tasks. The output from each task is given as a function of the number of qualified men assigned to it. The assignment model is to be solved by conventional linear programming approach or transportation model approach. It is a square matrix, having equal number of rows and columns. It enables the assignment of men to task and the objective is to assign one man from row to one task from column so as to maximize total output and minimize the total cost.

Keywords: Assignment Problem, Linear Programming, Operators, Task, Total Cost

Introduction

Assignment problem pertains to problem of assigning jobs to different machines. This model can be effectively used for any other problem in which m persons (or operators) are to be assigned to other n items so that each one of the first group is assigned to one distinct item from the second group. Assignment model can be solved by conventional linear programming approach or transportation model approach. It is a square matrix, having equal number of rows and columns. The objective is to assign one item from row to one item from column so that total cost of assignment is minimum. The assignment problem is concerned with an allocation technique to optimize a given objective. In linear programming we decide how to allocate limited resources over different activities so that, we maximize the profits or minimize the cost. Actually in assignment problem, the assignees are being assigned to perform different tasks. The assignees can be employees who need to be given work assignments, or they could be machines, vehicle, plants, time slots etc. to be assigned different tasks.

An algorithm for solving this general problem is given in which transfers like those used by Kuhn on the simple problem are selected using a node-labeling procedure on a related network. The algorithm yields for every k, $1 < k < m$, an optimal assignment of the first k men only, employing a single transfer to increase k by one.

In real life, we are faced with the problem of allocating different personnel/ workers to different jobs ([1] and [3]). Not everyone has the same ability to perform a given job. Different persons have different abilities to execute the same task and these different capabilities are expressed in terms of cost/profit/time involved in executing a given job. Therefore, we have to decide how to assign different workers to different jobs" so that the cost of performing such job is minimized [6] . They also stated that linear assignment problem requires the determination of an optimal permutation vector for the assignment of tasks to agents.

According to [2], some of the objectives of assignment problem include: the assignment of different jobs to different workers or different machines on one to one basis where time or cost of performing such job is given. The assignment of different personal to different location or service station with the objective to maximize sales/profit/consumer. The assignment of jobs to machines/workers so as to deal with a situation where number of jobs to be assigned do not match with number of machines/workers. The assignment of jobs to machines/workers so as to deal with a situation where some jobs can not be assigned to specific machines/workers

According to [4], a balanced or standard assignment problem refers to an assignment problem where numbers of rows are equal to number of columns. Consider a machine shop with 'n' jobs and 'm' machines, where number of jobs are equal to the number of machines i.e. n = m and this is known as balanced assignment problem.

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Unbalanced or non-standard assignment problem refers to an assignment problem where the numbers of rows are not equal to number of columns. Consider a machine shop with 'n' jobs and 'm' machines, where number of jobs is not equal to the number of machines i.e. 'n' is not equal to 'm'. This is known as unbalanced assignment problem.

Sahu and Tapadar, [5] generalized "Assignment problem" through genetic algorithm by basically using the "N men- N jobs" problem where a single job can be assigned to only one person in such a way that the overall cost of assignment is minimized. They solved the problem through genetic algorithm (GA), using a unique encoding scheme together with Partially Matched Crossover (PMX). The population size can also be varied in each iteration. In simulated annealing (SA) method, an exponential cooling schedule based on Newtonian cooling process is employed and experimentation is done on choosing the number of iterations (m) at each step [5].

Assumptions of an Assignment Problem:

An assignment problem should satisfy the following assumptions: (1) The number of assignees and number of task are the same (this number is denoted by n). (2) Each assignee is to be assigned to perform exactly one task. (3) Each task is to be performed by exactly one assignee. (4) There is a cost or profit associated with assignees performing different tasks. (5) The objective is to determine how all n assignments should be made to optimize the given pay offs which are expressed in terms of cost, time spent, distance, revenue earned, production obtained etc.

Mathematical Formulation Of Assignment Problem

Consider an assignment problem which consists of assigning m machines to n different operators as presented in Figure 1, where m equals 4 with 4 different operators. The model is a square matrix, having equal number of rows and columns. The objective is to assign one item from row to one item from column so that total cost of assignment is minimum.

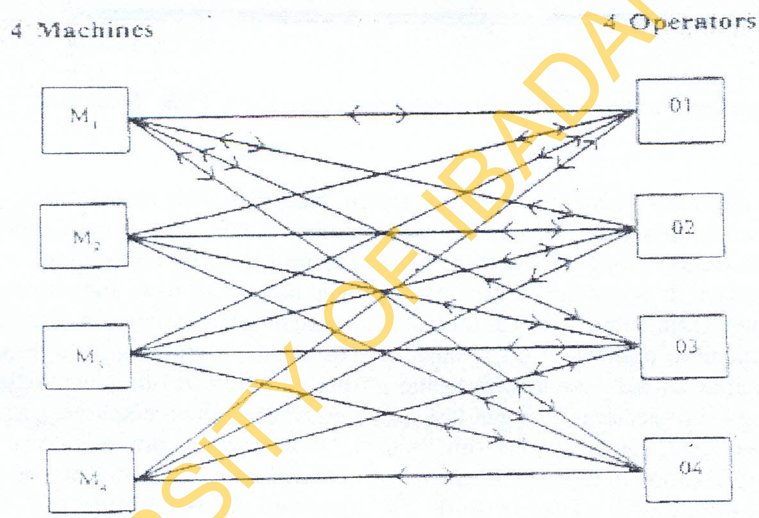

Fig. 1: Four Machines, Four Operators Assignment Problem

Let there be n jobs which are to be assigned to m operators so that one job is assigned to only one operator.

i = Index for job, $i = 1, 2, \dots n$
j = Index for operators, $j = 1, 2, \dots m$
C_{ij} = Unit cost for assigning job 'i' to operator 'j'
X_{ij} = 1 if job i is assigned to operator j
X_{ij} = 0 Otherwise

The objective is to minimize the total cost of assignment. If job I is assigned to operator 1, the cost is $(C_{11}X_{11})$. Similarly, for job 1, operator 2 the cost is $(C_{12}X_{12})$. The objective function is:

$$\text{Minimize} = \sum^{n}_{i=1} \sum^{n}_{j=1} C_{ij} X_{ij} \quad (1)$$

Since one job (i) can be assigned to any one of the operators, we have following constraint set:

$$\sum^{n}_{i=1} X_{ij} = 1; \text{ for all } j; j = 1, 2, \dots n \quad (2)$$

Similarly for each operator, there may be only one assignment of job. For this, the constraint set is:

$$\sum^{n}_{i=1} X_{ij} = 1; \text{ for all } i; i = 1, 2, \ldots n \quad (3)$$

The non-negativity constraint is:

$$X_{ij} \geq 0 \quad (4)$$

$$\text{Minimize } Z = \sum^{n}_{i=1} \sum^{n}_{j=1} C_{ij} X_{ij} \quad (5)$$

Subject to

$$\sum^{n}_{i=1} X_{ij} = 1; \text{ for all } j; j = 1, 2, \ldots n \quad (6)$$

$$\sum^{n}_{j=1} X_{ij} = 1; \text{ for all } i; i = 1, 2, \ldots n \quad (7)$$

$$X_{ij} \geq \text{ for all i and all j.} \quad (8)$$

Solution Method for Assignment Problem

The type of solution to assignment problem is obtained as presented the following flow chart as presented in Figure 2.

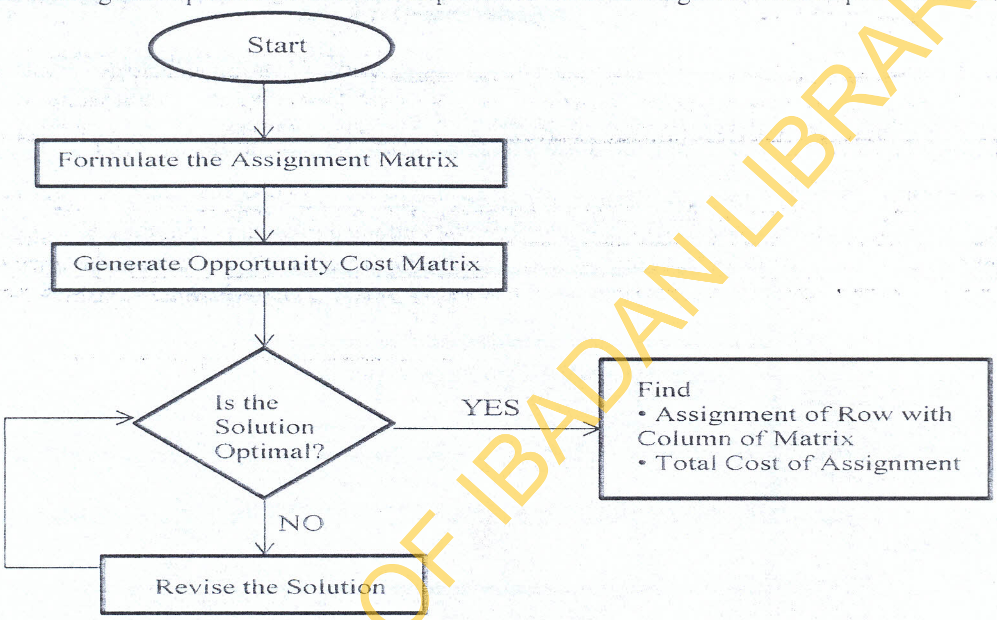

Fig. 2: Solution method of Assignment Problem

Algorithm to Solve Assignment Model

Let us understand it with an example. Let there be four machines and four operators. Operators I charges 6, 7, 7 and 8 units on machine I, II, III and IV respectively. Operator 2 charges 7, 8, 9 and 7 units, operator 3 charges 8, 6, 7 and 6 units and operator 4 charges 8, 7, 6 and 9 units respectively. The problem is to assign one operator on one machine so that over-all payment is least. The assignment model in the form of operator-machine matrix is shown in Figure 3. The entries in the matrix represent unit charge in naira per hour.

Operator \ ***Machine***	I	II	III	IV
1	6	7	7	8
2	7	8	9	7
3	8	6	7	6
4	8	7	6	9

Fig. 3: Representation of an Assignment Model

Opportunity Cost Approach

Opportunity cost is the cost ofpossible opportunity which is lost or surrendered. The given problem is related to assigning operators on machine for a least cost objective.

Consider that if operator 2 is assigned on machine I, it will cost ^ 7. With this, no other operators can be assigned machine I as one-to-one assignment is required. However, if operator 1 is assigned on machine I, it will cost ^6. Therefore, a potential saving of ^7 - ^6 = ^1 is possible, if instead of operator 2, operator 1 is assigned on machine 1. This is nothing but opportunity cost in case we assign operator 2 on machine 1. Similar logic may be put for opportunity cost of not assigning the least cost machine to an operator. So, to form a total opportunity cost matrix, we adopt a very simple two-step method.

Method to Find the Total Opportunity Cost Matrix

Step 1: Select any column. Subtract the lowest entry of this column from all the entries of this column and prepare a new column. Repeat this for all columns of the matrix and the result is presented in matrix in Figure 4 .

Operator \ Machine	I	II	III	IV
1	0	1	1	2
2	1	2	3	1
3	2	0	1	0
4	2	1	0	3

Fig. 4: Operator-Opportunity Matrix.

Step 2: Select any row of the revised matrix obtained in step 1. Subtract the lowest entry of this row from all the entries of this row. Prepare a fresh row.

Repeat this for rows of the revised matrix (operator-opportunity matrix). This would be the total opportunity cost matrix which is presented in Figure 5.

Operator \ Machine	I	II	III	IV
1	0	1	1	2
2	0	1	2	0
3	2	0	1	0
4	2	1	0	3

Fig. 5: Total Opportunity Cost Matrix

Optimality Test of Total Opportunity Cost Matrix

The optimality test of total opportunity cost matrix is performed through the following steps:

Step 3: Draw minimum number of possible horizontal and/or vertical lines so that all the zeros of the total opportunity cost matrix are covered.

If these lines are equal to the number of rows (or columns) then solution is optimal. Make assignment by following the steps as outlined in step 5.

If number of vertical and horizontal lines are less than number of rows, go to step 4, as the solution may be non-optimal.

Step 4: From the uncovered entries of step 3 (i.e., entries which are not struck by lines just drawn) select the lowest entry. Subtract this entry from all the entries of uncovered position.

Add this entry at intersection or junction points of lines just drawn. By junction point we mean entries where both horizontal and vertical lines meet.
Check for optimality as per step 3 and if optimal, go to step 5; otherwise repeat step 4.

Step 5: Optimal Assignment of the Matrix
Select row (or column), which has least-number of zeros (say, one zero). Note that all rows (or columns) will have at least one zero.
Make assignment of this row with corresponding column. Strike-off the already assigned row and column.
Now, select row and column which have minimum number of zeros. Make next assignment. Repeat the process till all rows are assigned to one column.

Illustration of Optimality Test and Assignment
The optimality test and assignment for the best result is conducted as follows:
Refer to Figure 5, we apply step 3 for the check of optimality. Draw minimum number of possible horizontal vertical lines to cover zeros. This can be done in no less than four lines to obtain the optimum assignment as presented in Figure 6.

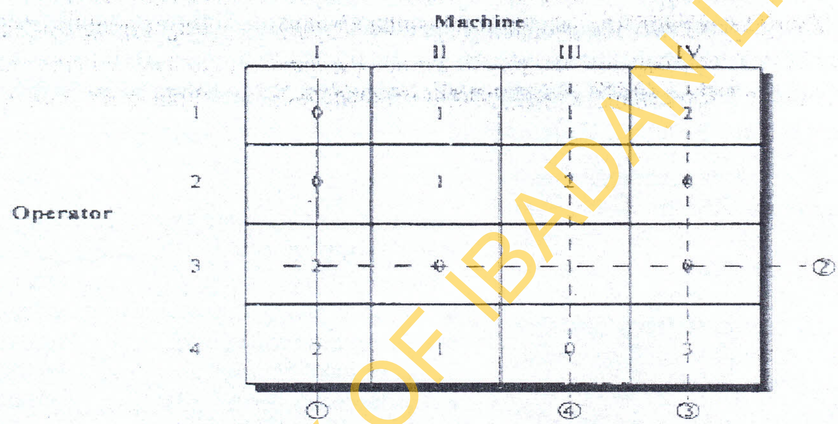

Fig. 6: Four lines needed to cover all zeros Fig 2

Therefore, the optimum assignment from Figure 6 (which is optimal) is carried out as follows::
Column II has only one zero, therefore, assign machine II to operator 3, while we remove column II and row 3.
From the remainder matrix, it is noticed that column III has only one zero, therefore, assign machine III to operator 4. We now remove row 4 and column III.
In the remainder matrix, only row 1 and 2 with column I and IV remain. In this case, column IV will have one zero at row 2. Therefore, assign machine IV to operator 2. The last assignment is the left-over machine 1 to operator 1.

Thus, the final assignment is:

Table 1: Optimum Assignment of Operators to Machines

Operator	Machine	Assign Operator To Machine	Unit Cost (^)
1	I	O1 ➡ MI	6
2	IV	O2 ➡ M2	7
3	II	O3 ➡ M3	6
4	III	O4 ➡ M4	6

Conclusion

The assignment problem as a special tool of linear programming problem has been successfully used to analysis a system of assignment involving the allocating of m operators to n machines given that each operator is qualified for certain of the tasks. In the allocation, a square matrix having equal number of rows and columns was developed to objectively assign the operators to the machines to get the possible minimum cost from the optimum assignment. The optimum assignment obtained allocate operator 1 to machine I, operator 2 to machine IV, operator 3 to machine II and operator 4 assigned to machine III.

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