

## OPTIMIZATION OF SINGLE-SOURCE MULTI-PRODUCTS MULTI-DESTINATION SUPPLY CHAIN SYSTEM

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### ABSTRACT

Optimal allocation of products to downstream locations is a major requirement for minimizing the distribution costs associated with supply chain systems. Unfortunately many supply chain managers rely on their intuition and feelings to make these allocation decisions. In this study a mathematical model was developed for minimizing the distribution cost in a multi-product 2-echelon supply chain system. The distribution system of a leading bottling plant in Nigeria was studied to understand the underlying supply chain system. Attempt was made to identify system parameters, variables, limitations, criteria so as to be able to define the distribution problem. The interactions and flow of products in the system were identified and characterized as a 2 echelon supply chain system. Mathematical model of the system was developed. The problem model, a linear program formulation with three major constraints; demand, availability and company policy requirements was parameterised based on demand data, product availability data, company policies and unit transportation costs to various downstream locations. The model was solved for a 12 product 8 destination case. It was observed that the model application produces 6% reduction in the distribution cost compared to the existing practice of the company. It is concluded that the model is effective to reduce or minimize distribution expenses for any multi-product multiple destination system and fulfilling demand at various destinations.

**Keywords:** Supply chain, Distribution, Optimisation, Bottling Company

### INTRODUCTION

A major challenge facing the manufacturing industries in today's business is how to consistently meet with customers' requirements and demands at the lowest possible cost in an increasingly competitive market. These require the optimisation of production process and the associated distribution operations. The optimisation of distribution operations however assumes greater importance in some industries such as the soft drink industry where distribution costs can take more than 70% of the value added costs of goods (Golden and Wasil, 1987).

Supply chain is the integration of key business processes across the supply chain for the purpose of adding value to customers and stakeholders (Lambert, 2008; Vidal and Goetschalckx, 1997). It is the combination of art and science that goes into the method of networking and supplier of input materials, retailers and customers, in the manner that will

ensure effective and coordinated flow of materials and information throughout the entire network (Slack, 1998, Beamon 1998). Supply chain exists in both service and manufacturing organization, although, the complexity of the chain varies from industry to industry. The design of a supply chain requires managers to determine the number, location, capacity, and type of manufacturing plants and warehouses to use; the set of suppliers to select; the transportation channels to use and quantities to ship among suppliers (Bilgen and Ozkarahan, 2004; Abdinnour-Helm, 1999). A great number of manufacturing companies or factories (single source multiple product factories) respond to diverse chain of product demand from customers or retailers thereby creating an intricate chain with far reaching effect on operation costs.

The gamut of decisions involved in the supply chain problem cut across the three levels of the planning function; the strategic, tactical, and operational level (Tony and Roy, 1989). Hence the



supply chain problem decision process cannot be effectively carried out based on the managers' intuition and experiences; the use of quantitative or decisions support tools has become imperative.

Many authors have addressed different faces of this problem. Chan and Chung (2005) developed an optimization algorithm to solve the problem of distribution in a given supply network, taking into account variables like demand allocation and production scheduling. They used a linear total cost function that has to be minimized; defining a genetic algorithm that first determines the demand allocation and transportation policy and secondly determines the production scheduling. Ambrosino and Scutellà (2004) studied the complex distribution network design problem that involves not only locating production plants and distribution warehouses, but also searching the best distribution strategy from plant to warehouses and from warehouses to customers minimizing the global costs given by the sum of six factors. These models do not address the variations and limitations on production capacities.

Axsater (2002) dealt with approximate optimization of reorder points for continuous review installation stock policies in a two-echelon distribution inventory system with stochastic demand, considering holding costs and shortage costs. The model does not consider the transport and handling (or replenishment) costs and assumes the delivery lead time as a constant.

Andersson and Marklund (1999) considered a system model that approximated holding costs and backlog costs with a stochastic lead time, and decomposing the problem with  $N$  retailers into an  $n + 1$  single-level problems. Also, Amiri (2004) defined an important strategic element as the best sites for intermediate stocking points, or warehouses introducing an MIP model that minimizes total costs on three different levels: costs to satisfy customers' demands from the warehouse, shipment costs from the plants to the warehouse, and costs associated with opening and operating both warehouses and the plant.

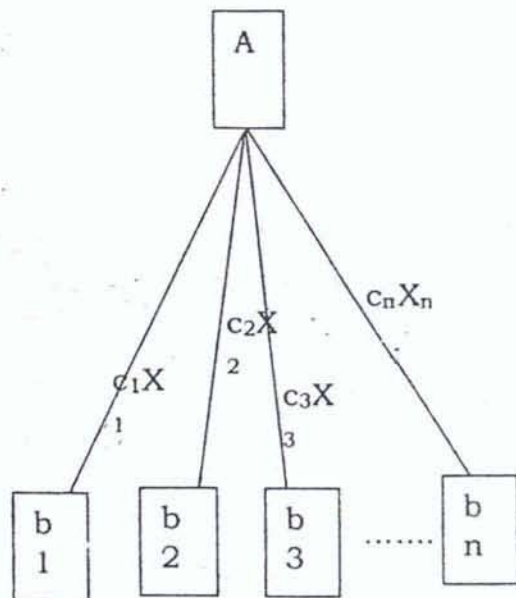
**PROBLEM DESCRIPTION AND MODEL DEVELOPMENT**

The product distribution networks are diverse and thus require different approaches of optimisation (Chan and Chung,2005; Axsater, 2002; Ambrosino and Scutellà, 2004). The general products distribution problem involves multiple products emanating from multiple sources going to multiple destinations over a

multi-echelon supply chain network (Bilgen and Ozkarahan,2003). However most real life situations in the literature are special cases of the general distribution problem; the multiple products from a single source to multiple destinations, the single product being shipped from multiple sources to multiple destinations, the problem of multiple products shipped from multiple sources to multiple destinations, and the problem of multiple products shipped from multiple sources to single destination are some examples. Based on number of sources, products and destinations, the general products distribution problem can be classified into 8 categories as summarised in Table 1. This study is focusing on number category no 7 which is further illustrated in Fig. 1.

**Table 1: Cases of Source-Product-Destination (SPD) Outlook**

s/n	Sources	Products	Destination
1	Single	Single	Single
2	Single	Single	Multiple
3	Single	Multiple	Single
4	Multiple	Single	Single
5	Multiple	Multiple	Single
6	Multiple	Single	Multiple
7	Single	Multiple	Multiple
8	Multiple	Multiple	Multiple



**Fig. 1: Shipping Multiple Products from a Single Source to Multiple Destinations**



**Notations and Terms Definition**

- m Total number of products to be distributed.
- n Number of destinations to be served
- i Index identifying products:  $i = 1, 2, \dots, m$
- j Index identifying destinations:  $j = 1, 2, 3, \dots, n$
- $A_i$  Quantity of product  $i$  available for distribution
- $D_{ij}$  Demand of product  $i$  at destination  $j$
- $b_j$  Fraction of demand  $D_{ij}$  that must
- $c_{ij}$  Unit cost of shipping product  $i$  to destination  $j$
- $X_{ij}$  The quantity of product  $i$  shipped to destination  $j$
- C Total distribution cost

**Problem Constraints**

The problem constraints and equations describing them were identified as follows:

Availability Limitation

In this model, one of the systems of constraints is related to product availability. This puts a limit on the total quantity of any product  $i$  that can be sent to all  $n$  destinations. If for each product  $i$ , the available quantity is  $A_i$ , then the availability constraint on  $i$  (for all  $i = 1, 2, \dots, m$ ) is shown in system of equation 1

$$\sum_{j=1}^n X_{ij} \leq A_i \quad \text{for all } i = 1, 2, \dots, m \quad \dots(1)$$

Demand Constraint:

This constraint requires that only what can be paid for by the depots is sent. This implies that not more than the respective depots demand  $D_{ij}$  should be sent. This is represented as system of equation 2

$$X_{ij} \leq D_{ij} \quad \text{for all } i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n, \dots(2)$$

Company policies

There are a number of company policies that guide the distribution of products. One of the critical policies is that there is a minimum quantity that must be sent to any given depot in order to keep customers loyalty. This is represented as the system of equation 3

$$X_{ij} \geq b_j D_{ij} \quad (0 \leq b_j \leq 1) \text{ for all } i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n, \dots(3)$$

Non-negativity constraint:

This is the variable type.

$$X_{ij} \geq 0 \quad (4)$$

The objective function

The objective is to minimize the total distribution cost.

$$Min C = \sum_{i=1}^m \sum_{j=1}^n C_{ij} X_{ij}$$

**The Model**

Aggregating all these set of equations result in the generic Linear Programme optimization model below:

$$Min C = \sum_{i=1}^m \sum_{j=1}^n C_{ij} X_{ij}$$

Subject to

$$\sum_{j=1}^n X_{ij} \leq A_i \quad \text{for all } i = 1, 2, \dots, m \quad \dots(1)$$

$$X_{ij} \leq D_{ij} \quad \text{for all } i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n, \dots(2)$$

$$X_{ij} \geq b_j D_{ij} \quad (0 \leq b_j \leq 1) \text{ for all } i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n, \dots(3)$$

$$X_{ij} \geq 0 \quad (4)$$

**APPLICATION AND RESULTS**

A major bottling plant in Ibadan produces 12 different bottled drinks which are distributed to 8 depots within South West region. Table 2 is the products availability and demands of depots, while Table 3 gives the decision on the distribution of these products.

Using the model above, the optimal solution is as shown in Table 4 with a total cost of N4, 073,410 as against the distribution cost of Table 3 (N4,318,445) where  $X_{11} = 1850$ ;  $X_{12} = 7,500$ ;  $X_{13} = 7,512$ ;  $X_{14} = 5,391$ ...  $X_{21} = 101$ ;  $X_{22} = 526$  etc

Table 2: Demand Matrix  $D_{ij}$

Product i	Destinations								Availability $A_i$
	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$D_6$	$D_7$	$D_8$	
1	1,850	1,005	222	24	114	5,587	372	552	8,235
2	7,636	5,263	9,760	2,056	4,627	5,256	6,020	4,919	21,544
3	7,512	485	175	1,494	1,129	5,708	4,084	2,337	18,617
4	8,290	163	703	11	96	348	834	419	6,608
5	960	90	361	346	200	386	998	410	6,240
6	780	275	755	278	102	411	78	348	5,565
7	1,152	228	989	137	287	431	658	311	1,596
8	5,130	2,296	1,841	1,437	2,154	3,805	2,875	2,565	8,543
9	2155	2060	2575	515	847	1607	963	1692	7,504
10	4254	28	41	373	625	3708	2015	308	8212
11	534	240	264	46	78	480	120	76	1536
12	35	8	16	3	10	48	9	9	1,008
Unit cost	35	70	70	60	55	55	45	70	
Policy	0.1	0.1	0.2	0.2	0.3	0.4	0.7	0.9	

Table 3: Quantity of Products Distributed to destination

	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$D_6$	$D_7$	$D_8$
1	845	1,005	22	2	114	5277	372	497
2	2,373	5,263	976	411	1388	2102	4214	4291
3	7,027	485	18	299	1129	3423	4084	2103
4	5,605	163	70	2	29	139	584	0
5	375	90	48	346	60	154	0	410
6	116	275	76	100	31	120	0	55
7	500	228	249	27	86	172	0	311
8	1,365	2,296	184	287	646	1522	2013	0
9	95	2,060	258	103	847	1449	963	1523
10	4,226	28	4	75	188	1483	1928	277
11	0	240	145	9	23	192	84	0
12	4	8	2	1	3	19	6	0
<b>Total cost of distribution</b>						<b>4,318,445</b>		

Table 4: Optimal distribution plan

Product i	Destinations								Availability $A_i$
	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$D_6$	$D_7$	$D_8$	
1	1,850	101	22	2	144	5277	372	497	8,235
2	7500	526	976	411	1388	2102	4214	4427	21,544
3	7512	49	18	299	1129	3423	4084	2103	18,617
4	5391	16	70	2	29	139	584	377	6,608
5	960	90	361	346	200	386	998	410	6,240
6	780	275	755	278	102	411	78	348	5,565
7	448	23	99	27	86	172	461	280	1,596
8	1352	230	184	287	646	1522	2013	2309	8,543
9	2155	206	258	103	847	1449	963	1523	7,504
10	4254	3	4	75	188	1483	1928	277	8212
11	534	240	264	46	78	480	120	76	1536
12	35	8	16	3	10	48	9	9	1,008



**CONCLUSIONS AND RECOMMENDATIONS**

This paper models the problem of minimizing the cost of distributing multiple products from a single source to multi-destinations. This outbound supply chain network in a two echelon supply chain system has practical applications in many production/inventory systems, such as bottling companies, fruit juice companies, foods and Pharmaceutical industries, where demand of their multiple products are required at various distribution depots. The effectiveness of the model was demonstrated with a real life problem where a cost reduction of about 6% or absolute sum of N244, 815 in a one week planning horizon, which translates to over N12, 000,000 savings per annum, was achieved.

However, the assumption of linearity of the shipping cost is a major limitation of the model hence further work based on some other realistic cost functions should be considered. a multiple-source multiple-product to multiple-distribution should also be considered. Also, while the model will be effective in managing the two echelon supply chain system this work will require further extensions in order to effectively handle a three or more echelon supply chain systems

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