# Mathematical Modelling of the Road Bumps Using Laplace Transform

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**Abstract:** The contribution of this paper to traffic engineering is the application of Laplace Transform to the quantification of speed control in the modelling of road bumps with hollow rectangular shape. The paper simulates the behaviour of a vehicle when passing over a series of bumps. In many countries the current practice used for lowering the vehicle speed is to raise road bumps above the road surface. If a hollow bump is used it may be economical and offers other advantages over road bumps raised above the road surfaces. The method models the vehicle as the classical one-degree-of-freedom system whose base follows the road profile, approximated by Laplace Transform. Then, a traditional vibration analysis is carried out and the isolation factor is calculated. A case study application is presented to substantiate the model developed. This case indicates how much difference the current model has compared to other existing models. Therefore, a relationship is established between the characteristics of the road profile, and the vehicle vibration response. Thus, the model's impact on the vehicle speed control is specified.

**Keywords:** Laplace Transform, traffic engineering, vibration analysis, speed limit, accidents prevention.

## Laplace Dönüşümü Kullanılarak Yol Çarpışmalarının Matematiksel Modellenmesi

**Özet:** Bu makalenin trafik mühendisliğine katkısı çukur dikdörtgen biçimindeki şekille yol çarpışmalarının modelinde hız kontrolünün niceline Laplace dönüşümünün bir uygulamasıdır. Makale bir seri çarpışmalar geçtiğinde bir taşıtın davranışları gibi görünüyor. Birçok ülkede ataç hızını azaltmak için kullanılan şu anki uygulama yol yüzeylerinin üstünde yol çarpışmalarını artırmaktadır. Eğer bir çukur çarpışması kullanılırsa, ekonomik olabilir ve yol yüzeyinin üstünde yol çarpışmalarını artırmasından öte diğer avantajlar önerir. Tabanı yol profilini takip eden birinci dereceden bağımsız sistem klasik olarak bir aracın metot modellerine Laplace dönüşümü ile yaklaştı. O zaman, geleneksek bir titreşim analizi yapıldı ve izolasyon faktörü hesaplandı. Bu durum çalışmasının uygulaması model gelişimini kanıtlamak için sunulur. Bu durum diğer var olan modellerle karşılaştırıldığında son modelin ne kadar farklı olduğunu belirtir. Bundan başka, yol profilinin karakteristikleri ve taşıt titreşimlerinin karşılığı arasında bir ilişki kuruldu. Böylece, taşıt hız kontrolü üzerinde modellerin çarpışması kesinlikle belirtildi.

**Anahtar Kelimeler:** Laplace Dönüşümü, trafik mühendisliği, titreşim analizi, hız limiti, kazaların önlenmesi

## 1. Introduction

The introduction of road bumps in road networks has important implications for traffic safety [1-3]. The main purpose of road bumps is traffic calming as a physical method of traffic control [4]. They are the most effective measures used to lower the vehicle speed, encouraging motorists to drive carefully and to reduce accident numbers and severity [5,6]. The aim of road bumps is to break the speed of vehicles (especially the fast ones) in order to prevent them from going beyond a certain limit in some areas [7]. Another goal is to ultimately reduce the rate of accidents usually caused by over-speeding. In sum, the development and implementation of road bump models allows

control of vehicular speed in different locations. It provides information on the effects of road bumps on vehicular systems, and budgetary information for construction and maintenance purposes. Road bumps are varied in geometry (Figure 1), and are usually a few meters apart in some communities. They have a minimum distance of about 10 meters and a maximum distance of about 100 meters where other ranges come in-between.
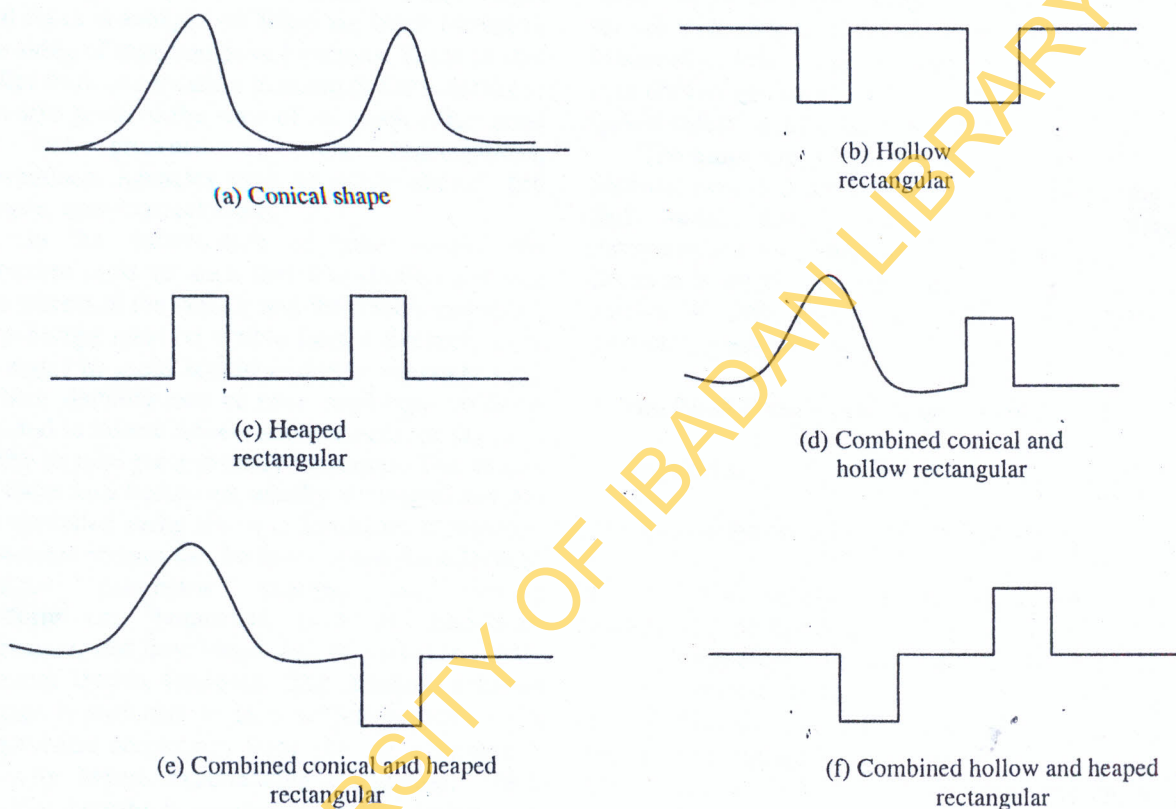


(a) Conical shape

(b) Hollow rectangular

(c) Heaped rectangular

(d) Combined conical and hollow rectangular

(e) Combined conical and heaped rectangular

(f) Combined hollow and heaped rectangular

**Figure 1:** Various geometries of road bumps

The geometries of road bumps are classified according to the shapes roads bumps in two consecutive positions. To the best of the author's knowledge, the following possible classifications are available: (i) conical shape; (ii) hollow rectangular; (iii) heaped rectangular; (iv) combined conical and heaped rectangular; (v) combined conical and hollow rectangular; and (vi) combined hollow and heaped rectangular. The literature has only accounted for the conical shaped geometry. Even at this, only Fourier series has been applied as an approximate function to evaluate the effective distance between two consecutive roads bumps. Thus, an opportunity to utilise other functions such as Laplace Transform on all possible geometries is created. With thus, it would be possible to compare results. However, in order to avoid complexity of analysis, only the

hollow rectangular geometry is treated here. The result of this is then compared to those on conical shape, which is available in the literature.

Depending on the density of traffic, types of vehicles expected to ply route (i.e. heavy duty vehicles, cars, lorries, buses, etc.) the minimum and safest distance between bumps can be determined [8,9]. Other factors include: the type of environment (i.e. residential neighbourhood, commercial centers), population density, availability of infrastructure (such as traffic light and zebra crossing). All these are input factors in the siting of road bumps in locations. These factors differ from one location to another. Considerations are also given to the state of the road, either good or bad, presence of some organisations, government agencies such as police station, fire station, army barracks, etc.

In the construction of road bumps, the structure must be such that low friction between the wheels of the vehicle and the road is permitted. The bumps must be visible from a distance (even at night) to avoid accidents due to oversight [10]. Where visibility may be poor, road signs could be erected to inform drivers about the bumps ahead in order to take precautionary measures. The shapes of these road bumps are usually sinusoidal and can be modelled using the sine functions. Computer-generated images can be made to see the effects of various parametric changes on vehicle performance. Parameters such as amplitude, curvature, and base length can be varied to obtain optimal system functions. The distance between bumps is such that when a vehicle passes over it the vehicle accelerates from zero to a maximum velocity before experiencing retardation, which enables the vehicle to safely pass over the bump. It is assumed that the vehicle has sufficient shock absorbing device to minimize the shock [11]. The altitude should not be more than 1 inch. Some road bumps are constructed with a flat surface having a lengthy base while others have short bases. Flat surface road bumps have an advantage in not being easily affected by wear at the top surface unlike the curved surface bumps, which are prone to erosion especially at the top [12].

Research on road bumps in recent times has been empirical. The growing research on road bumps initially focused on the development of conical-shaped road bumps. The current authors are concerned with road bumps research with the use of Laplace Transform since no reported cases have been documented on this. Although Fourier series presents an interesting research method in road bump design, Laplace Transform may be a more exciting tool for mathematical analysis of the road bump problem. The current work presents a mathematical model for a hollow rectangular bump with a particular design that could be used for effective control of vehicle over-speeding [13]. Practically, this work develops a new set of quantitative measures that would be valuable to typical safety professionals and researchers.

The following is a summary of the succeeding sections. Section 2 presents the necessary notations and model description. In section 3 the determination of isolation factor and effective distance is made. The case study is introduced in section 4 with results illustrated numerically. Section 5 concludes on the study.

## 2. The Road Bump-Vehicle Interaction Model

### 2.1 Notations

The following notations are used in the model:

$v$ — linear velocity at which vehicles are moving over bumps

$f$ — frequency at which vehicles are moving over bumps

$\lambda$ — distance between two consecutive points on road (wavelength)

$T$ — period, which is the time taken to complete a cycle

$h(t)$ — height of undulation of the bump at a particular time

$t_n$ — time of oscillations

$x$ — vertical displacement of body due to undulation

$k$ — spring constant (stiffness)

$I_f$ — Isolation factor

$cx = \gamma$ — damping force

$kx$ — spring force

$d$ — linear displacement

$L$ — general Laplace Transform notation

$\omega^2$ — natural frequency (rad/s or l/s)

$s$ — complex Laplace operator

S,J,n, and H    constants
Q    Laplace operator

## 2.2 Model development

In considering the pictorial representation of the road bump shown below, a clear understanding of the road bump geometric layout gives us some insight into the governing equations for the road bump designs which will further point out its interaction with the mechanical property of the vehicle suspension system. The geometry discussed in this work is a variance of existing studies on road bumps in terms of bump geometry.
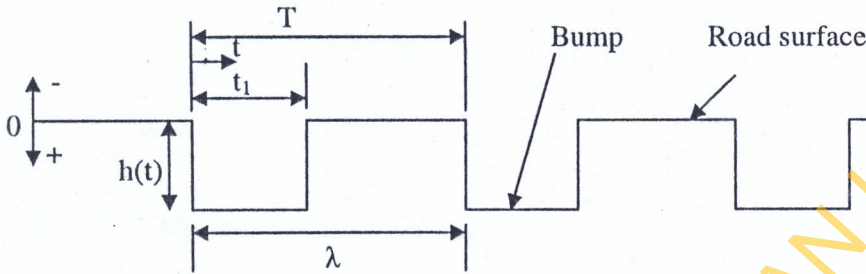


**Figure 2:** The road bump geometric shape (hollow rectangular)

From figure 2, a number of observations could be made. Firstly, movement of vehicles is considered over and below the ground surface. Consideration is given for movement below the ground surface due to the hollow design of the road bump that makes the tyre movement below the road surface. Secondly, a complete motion of the vehicle is divided into two stages. Stage 1 considers motion of the vehicle over the road bump while the second stage visualises motions of vehicle in-between two consecutive road bumps. From the above consideration, the development of the governing equations for this road bump problem presents a challenging mathematical debate. Therefore, in analysing the periodicity of the model, we apply the mathematical tool of Laplace Transform. For the first stage, $f_1(t) = h$, where t ranges from 0 to $t_1$. In considering the second stage, $f_2(t) = 0$, where $t_1 < t < T$.

We then have
$$L[F(t)] = \frac{1}{1-e^{-\alpha S}}\left[\int_0^{\infty} e^{-st}.F(t)dt\right] \tag{1}$$

where T is considered as the period of motion, which is the angular velocity. Further analysis of equation (1) gives

$$L[F(t)] = \frac{1}{1-e^{-TS}}\left[\int_0^{t_1} e^{-st}hdt + \int_{t_1}^{T} e^{-st}.0dt\right] = \frac{h\left(1-e^{-st_1}\right)}{s\left(1-e^{-TS}\right)} \tag{2}$$

This equation (1) represents the Laplace Transforms of the periodicity of the model. It is therefore used in the determination of the inverse of the Laplace Transforms of equation (2).

$$L^{-1}\left[\frac{h\left(1-e^{-st_1}\right)}{s\left(1-e^{-TS}\right)}\right] = f(t) = \frac{h}{s}\left(1 + e^{-TS} + e^{-2TS} + e^{-3TS} + \ldots - e^{-st_1} - e^{-(T+t_1)S} - e^{-(2T+t_1)S} - e^{-(3T+t_1)S}\right) \quad (3)$$

$$L^{-1}\left[\frac{h\left(1-e^{-st_1}\right)}{s\left(1-e^{-TS}\right)}\right] = \frac{h}{s}\left[Q - e^{-(L)S}\right] = \frac{h}{S}\left[\sum_{n=0}^{\infty}Q - e^{-(L)S}\right] = h\left[\sum_{n=0}^{\infty}\frac{Q}{S} - \frac{e^{-(L)S}}{S}\right]$$

$$= h\left[\sum_{n=0}^{\infty}\left(H(J) - H(t - nT - t_1)\right)\right] = L^{-1}(F(s)) \quad (4)$$

Equation (4) is the result of the inverse of Laplace Transform. Now, a relationship between the vehicular body suspension spring of the vehicle and the damper (shock absorber) could be established. Here, vibration analysis of the problem is described as illustrated by the diagram in Figure (3).
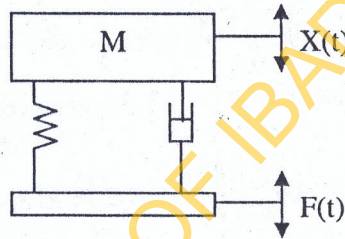


**Figure 3:** Moving support problem

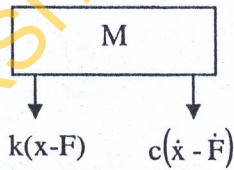We could develop a free body diagram for the system as shown in Figure (4).



Figure 4: Free body diagram for vehicle - road interaction system

When the mechanical property of the free body diagram was analysed, equation (5) was obtained so that

$$-k(x - F) - c\left(\dot{x} - \dot{F}\right) = m\ddot{x} \quad (5)$$

By expanding equation (5) and re-arranging, we obtain

$$m\ddot{x} + c\dot{x} + kx = c\dot{F} + kF \quad (6)$$

This shows the relationship between damping force, cx, and the spring force, kx. From equation (2),

$$L[F(t)] = \frac{h}{s}\left(1 - e^{-st_1}\right)\left(1 - e^{-TS}\right) = F(s) = \frac{h}{s}\left[\sum_{n=0}^{\infty}Q - e^{-(L)}\right] \quad (7)$$

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$$L[F'(t)] = S\overline{F} - F_o, \qquad F_o = F(o)$$
$$L[F(t)] = \overline{F}, \qquad L[F(t)] = \overline{F}$$
$$\dot{F} = F^I$$
$$L[F'(t)] = h\left[\sum_{n=0}^{\infty} Q - e^{-(L)}\right] - F_o \qquad (8)$$

To know the value of $F_o$, it was stated in the assumption that the road is smooth and that the only major source of vibration is bump. Therefore, at t = 0, F(0) = 0 because there is no displacement yet.

$$L[F'(t)] = h\left[\sum_{n=0}^{\infty} Q - e^{-(L)}\right]$$

Substituting for $\dot{F}$ and F in equation of motion, dividing through by m, and putting $\omega^2 = k/m_1$ $\gamma = c/2m$, we have $\ddot{x} + 2\gamma\dot{x} + \omega^2 x = \left(2\gamma h + \dfrac{\omega^2 h}{2}\right)\sum_{n=0}^{\infty}\left(Q - e^{-(L)S}\right)$ (9)

Note that $\omega^2$ is called natural frequency and $\gamma$ is called the damping factor. But $L[\ddot{x}] = S^2\overline{x} - Sx_0 - x_1$, $L[\dot{x}] = S\overline{x} - x_0$, and $L[x] = \overline{x}$. Then substituting in equation (9) to get

$$\left(S^2\overline{x} - Sx_0 - x_1\right) + 2\gamma\left(S\overline{x} - x_0\right) + \omega^2\overline{x} = \left(2\gamma h + \frac{\omega^2 h}{s}\right)\sum_{n=0}^{\infty}\left(Q - e^{-(L)S}\right) \qquad (10)$$

From the boundary conditions of the system at t = 0, $x_{(0)} = x_0 = 0$, and $x'_{(0)} = x'_{(0)} = 0$

Then, $\left(S^2\overline{x} - Sx_0 - x_1\right) + 2\gamma\left(S\overline{x} - \dot{x}_0\right) + \omega^2\overline{x} = \left(2\gamma h + \dfrac{\omega^2 h}{s}\right)\sum_{n=0}^{\infty}\left(Q - e^{-(L)S}\right)$

Knowing that $x_0 = 0$, and $x_1 = 0$, we may substitute into the above equation to obtain $\left(S^2\overline{x} - 0 - 0\right) + 2\gamma\left(S\overline{x} - x_0\right) + \omega^2\overline{x} = \left(2\gamma h + \dfrac{\omega^2 h}{s}\right)\sum_{n=0}^{\infty}\left(Q - e^{-(L)S}\right)$. This reduces to

$$S^2\overline{x} + 2\gamma S\overline{x} + \omega^2\overline{x} = \left(2\gamma h + \frac{\omega^2 h}{s}\right)\sum_{n=0}^{\infty}\left(Q - e^{-(L)S}\right) \qquad (11)$$

Thus, we have eliminated the vertical displacement of body due to undulation, x. It means that

$$\overline{x} = 2\gamma h \sum_{n=0}^{\infty}\left(\frac{Q}{M+N} - \frac{e^{-(L)S}}{M+N}\right) + \omega^2 h \sum_{n=0}^{\infty}\left(BQ - Be^{-(L)S}\right) \qquad (12)$$

Equation (12) gives an expression for $\bar{x}$ where, $B = \dfrac{1}{[M+N]}$. By using partial fraction method to split the equation, we have $B = \dfrac{1}{S[M+N]} = \dfrac{c}{s} + \dfrac{D_s + E}{M+N}$. This could further be stated as: $B = \dfrac{1}{\omega^2 S} -$

$$\dfrac{\dfrac{s}{\omega^2}}{M+N} - \dfrac{\dfrac{2\gamma}{\omega^2}}{M+N} \tag{13}$$

If the angular velocity is equal to the distance between two consecutive points on the road, that is $\omega = \gamma$, and the square of the angular velocity equal to the square of the distance between two consecutive points on the road (wavelength) ($\omega^2 = \gamma^2$), it implies that there is no oscillation in the system. Therefore, the system

frequency is equal to zero. However, $f = 0$, and $f = \dfrac{1}{T}$. The expression between $f$ and $T$ could be written as $T = \dfrac{1}{f}$. Therefore, at $f = 0$, $T = \infty$, then,

$$\bar{x} = 2\gamma h \sum_{n=0}^{\infty} \left( \dfrac{Q - e^{-(L)s}}{M+N} \right) + \omega^2 h \sum_{n=0}^{\infty} \left( GQ - Ge^{-(L)s} \right) \tag{14}$$

When $T = \infty$, $\bar{x} = 2\gamma h \sum_{n=0}^{\infty} \left( \dfrac{e^{-\infty}}{M} - \dfrac{e^{-\infty}}{N} \right) + \omega^2 h \sum_{n=0}^{\infty} \left( Ge^{-\infty} - Ge^{-\infty} \right) \tag{15}$

However, $\bar{x} = 0$ as earlier shown. Therefore, if angular velocity is less than wavelength, it implies that the square of angular velocity is less than the square of wavelength. Mathematically, If $\omega < \gamma$, then $\omega^2 < \gamma^2$. It means that the system is overdamped which resulted in no oscillation of the

system. It implies that the frequency at which vehicles are moving over bumps is equal to zero, then, period, $T = \infty$ since $T = \dfrac{1}{f}$. However, $\bar{x} = 0$, and if angular velocity is greater than the wavelength, it implies that $\omega^2 - \gamma^2 > 0$. Now, let

$\omega^2 - \gamma^2 = a^2$. Then from equation (15), $\bar{x} = 2\gamma h \sum_{n=0}^{\infty} \left( \dfrac{Q - e^{-(L)s}}{M+N} \right) + \omega^2 h \sum_{n=0}^{\infty} \left( BQ - Be^{-(L)s} \right)$ (16)

where, $B = \dfrac{1}{S[M+N]}$. Therefore $\bar{x} = 2\gamma h \sum_{n=0}^{\infty} \left( \dfrac{Q - e^{-LS}}{M+N} \right) + \omega^2 h \sum_{n=0}^{\infty} \left( DQ - De^{-LS} \right)$ (17)

where, $D = \dfrac{1}{S[P]}$. By using partial fraction for $D = \dfrac{1}{S[P]} = \dfrac{E}{S} + \dfrac{GS+N}{P}$, we obtain

$D = \dfrac{1}{B} - \dfrac{S}{B(P)} - \dfrac{2\gamma}{B(P)}$ . Therefore,

$$\bar{x} = \frac{2h\gamma}{a} \sum_{n=0}^{\infty}\left(\frac{aQ}{P} - \frac{ae^{-LS}}{P}\right) + \frac{\omega^2 h}{B}\sum_{n=0}^{\infty}\left(\frac{Q}{S} - \frac{SQ}{P} - \frac{2\gamma Q}{P} + \frac{e^{-L)S}}{S} - \frac{Se^{-LS}}{P} - \frac{2\gamma e^{-LS}}{P}\right) \quad . \quad (18)$$

Converting the equation above from Laplace Transform function of f(s) to function of f(t) by using Laplace inverse. For simplicity, let $Q = e^{-nTS}$ and $P = (S+\gamma)^2 + a^2$, then:

$$\bar{x} = \frac{2h\gamma}{a} \sum_{n=0}^{\infty}\left(\frac{aQ}{P} - \frac{ae^{-LS}}{P}\right) + \frac{\omega^2 h}{B}\sum_{n=0}^{\infty}\left(\frac{Q}{S} - \frac{(S-\gamma)e^{LS}}{P} - \frac{3\gamma Q}{P} + \frac{e^{-LS}}{S} - \frac{(S-\gamma)e^{-LS}}{P} - \frac{3\gamma e^{-LS}}{P}\right) \quad (19)$$

By further simplification, let $J = (t-nT)$ and $K = (T-nT-t_1)$

$$\bar{x} = \left(\begin{array}{l}\dfrac{2h\gamma}{a}\sum\limits_{n=0}^{\infty}\left(HJ\,e^{-\gamma J}\,SinaJ - HK\,e^{-\gamma K}\,SinaK\right) + \\[2mm] \dfrac{\omega^2 h}{B}\sum\limits_{n=0}^{\infty}\left(HJ\,(1-e^{-\gamma J}\,CosaJ - 3HJ\,e^{-\gamma J}) + HK\,(1-e^{-\gamma K}\,CosaK - 3HK)\right)\end{array}\right) \quad (20)$$

However it has been defined earlier that T is the period of the motion and $T = \dfrac{1}{f}$. From the Figure 2, it can be stated that $v = \dfrac{d}{t_1}$, which could be re-written as $d = vt_1$ or $t_1 = d/v$, where d is the length of the rectangular bump in road direction. The assumption is that v is constant. Therefore the displacement equation of the system can now be re-written as:

$$x = \left(\begin{array}{l}\dfrac{2h\gamma}{a}\sum\limits_{n=0}^{\infty}\left(HZ\,e^{-\gamma Z}\,SinaZ - JY\,e^{-\gamma Y}\,SinaY\right) + \\[2mm] \dfrac{\omega^2 h}{B}\sum\limits_{n=0}^{\infty}\left(HZ\,(1-e^{-\gamma Z}\,CosaZ - 3e^{-\gamma Z}) + HY\,(1-e^{-\gamma Y}\,CosaY - 3e^{-\gamma Y}\,SinaJ)\right)\end{array}\right) \quad (21)$$

The equation (21) above represents the displacement response of vehicular system as it passes over road with bumps.

## 3. Determination of Isolation Factor and Effective Distances

The literature has determined effective distances between two consecutive on roads with bumps using the relationship between velocity, frequency, and wavelength [3]. The idea is to measure the liner velocity at which vehicle move over bumps, then derive the distance (equivalence of wavelength) form the expression $V = f\lambda$. This simple relationship has been extended in the literature to capture more variables [4]. The frequency used in the calculation of effective distance between two consecutive bumps is usually determined from the values of Isolation Factor. Usually. It is practically accepted that in order to determine what frequency is best for vehicles to travel over road bumps a graph should be constructed from which corresponding values

frequency and isolation factors could be interpreted. It is generally accepted that a isolation factor of 80% and above is considered excellent.

One problem with reading and interpreting the values of isolation from graphs is the limitation of inability to alter the component variables of isolation factor to desired values. The motivation to have control over the generation of isolation

factor has party motivated this study. Thus, an extension to the literature sought have is to reformulate the isolation factor. A relation factors, system displacement, and road bump displacement. In the reformulation, it is acknowledged. That ride harshness isolation depends greatly on the stuffiness of the tyres, the sprung mass, loose or damaged components the friction in the suspension depends moderately on the unsprung mass and dumping characteristics of the tyres. Ride harshness isolation is a measurement of the percentage of the road disturbance input that is dissipated by the suspension system. The function, which described the bump, is a function of distance. This is obtained by converting the true and velocity of motion of vehicle to distance. The assumption here is that the velocity is constant. This is true at the instance that the vehicle approaches road-bump. At this point, the vehicle rides over the road bumps with a relatively how but constant speed. However, for some vehicles, the velocity of motion of vehicle is variable. The formulation of mathematical model to represent this is beyond the scope of this paper.

In the determination of isolation factors, a relationship is established between system displacement and road bump displacement. The expression for the former is stated in equation (21) while that of the later is in equation (3). Thus, we have the mathematical statement for isolation. Factors as: $I_f = \dfrac{\text{System}}{\text{Road}} = \dfrac{x(t)}{f(t)}$

(22)

However, we need to simplify this expression (22) to make it easy to understand. The changes sought in the above expression are to express T as $\dfrac{1}{f}$, $t_1$ as $\dfrac{d}{V}$. Also, Y is expressed as $\left(t\dfrac{-n}{f}\dfrac{-d}{V}\right)$. and Z = $\left(t\dfrac{-n}{f}\right)$. In summary, $f(t) = h\left(\displaystyle\sum_{n=0}^{\omega}\right) = h\left(\displaystyle\sum_{n=0}^{\omega}(HZ - HY)\right)$.

Thus, $I_f = \dfrac{\text{System displacement}}{\text{Road bumps displacement}} = \dfrac{x(t)}{f(t)} = \dfrac{\text{equation (21)}}{\text{equation (3)}}$ \qquad (23)

As stated earlier, $f(t) = h\left(\sum_{n=0}^{\infty}(H(t-nT) - H(t-nT-t_1))\right)$. By changing $T = \dfrac{1}{f}$ and $t_1 = \dfrac{d}{v}$. The equation

can be re-written, giving us $f(t) = h\left(\sum_{n=0}^{\infty}(HZ - HY)\right)$

But if $Y = \left(t - \dfrac{n}{f} - \dfrac{d}{v}\right)$; and $Z = \left(t - \dfrac{n}{f}\right)$, then

$$I_f = \left( \dfrac{\dfrac{2\gamma}{a}\sum_{n=0}^{\infty}\left(HZ\,e^{-n}\,SinaZ - HY\,e^{-N}\,SinaY\right) + \dfrac{\omega^2 h}{B}\sum_{n=0}^{\infty}\left(HZ\,(1-e^{-Z}\,CosaZ - 3e^{-Z}) + HY\,(1-e^{-N}\,CosaY - 3e^{-N}\,SinaY)\right)}{h\sum_{n=0}^{\infty}(HZ - HY)} \right) \quad (24)$$

The equation (24) represents isolation factor. For this to be possible, all other variables in the isolation factor should be fixed, depending on the vehicles characteristics except frequency, f. Thus, we state that $I_f = F(f)$. However, from the expression $V = f\lambda$, we obtain $\lambda = \dfrac{v}{f}$, which is a component of the effective distance. When d is introduced as in $\lambda - d$ or $\dfrac{v}{f} - d$, we obtain the effective distance.

## 4. Case Study

In order to substantiate the model with results, a hypothetical case study is used. This demonstrates the usefulness of the model where the calculations are applied and the whole logic of the paper is explained. Consider a hypothetical case of a road called University of Ibadan (UI) - Ojoo road located in front of a higher institution. The road also has many commercial shops and offices on both sides, making it very busy for pedestrians. Thus, pedestrian safety on the road is assumed by installing bumps at designated point so as to enforce speed breaks. Consequently, a committee has been constituted by the Federal Government of Nigeria to design, install and implement road bumps at the desired locations. To this end, this technical committee wishes to determine the road bump parameters in order to determine the cost requirements of labour materials, and maintenance of the bump (Figure 5).
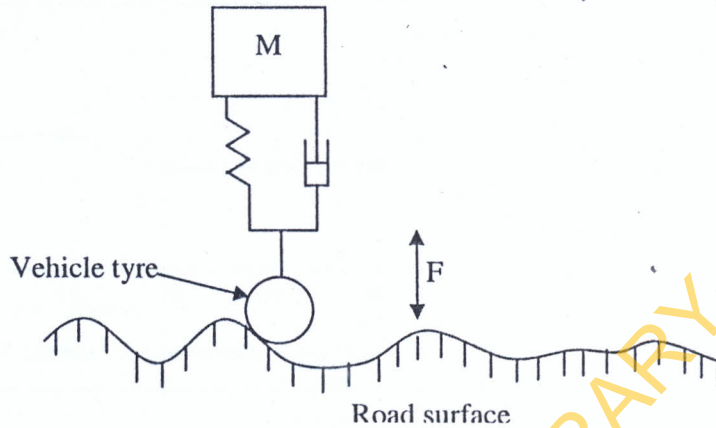
**Figure 5:** Model example of the road-bump-vehicle interaction problem

The application of the mathematical model presented in this work is made as follows. The first concerns the simulation of values for equation (2) and the generation of results. The other application is the simulation of values for equation (27) and the generation of corresponding results. The Figure 6 shows the variation of Laplace Transform index with the height of undulation of the bump at a particular time, t = 10seconds. Using equation (2), the inverse of the Laplace Transform index is plotted against the height of undulation ranging from 0.2m to 3m. It could be observed that the curve gradually falls from left to right, showing that the slope becomes more negative as the height of undulation of the bump increases.
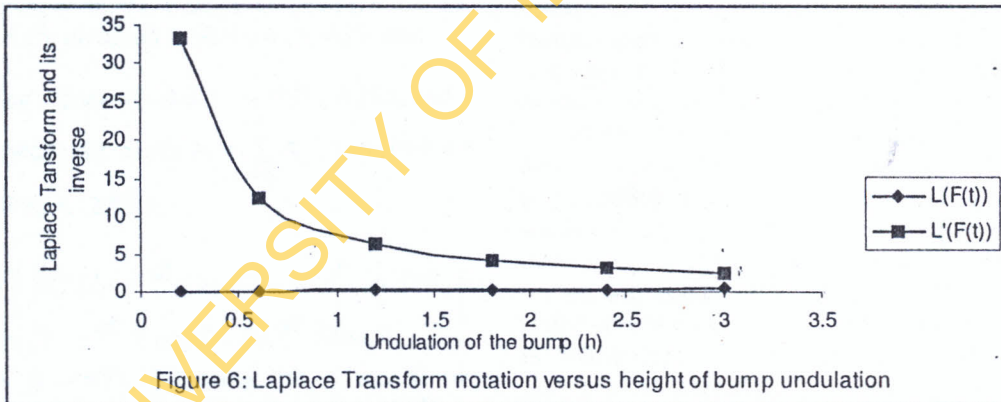


Figure 6: Laplace Transform notation versus height of bump undulation

Figure 7 shows the variation of Laplace Transform index with time using equation (2). The vertical axis was obtained using the inverse of Laplace Transform index while the horizontal axist is the range of time from o to 30 seconds. It could be observed that between 0 and 10 seconds, the slope of the curve is negative while the slope is zero between 10 seconds and 15 seconds. This trend of negative and zero value of slope is continued from left to right.
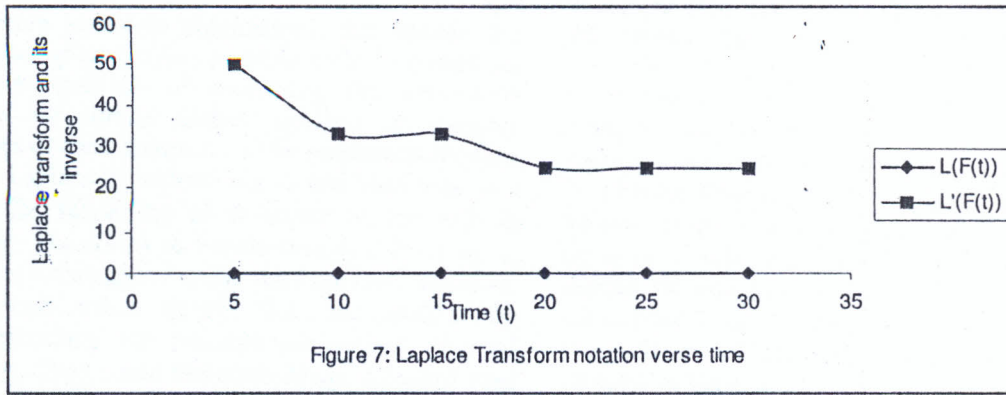
Figure 7: Laplace Transform notation verse time

Equation (21), which is an important equation that reflect the displacement response of vehicular system as it passes over the road with bumps is tested with the following values in order to evaluate its validity: H = 0.1m, Z = 20, $\gamma$ = 0.001, a = 0.05, T = 0.2s, t – 10s, Y = 4, n = 0, 5, 10, 15, 20, and 25.

For simplicity of computation, let A = $\left(HZ e^{-n} SinaZ - JY e^{-\gamma Y} SinaY\right)$. Now, A can be solved for using different values of n. For n = 0, J = (t – nT) = 0, we have $A_0$ = -0.109. When n = 5, and J = 9, $A_5$ = -0.091. Computation of n = 10, 15, 20, and 25 are obtained as A10, A15, A20, and A25 having values of –0.077, -1.037, -0.884, and – 0.731 respectively. Since A = $\sum_{n=0}^{25} A_n$, therefore the sum of all $A_n$ = -2.93m.

Also, let

$$D = \sum_{n=0}^{25} \left( \frac{HZ (1 - e^{-\gamma Z} CosaZ - 3e^{-\gamma Z}) + HY}{(1 - e^{-\gamma Y} CosaY - 3e^{-\gamma Y} SinaJ)} \right).$$

Also J = (t – nT). By using the values of the variables earlier introduced i.e. n = 0, J = 10, $D_0$ to $D_{25}$ could also be computed. Thus, when n = 0, 10, 15, 20, and 25, with J = 10, 9, 8, 7, 6, and 5, the corresponding values of $D_0$, $D_5$, $D_{10}$, $D_{15}$, $D_{20}$, and $D_{25}$ are –45.892, -45.89, -45.90, -45.89, -45.90, and –45.84 respectively. Thus, D = $\sum_{n=0}^{25} D_n$ = - 229.46. It follows that

$x = \frac{2h\gamma}{a} A + \frac{\omega^2 h}{B} D$. Assuming that B = 0.650, $\omega$ = 0.3 rads$^{-1}$, and h = 0.2m, then, x = - 6.38m. Since X represents the displacement response of vehicular system as it passes over road with bumps, it could be deduced that the displacement is in opposite direction since it is negative.

## 5. Conclusion

The work as attempted to investigate into the relationship between road bumps at the road surface and the tyre that rides through the road bumps with a view to identify some peculiar characteristics of this interaction. In particular, the model evaluates the effective distance that should be installed between two road bumps. This would help in controlling the speed of vehicles, reduce noise pollution due to vehicle movement and sudden break application, and maintain minimum impact on the vehicles. Previously developed road bumps are conical shaped as against the hollow shape presented here. The usefulness of the model is many-sided. The mere knowledge of the presence of calming road bumps on the road instills a sense of discipline in the drivers. Complex-shaped bumps are sometimes not periodic. Thus, they may at best be modelled using Laplace Transform instead of Fourier series that is used to model periodic-shaped bumps. Since the hollow design is used in this work, it seems that the impact of the road bump on the passing vehicles is least for the designed bump category. This would extend vehicle life and make the

vehicle more economical to use. Since the literature contains publications that solved the road-bump-interaction problem with the traditional simple approach of estimating the parameters based on simple shapes, analysis of complex shapes of road bumps could be effectively handled with Laplace Transform which has been used as a tool for modelling of response of the vehicle model. The ability to handle complex situations is a great advantage of using the Laplace Transform. Engineers would benefit from the analysis by understanding the possible dimensions of road bumps. They could therefore design standard road bumps with related characteristics to the population of vehicles in the environment in which the road bump is to be installed.

The paper succeeds in presenting a mathematical viewpoint for this interacting surfaces from an extend viewpoint of the documentation that exists in the literature. With this formulation, there is a wide research opportunities that are opened for future investigators in which future works on the road bump-vehicle interaction problem should be focused. The model of two rectangular pulses and its Laplace Transform as a kinematics input to the vehicle model would bring new insights into the problem of road-bump-vehicle interaction if a 2-degree of freedom analysis of the vehicle is considered. This is necessary in view of the type of tyre-road surface contact involved and the sudden changes in kinematics excitation. The radius of the wheel should be taken into account. Further, only suspension model is considered, and no model of wheel is used. Tackling this challenge would open up series of interesting results. From the analysis, the suspension is in direct contact with the road surface. This idea could be changed.

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