Full Paper

DEVELOPMENT OF A COMPUTER AIDED SOFTWARE FOR POWER TRANSMISSION SHAFT DESIGN WITH MULTIPLE CRITERIA

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NOMENCLATURE

d	shaft diameter, mm
τ	design shear stress, N/m2
Mb	bending moment, Nm
Mt	torsional moment, Nm
Kt	combined shock and fatigue factor applied to torsion
Kb	combined shock and fatigue factor applied to bending
R	ratio of inside diameter to outside diameter
K	radius of gyration, mm
I	rectangular moment of inertia, mm4
di	internal diameter, mm
D	outer diameter, mm
L	length of shaft between two bearings, mm
N	constant for the type of end supports
Rg	gear pitch radius, mm
Rp	pulley radius, mm
Wg	gear weight, N
Wp	pulley weight, N
Wi	weight of members on shaft pulley, gear, etc, N
T1	tension in the tight side of the belt on pulley, N
xn	distance between mounted member and bearing, mm
Ø	gear pressure angle, deg
α	column factor
β	pinion cone angle for bevel gear (o)
P	power, Kw
RPM	rotational speed, rev/min
E	modulus of elasticity, N/m2
g	acceleration due to gravity
ω_C	critical speed, rad/sec.
yi	deflections, mm
G	modulus of rigidity, N/m2
θ^o	angle of twist
Fv	vertical forces
Fh	horizontal forces
Fa	axial load on the shaft, N
$\sigma_{..}$	yield strength, N/m2
f	factor of safety
σ_{e}	endurance limit, N/m2
T2	tension in the slack side of belt, N
di	inner shaft diameter, mm
do	outer shaft diameter, mm

ABSTRACT

Power transmission shafts, such as the crankshaft, impeller shaft, propeller shafts, camshafts etc, are essential machine elements with wide application in mechanical systems. The manual shaft design procedure is known to be tedious and complex because of large numbers of formulas, many computations and iteration procedures involved in the design. However, the use of Computer Aided Software (CAS) offers improved accuracy, high speed computations, reduced rigorousness and cumbersomeness involved in manual design procedure. This paper discusses the development of a Computer Aided Design and Drafting (CADD) of power transmission shafts with multiple design criteria (strength, torsional rigidity, critical speed, Soderberg and lateral rigidity). Visual Basic 2008 was used for the implementation of the algorithm, while Fireworks was used for the development of the Graphical User Interface (GUI). The developed CADD is called 'UI-CADSHAFT'. The software was applied for a typical case study and validated with manual design. Accurate results with increased productivity about twenty folds over the manual design are obtained. The software is fully interactive, user-friendly and runs stand-alone on Microsoft Windows.

Keywords: *Shaft Design, Drafting, Computer-Aided, Software development.*

1. INTRODUCTION

Machines are combination of mechanical parts which transmit motion and/or forces in order to do work. The Power transmission shaft constitutes one of the most important machine elements in mechanical systems. Shafts are most widely used machine elements in various mechanical engineering applications, especially in power transmission. Generally, transmission shafts are used to transmit power between the power producing unit and power absorbing unit. They are found in applications such as the crankshaft, impeller shaft, propeller shafts, camshafts etc. The application of a shaft usually determines its nomenclature. Shafts are usually cylindrical in shape

and they provide supports for the other machine elements such as bearings, pulleys, wheel, sprocket and pinion which may be mounted on shafts. They may be subjected to both torque and bending forces. Power is delivered to the shaft by tangential force and the resultant torque (or twisting moment) set up within the shaft, this power is then transferred to various machine elements linked up to the shaft. Shafts can either be solid or hollow, uniform or variable cross sections, with keyways or splines.

Transmission shafts, spindle, axle and machine shafts are the different types of commonly used shafts in mechanical systems. They are commonly made of steel and its alloys. The material selection for the transmission shaft is mainly determined by the material heat treatability, wear resistance, machinability, notch sensitivity factor, and strength [1-5].

The design and production of transmission shafts are deeply entrenched in mechanical design and engineering principles; it involves rigorous analysis to determine the stress, geometry and dimensions of the shaft. Designers are faced with the problems of working with large numbers of formulas, many computations and iteration procedures involved in the design. These have made the orthodox design procedures of shafts cumbersome, rigorous and time consuming.

However, these problems can be overcome using the various advantages, utilities and flexibilities which modern high-speed micro-computers offer for the design of shafts. Computer Aided Design (CAD) is defined by Ray [6] as any design activity that involves the effective use of computer to create, modify, analyse, or optimize an engineering design. According to Serope [7], Computer Aided Drafting commonly refers to the actual technical drawing of component using computer rather than a traditional drawing board. The use of CAD is becoming increasingly popular in many fields of engineering design. Different CAD-based software are now commercially available, while efforts are still being made by many researchers in the development of new and more sophisticated software for different applications in engineering design, such as mould design [8, 9], cam profile design [10, 11], leaflet blood-pump valves [12], while Ogedengbe and Aderoba [13] developed a CAD software using Visual Basic 6.0 for design of shafts based only on strength criterion when the shaft has no axial loads. The aim of this paper is to develop a Computer Aided Design (CAD) software for the design and drafting of power transmission shafts based on multiple design criteria.

2. MATERIALS AND METHODS

In developing this software, five design criteria: (1) strength, (2) torsional rigidity, (3) critical speed or vibration, (4) Soderberg and (5) lateral rigidity were considered in order to determine the optimum method for transmission shaft design. Relevant design equations based on different criteria were compiled from literatures. This was followed by the implementation of the algorithm using Visual Basic 2008 and Fireworks for the development of a user-friendly application software. The developed CADD is called 'UI-CADSHAFT'. The procedures for the design calculations are discussed in the following sections.

2.1 Determination of torsional and bending moments

Bending and torsional moments are the main factors influencing shaft design. First step in shaft design for any given design criterion is to calculate the torsional moment on the shaft followed by finding the maximum bending moment. The torsional moment, M_t acting on the shaft is expressed as [14]:

$$M_t = \frac{9550 * P}{RPM} \qquad (1)$$

If the load mounted on the shaft is gear, then vertical F_v and horizontal F_h components of load on the shaft are given as [13]:

$$F_v = \left(\frac{M_t}{R_g}\right) Tan\varphi Cos\beta + W_g$$

$$F_h = \frac{M_t}{R_g} \qquad (2)$$

Else, if the load mounted on the shaft is pulley, then vertical F_v and horizontal F_h components of load on the shaft are given as [13]

$$F_h = (T_1 + T_2) Cos\theta$$

$$F_v = W_p - (T_1 + T_2) Sin\theta \qquad (3)$$

After the determination of F_v and F_h, the bending moment acting on the shaft is determined by drawing and analyzing the bending moment diagram of the loaded shaft. The design calculations based on the five criteria are discussed as follows:

2.2 Design of shaft for strength

The most common shaft loading condition is constant torque and reversed bending. The diameter, d of a shaft subjected to torsion and bending loads having little or no axial loading is calculated from the ASME code [14] as:

$$d = \left[\frac{16}{\pi\tau(1-R^4)}\left[\sqrt{(K_b M_b)^2 + (K_t M_t)^2}\right]\right]^{1/3} \qquad (4)$$

On the other hands, if the shaft orientation is vertical such that it has significant axial loading, equation (4) is expressed as [1]:

$$\tau = \left[\frac{16}{\pi d^3(1-R^4)}\sqrt{\left[K_b M_b + \frac{\alpha F_a d(1+R^2)}{8}\right]^2 + (K_t M_t)^2}\right] \qquad (5)$$

For solid shafts, $R = \frac{d_i}{d_o} \Rightarrow 0$ because $di = 0$. Else, for hollow shafts, the value of parameter R must be given.

For vertically orientated shafts with axial loading, F_a is initially assumed to be negligible, as a result, the term as $Fa \underset{\rightarrow 0}{\lim} \frac{\alpha F_a d(1+R^2)}{8}$. Hence, Equation (5) is reduced to Equation (4) and d is determined. For shafts with or without keyways, initial value of τ is taken as 56 or 42 MN/m^2, respectively. Then the radius of gyration (K) is evaluated using Equation (6) and α is then evaluated for compressive loading condition using Equation (7) for different ranges of $\frac{L}{K}$, while for tensile loads, α is taken to be equal to 1 and new value of τ is determined using Equation (5). If the new value of τ is greater than initial τ, it means the design is unsafe. Iteration process starts using higher value of diameter, d until new τ is less than initial τ, at this stage the design is now safe. That is, the value of diameter used to obtain lesser τ is the required diameter of shaft for the vertical orientation. [1]:

$$K = \frac{d}{4} \qquad (6)$$

$$\alpha = \frac{1}{1 - 0.0044\left[\dfrac{L}{K}\right]} \qquad for \ \frac{L}{K} \le 115$$

(7)

$$\alpha = \frac{\sigma_y}{\pi^2 nE}\left[\frac{L}{K}\right]^2 \qquad for \ \frac{L}{K} > 115$$

Where n = 1 for hinged ends, n = 2.25 for fixed ends, n = 1.6 for partially restrained ends. i. e. bearings

2.3 Design of shaft for Torsional rigidity

Design torsional rigidity is in accordance with the ASME code. The angle of twist is determined using the expression [14]:

$$(d^4 - d_i^4) = \frac{584 M_t L}{G\theta^o}$$

(8)

2.4 Design of shaft for critical speed or vibration

The equation used to determine the diameter of shaft based on critical speed or vibration is expressed as [15]:

$$\omega_c = \frac{30}{\pi}\left[\frac{g\sum\limits_{n-1}^{N} W_n y_n}{\sum\limits_{n-1}^{N} W_n y_n^2}\right]^{\frac{1}{2}}$$

(9)

Where the deflection, y_n is expressed as

$$y_n = \frac{32 M_{b_n} x_n}{E\pi d^4}(L - x_n)$$

(10)

and the diameter is evaluated as:

$$d = \left|\frac{\omega_c}{30}\left[\frac{32\Pi}{gE}\left(\frac{\sum\limits_{n=1}^{N} W_n\left(M_{bn}x_n\left(L-x_n\right)\right)^2}{\sum\limits_{N=1}^{N} W_n M_{bn}x_n\left(L-x_n\right)}\right)\right]^{\frac{1}{2}}\right|^{\frac{1}{2}}$$

(11)

2.5 Design of shaft for Soderderg

Soderderg criterion is based on distortion energy and maximum shear theories [16]. For distortion energy theory, the diameter, d of a shaft is expressed as [13]:

$$d = \left[\frac{32f}{\pi\sigma_y}\left[\sqrt{\left(\frac{M_b}{\sigma_e}\right)^2 + \frac{3}{4}\left(\frac{M_t}{\sigma_y}\right)^2}\right]\right]^{\frac{1}{3}}$$

(12)

For maximum shear stress theory, the diameter, d of a shaft is expressed as [13]:

$$d = \left[\frac{32f}{\pi\sigma_y}\left[\sqrt{\left(\frac{M_b}{\sigma_e}\right)^2 + \left(\frac{M_t}{\sigma_y}\right)^2}\right]\right]^{\frac{1}{3}}$$

(13)

2.6 Design of shaft for Lateral rigidity

The lateral deflection of the shaft is determined from bending moment equation using the double integration method. The lateral deflection is expressed as [17]:

$$y_{max} = \frac{32 M_{bmax} * x}{E\pi d^4}[L - x]$$

(14)

The procedural steps taken in designing the shaft based on lateral rigidity are:

- The highest of the values of diameter of the shaft is selected from the diameter obtained using the previous design criteria (strength, torsional rigidity, critical speed and Soderberg) is selected as initial value and Equation (14) is used to calculate maximum deflection, y_{max}.

- The procedure in Step 1 is repeated using 5 mm incremental value of the diameter to determine the new y_{max}, this process is continued until new y_{max} is less than initial y_{max} obtained in step 1.

2.7 Description of the software algorithms

Design calculations for the five criteria, material selection, loading and operating conditions constitute the basic input parameters. The flowchart of the algorithm is shown in Figure 1. The UI-CADSHAFT software incorporates a material property library of commonly used engineering material such as steels and aluminum. New material property data can also be added to the library by the user. Possible loading conditions considered are belt, gear and direct power transition drives; while operating conditions are power, rotational speed, length of shaft, distance between bearings, and number of members mounted on the shaft. In order to make the software user-friendly, graphical user interfaces (GUIs) are designed using Fireworks. The software is capable of the followings: (i) estimating torsional moment, (ii) estimating tensions in the slack and tight sides of a belt drive (provided the ratio is known), (iii) estimating horizontal and vertical force(s) that correspond to the input parameters of the load(s), (iv) determining maximum bending moment due to force(s), (v) estimating distance between end points and location of maximum deflection, (vi) calculating diameter of the shaft based on different design criteria, taking the maximum out of them and then selecting the next available standard bore, (vii)

$$\alpha = \frac{1}{1-0.0044\left[\frac{L}{K}\right]} \qquad for \frac{L}{K} \le 115 \tag{7}$$

$$\alpha = \frac{\sigma_y}{\pi^2 nE}\left[\frac{L}{K}\right]^2 \qquad for \frac{L}{K} > 115$$

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Where the deflection, y_n is expressed as

$$y_n = \frac{32 M_{b_n} x_n}{E\pi d^4}(L - x_n) \tag{10}$$

and the diameter is evaluated as:

$$d = \left[\frac{\omega_c}{30}\left[\frac{32\Pi}{gE}\left(\frac{\sum_{n=1}^{N} W_n\left(M_{bn} x_n (L - x_n)\right)^2}{\sum_{N=1}^{N} W_n M_{bn} x_n (L - x_n)}\right)\right]^{1/2}\right]^{1/2} \tag{11}$$

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For maximum shear stress theory, the diameter, *d* of a shaft is expressed as [13]:

$$d = \left[\frac{32f}{\pi\sigma_y}\left[\sqrt{\left(\frac{M_b}{\sigma_e}\right)^2 + \left(\frac{M_t}{\sigma_y}\right)^2}\right]\right]^{1/3} \tag{13}$$

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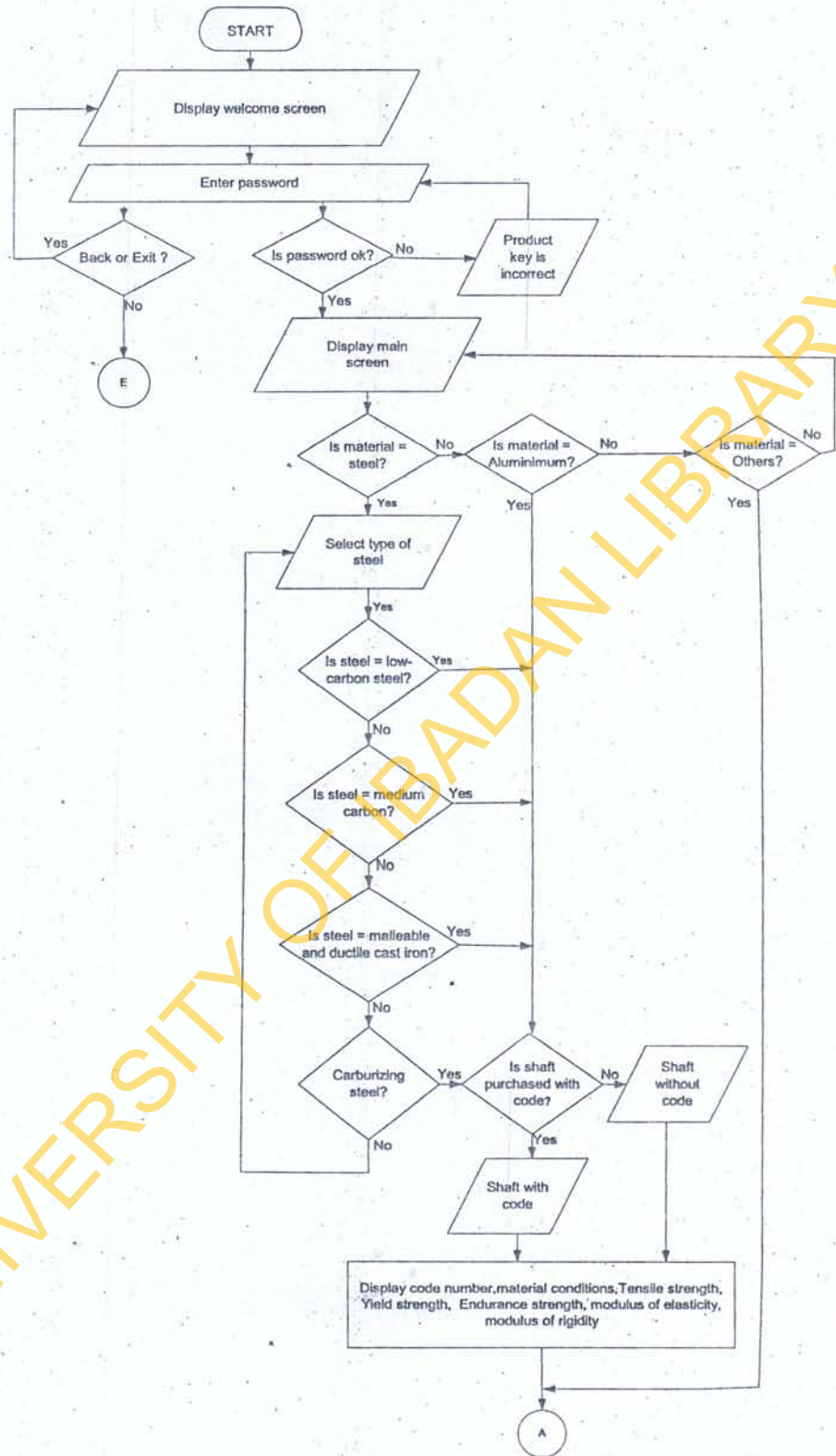
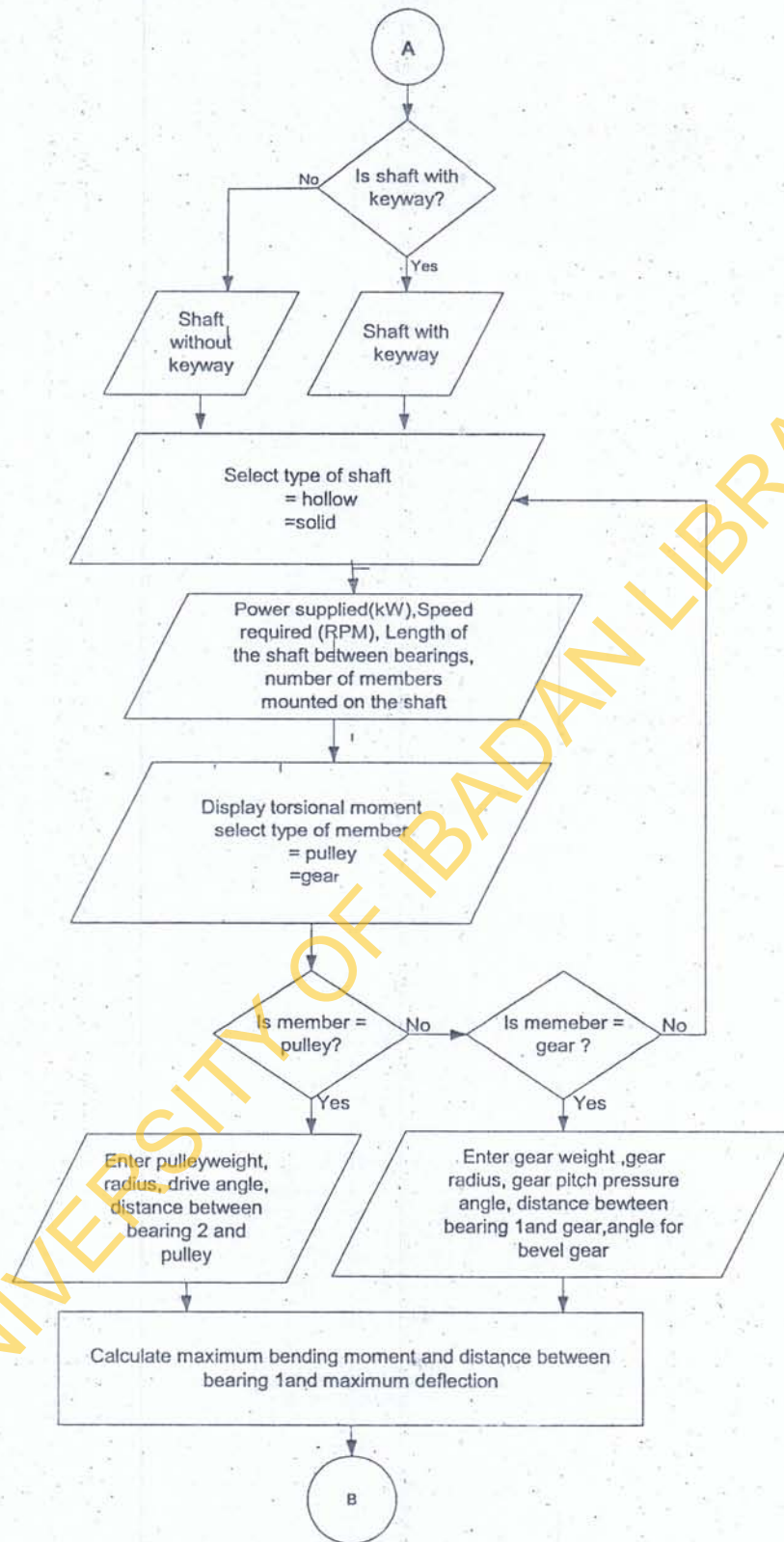Fig. 1: Flow chart for the computer-aided shaft design (UI-CADSHAFT)
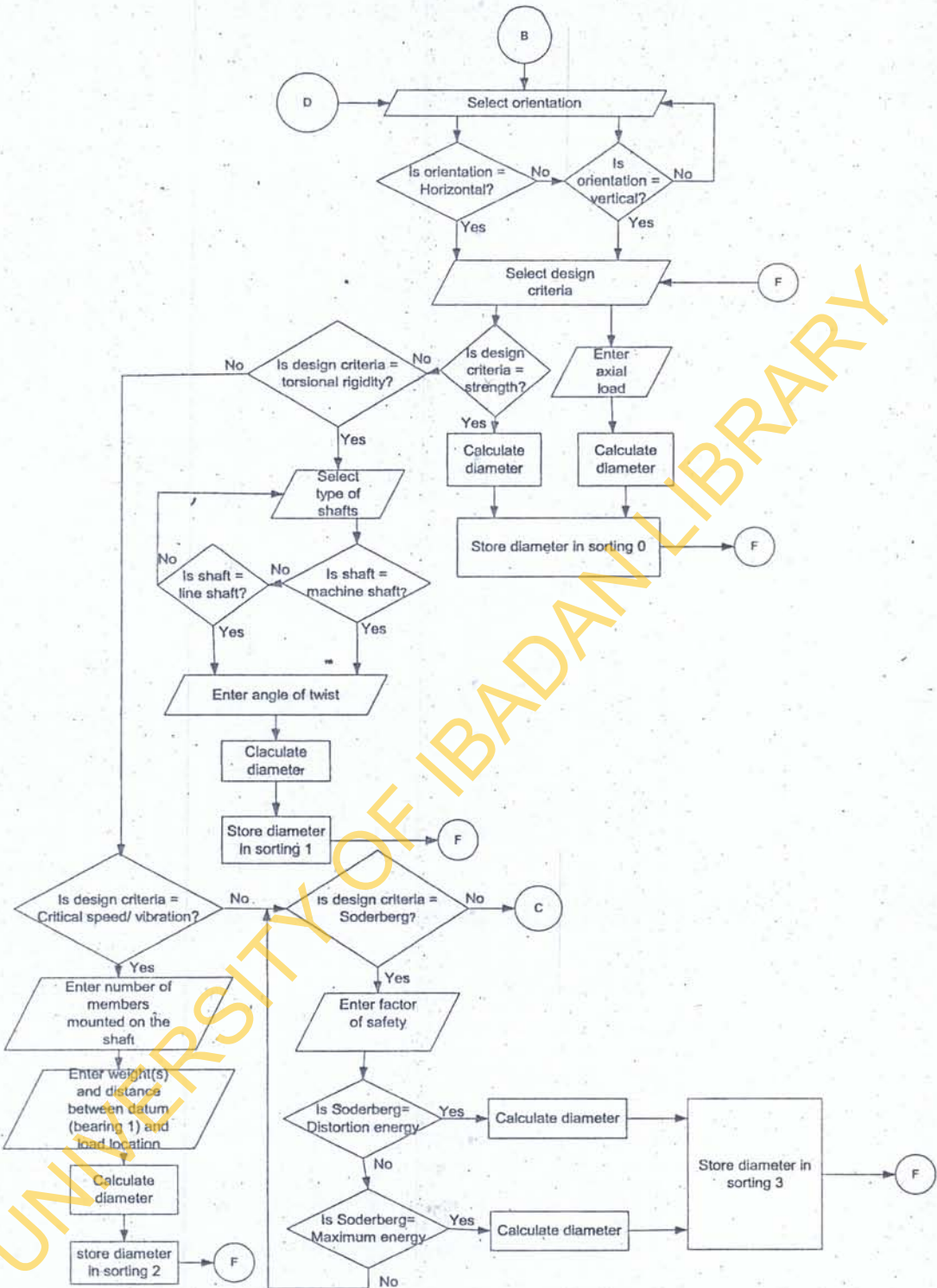
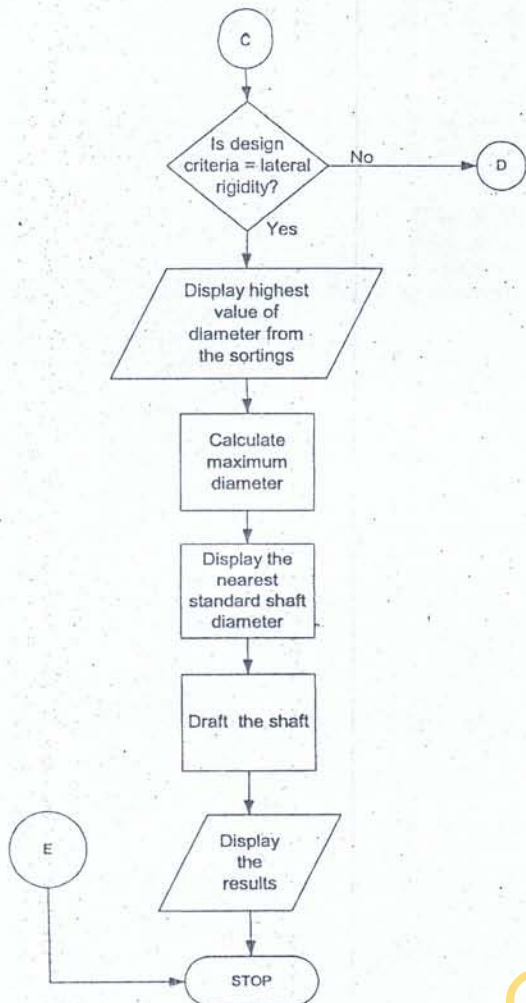Fig. 1: Continued

Fig. 1: Continued

Fig. 1: Continued

displaying the result, (viii) drawing the expected shaft with label of mounted members automatically.

2.8 Validation of the software

The software was applied for a typical case study for design of a power transmission shaft with design parameters shown in Table 1, for a selected material made of AISI 1112 with mechanical properties shown in Table 2. Results obtained from the software were validated with manual calculations.

3. RESULTS AND DISCUSSION

Figure 2 shows the main window for the case study. The main window is divided into two parts: input and output section. In the input section, AISI 1112 steel, hot-rolled, solid shaft with keyway for horizontal orientation is selected from material library pull down menus. The mechanical properties of the selected material are displayed after the material selection. The operating parameters such as: power to be transmitted, rotational speed, length of the shaft, distance between bearing, and number and type of members mounted on the shaft are entered. Torsional and maximum bending moments, and distance between end points and maximum deflection are calculated and displaced in the output section on clicking the

"calculate" button. Then the desired criterion is selected from the "design criterion" pull-down menu and design diameter of the shaft is displayed in the output section. For this case study, the design diameters of the shaft based on five design criteria: strength, torsional rigidity, critical speed or vibration, Soderberg and lateral rigidity are 57.1334 mm, 62.1006 mm, 11.3355 mm, 0.1272 mm and 67.1006 mm, respectively. The highest calculated diameter of shaft is displayed and the standard shaft diameter of 71.0 mm is recommended.

Table1: Design input parameters used for the case study

Parameter Description	Nature/Value
Shaft:	
Material	AISI 1112 steel
Types:	Solid with keyway
Orientation:	Horizontal
Condition	Hot-rolled
Loading Conditions:	
Gear:	
Weight	900 N
Pitch radius	375 mm
Pressure angle	20°
Distance from end point	30 mm
Angle for bevel gear	0°
Pulley:	
Weight	2700 N
Radius	625 mm
Drive's angle	0°
Distance from end point	1300 mm
Tension ratio	2.5
Operating Parameters:	
Power	18.75 kW
Rotational speed	150 rpm
Length of shaft	1600 mm
Distance between bearings	1000 mm
Number of mounted members	2
Angle of twist	1 deg/m
Factor of safety	7

Figure 3 shows the summary window, which is divided into three sections. The fist section shows the design parameters used in the case study, the operating parameters are shown in the second section, while the design criteria used and calculated shaft diameters are shown in the third section. Figure 4 shows the drafting window, which displaces the front elevation of the designed shaft, and also indicates locations of gear, pulley and bearings on the shaft. The drafting scale, calculated shaft diameter and length of the shaft are also shown on the drafting window.

The developed software was validated with manual computation for the same case study using the design criteria for strength and lateral rigidity. The calculated shaft diameter for manual computation is found to be 57.13 mm and 59.49 mm respectively, while the application of the software produces exact result with higher decimal number. The software was also validated with existing software called SHAFTCAD [13] using design input parameters in Table 2. The torsional moment, bending moment, calculated diameter and selected shaft diameter of 238.75 Nm, 386.67 Nm, 37.76 Nm and 40 mm respectively were obtained usig the UI-CADSHAFT. The results were exactly the same with that of existing software. Increased productivity of about twenty folds over the manual design is obtained with the application of the software in this case study. This because manual calculation lasts for twenty

three minutes and software computation takes about one and half minutes.

Table2: Design input parameters used for the validation of UI-CADSHAFT.

Parameter Description	Nature/Value
Shaft:	
Material	AISI 1045 steel
Types:	Solid with keyway
Orientation:	Horizontal
Condition	Hot-rolled
Loading Conditions:	
Gear:	
Weight	0 N
Pitch radius	100 mm
Pressure angle	20°
Distance from end point	250 mm
Angle for bevel gear	0°
Pulley:	
Weight	800 N
Radius	225 mm
Drive's angle	60°
Distance from end point	1250 mm
Tension ratio	3
Operating Parameters:	
Power	15 kW
Rotational speed	600 rpm
Length of shaft	1250 mm
Distance between bearings	1000 mm
Number of mounted members	2
Factor of safety	6

4. CONCLUSION

The development and application of CAD software (UI-CADSHAFT) for design of power transmission shaft with multiple design criteria has been presented. The result was found to be accurate and the shaft was drafted and labeled neatly, with the developed software, the user is able to design and draft shaft at an extremely fast rate. The design was found to increase productivity in the design function by over twenty folds above manual design. Hence the software can be used in the industries, as this will improve the productivity of design engineers.

Table 2: Mechanical Properties of the AISI 1112 steel

Tensile strength σ_u x10^8 (N/m^2)*	Yield strength, σ_y x10^8 (N/m^2)*	Endurance limit, σ_e x10^8 (N/m^2)*	Modulus of elasticity, Ex10^9 (N/m^2)**	Modulus of rigidity, G x10^9 (N/m^2)**
5.688	4.895	0.954	205	75

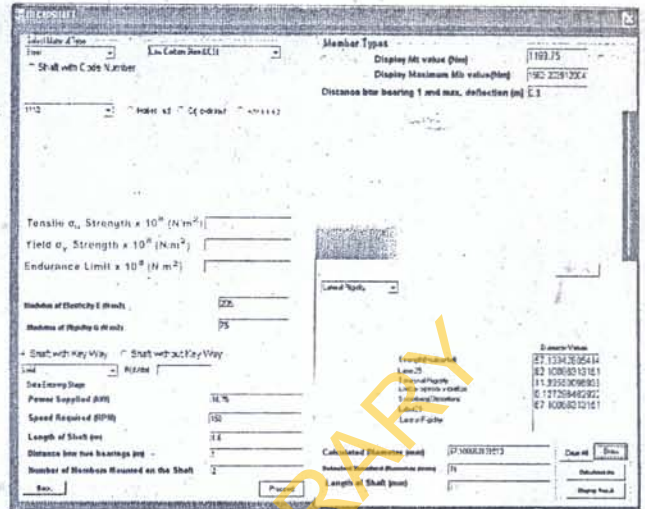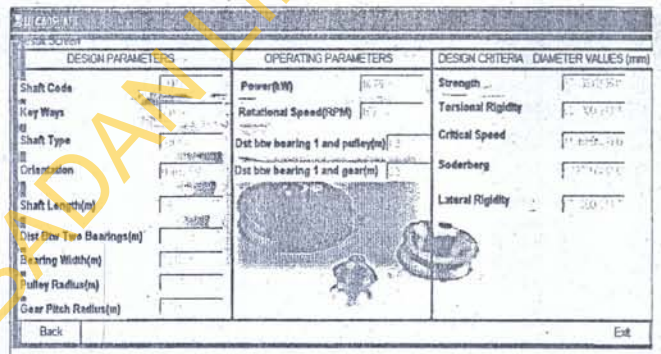Sources: *Spotts (1988), **Robert (2000)

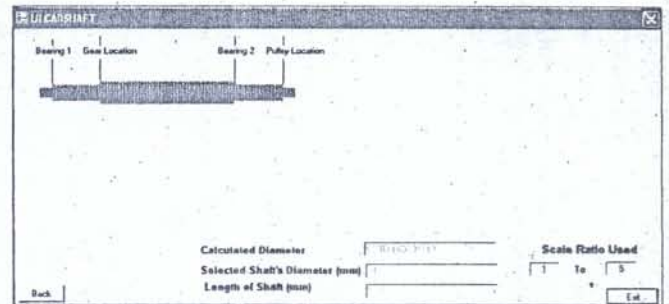Fig 2: Main window

Fig.3: Summary window

Fig.4: Drafting window

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