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FRACTAL ANALYSIS OF AREA-PERIMETER RELATIONSHIP OF SOME SELECTED COUNTRIES OF THE WORLD

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ABSTRACT

Boundaries of six selected countries one from each continent were studied, using fractal dimension technique. The Area – Perimeter relation method was used, where land area covered and boundary length of designated countries were measured in the unit of known box size (Box counting). The log-log plots of these variables were obtained and the slopes were measured as the required dimensions. Fractal dimensions determined lie between 1.19 and 1.55. The scale size of the maps, and the number of states both had no significant effect ($p>0.05$) on the fractal dimensions obtained.

Keywords: Fractal dimension, Area- Perimeter relation, box counting, self similarity.

INTRODUCTION

Many patterns of nature are often irregular to such an extreme degree that Euclidean (or classical) geometry cannot describe their form (Mandelbrot, 1977). Klonowski (2000) reported that during the last decade it has widely been recognized by physicists working in diverse areas that many of the structures common in their experiment possess a rather special kind of geometrical complexity. This awareness is largely due to the attention given to the particular geometrical properties of such objects as the shores of continents, the branches of trees, or the surface of clouds. Takayasu (1990) contributed that any shape can be characterized by whether or not it has a characteristic length. Characteristic length is a convenient reference length (usually constant) of a given configuration, such as overall length of an aircraft, the maximum diameter or radius of a body of revolution, or a chord or span of a lifting surface. Characteristic length can define any regular shape geometry, such as a sphere, cube, e.t.c., and the shapes with characteristic length have an important common peculiarity of smooth surface. Shapes having no characteristic length are self-similar or scale- invariant since these shapes does not change under a different change of observation scale.

Most branches of science and engineering are now using fractal analysis for characterizing natural or synthetic particles, and complex physical or chemical processes. Fractal dimensions have been successfully used to describe the ruggedness and geometric complexities of both natural and synthetic particles (Graf, 1991; Peleg and Normand, 1985; Nagai and Yano, 1990; Yano and Nagai, 1989). In addition to its application to problems in engineering and physical sciences, the fractal dimension has a rapidly increasing variety of uses in contexts ranging from urban and landscape planning (Milne, 1991) to oceanography and Meteorology (Jain, 1986; Morrrison and Srokosz, 1993).

The fractal dimension (D) can be estimated by structured walk (Richardson's plot), dividers (compass) method, grid (Box counting) method, probability-density function, size-frequency distributions, branch order relationships, spatial and temporal series and two-surface method.

Previous researchers (Smith Jr. et al., 1989: Alabi, 2001: Peleg and Normad, 1985) discovered that D values lies between 1.0 and 2.0 for fractal images in 2-dimensional plane. The objective of this study is to determine the fractal dimension of the countries selected from each of the continents in terms of their perimeter-area covered on the map and associate physical meaning and interpretation using grid method.

MATERIALS

Materials and material preparation

Materials used for this study include the following;

(i) Scale map of Nigeria (http:// www.Lib.u texaz-education /maps/ Africa/Nigeria –Pol96 jpg.)

(ii) Scale map of Australia (http: //www. World time zone .com/time – Australia.htm)

(iii) Scale map of Mexico (http: // www .maps – of – Mexico .com/)

(iv) Scale map of Argentina (http: //www. liibutexas.edu/maps/America/argentine –Pol96 .jpg.)

(v) Scale map of Germany (http:// www.Ulib.Iupui.education /kad/name word/map4 html)

(vi) Scale map of China (http: //www > Fotw. Net/flags/Mxhtm# map)

(vii) Grid of 1mm by 1mm size was prepared to cover size A4 paper using Microsoft Word Processor followed by photocopying the hard copy of same onto transparent sheet at room temperature.

MODEL AND METHOD

The Area-Perimeter relationship method given by Eqn.(1) was used for this study. This method measures the extent at which the states/provinces boundaries "fill' the two dimensional plane.

$$P = KA^{D/2} \qquad (1)$$

where, the area (A) is the number of 1mm by 1mm square boxes making up a given state land area, the perimeter (P) is the count of the number of 1mm by 1mm square that falls on the edges of the state boundary. D is the fractal dimension and K is the proportionality constant.

A fragmented scale map of the country to be analyzed was placed on the drawing board and the transparent grid of size 1mm by 1mm was arbitrarily and firmly placed on it. The numbers of squares that fell on each of the state of the country were then counted. Likewise the numbers of the squares that fell on the edge of the boundary were counted as an estimate of the perimeter. The number of states or provinces as each case may be, determined the number of solution points on a log P versus $\log A^{1/2}$ plane as given by Eqn. (2)

$$\text{Log } P = D \log A^{1/2} + \log K \qquad (2)$$

Eqn. (2) was obtained by taking log of both sides of Eqn. (1).The slope of the resulting graph of Eqn. (2) gives a measure of D. The above procedures were carried out at two different levels of map enlargement, and replicated for each of the selected maps. The mean data were then used for the analysis.

ANALYSIS OF RESULTS

(a) Values of P and A for all states in each country were plotted on log-log graph using Eqn (2). The slope of the graph of Eqn.2 was taken as the fractal dimension D.

(b) The D values obtained for each set of experimental run were subjected to a t-Test to determine the difference in means for the two cases considered (Munro, 2001).

DISCUSSION

Fig. 1 and Fig. 2 show the map of Nigeria drawn to scales1.1cm:100km and 1.32 cm: 100km respectively. The Perimeter/Area obtained for the two scaled maps of Nigeria (Fig. 1 and Fig. 2) including F.C.T is as shown in Table 1.

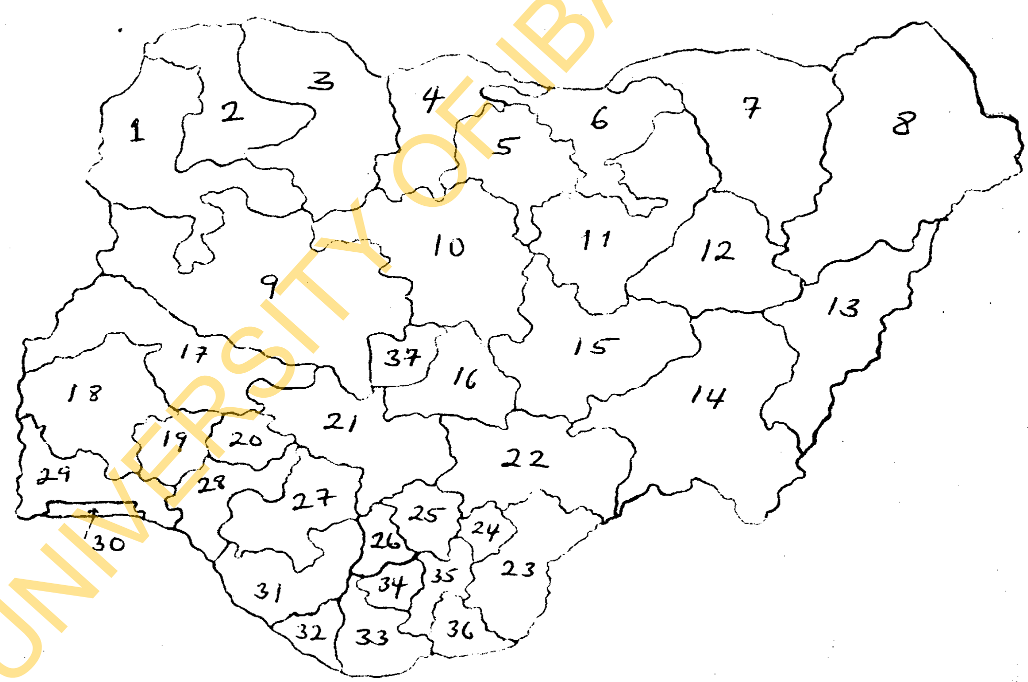

Fig. 1. Map of Nigeria drawn to Scale 1.1cm: 100km

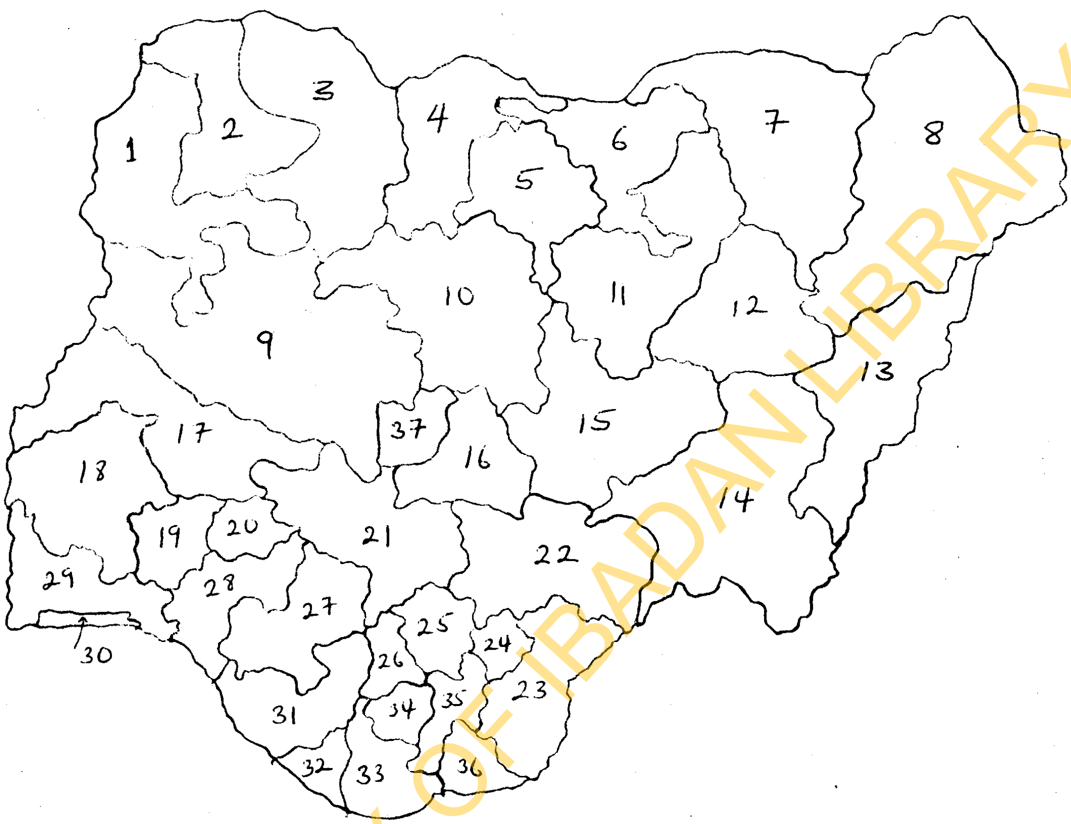

Fig. 2. Map of Nigeria drawn to Scale 1.32cm: 100km

Note that the numbering of the states in both Fig. 1 and Fig. 2 is arbitrary

Table 1: Perimeter/Area of Nigerian States including FCT (In terms of Box Counting)

States	Scale:1.1cm:100km		Scale:1.32cm:100km	
	Perimeter	Area	Perimeter	Area
1	63	98	75	131
2	39	61	47	85
3	44	98	55	138
4	41	60	48	83
5	32	56	38	73
6	43	60	50	88
7	49	106	60	145
8	58	158	68	182
9	76	165	81	193
10	48	101	51	142
11	56	97	60	130
12	33	68	36	92
13	49	86	60	117
14	59	138	72	185
15	46	92	54	125
16	25	42	28	57
17	53	92	63	122
18	37	72	47	92
19	19	27	24	37
20	16	21	18	27
21	46	72	53	98
22	41	76	49	104
23	36	56	44	74
24	14	14	18	22
25	18	25	20	31
26	17	19	20	24
27	33	51	38	68
28	31	40	36	54
29	34	46	47	63
30	12	11	15	14
31	28	47	38	68
32	15	18	18	24
33	25	34	27	42
34	15	16	15	19
35	22	22	26	85
36	18	21	18	26
37	15	21	20	30

The values obtained for the Perimeter/ Area measured for scale 1.1cm: 100km was always less than that of scale 1.32cm: 100km as expected. These values were plotted on a log-log scale as shown in Fig.3 and Fig.4 to determine the fractal dimension D from the slope of the line of best fit. The coefficient of determination (R^2) was always greater than 0.90 in all cases considered as shown in Table 2, showing a good correlation between experimental and predicted values.

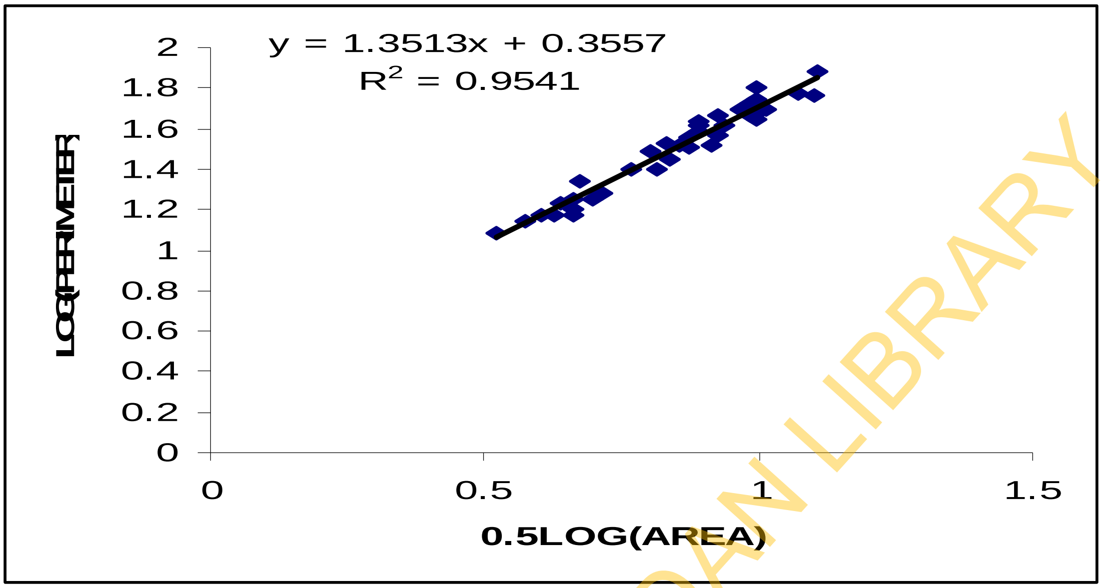

Fig.3. Graph of LOG (PERIMETER)/LOG (AREA) of Nigerian states including F.C.T. (scale 1.1cm: 100km)

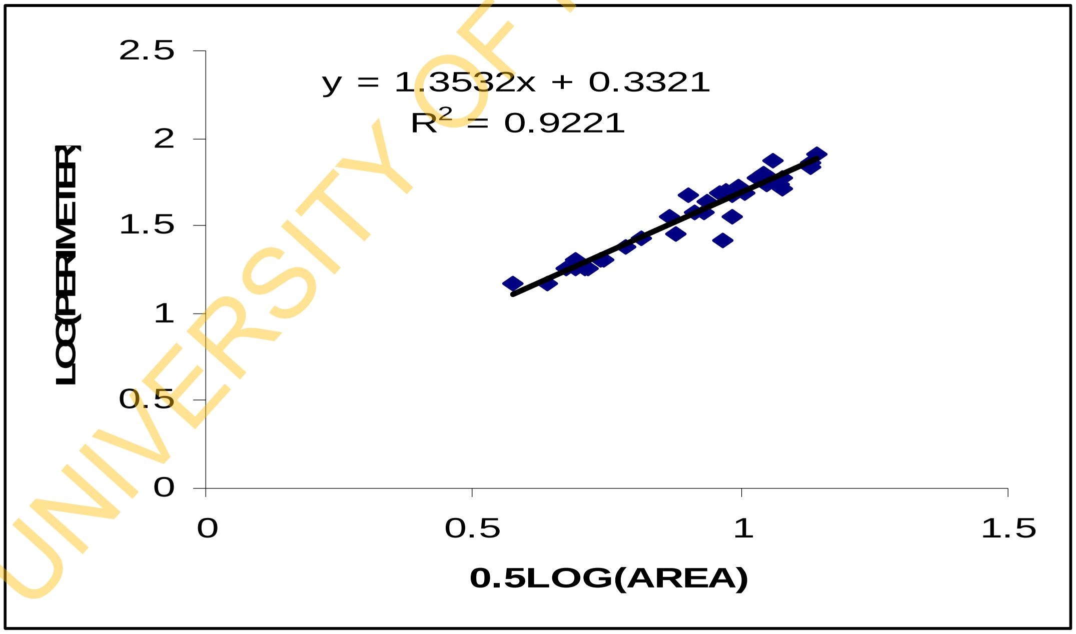

Fig. 4. Graph of LOG (PERIMETER)/LOG (AREA) of Nigerian states including F.C.T (Scale: 1.32cm: 100km).

Table 2: Fractal Dimensions of the Countries Analyzed.

Countries	No. of states as of 2005	Fractal Dimension (D)		(R2)	
		Scale 1	Scale 2	Scale 1	Scale 2
China	31	1.29	1.29	0.92	0.94
Argentina	23	1.23	1.22	0.90	0.90
Nigeria	37	1.36	1.35	0.95	0.92
Germany	16	1.42	1.37	0.99	0.99
Australia	7	1.19	1.21	0.98	0.98
Mexico	32	1.54	1.55	0.96	0.96

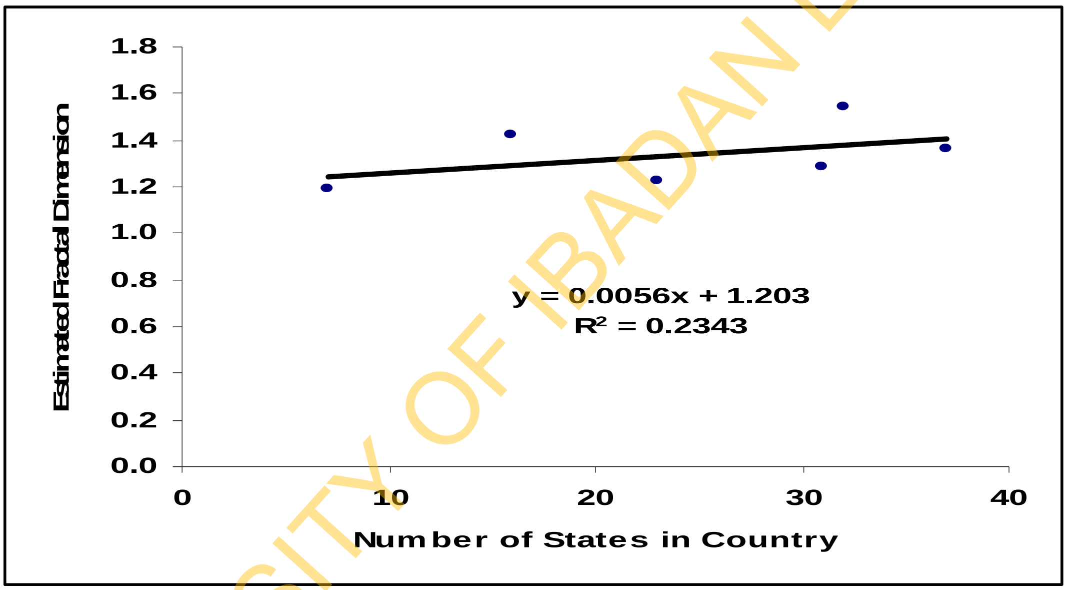

Fig. 5. Graph of Number of States versus Estimated Fractal dimension at Scale 1

Figure 5 depicts the relationship between the number of state and the fractal dimension D. There was a poor correlation between the two variables where the coefficient of determination (R^2) was below 0.5**.** This implies that the fractal dimension D is independent of number of states.

The fractal dimension of the boundary of the countries studied lie between 1.19 and 1.55 as shown in Table 2 which is in consonance with previous researches (Smith Jr. et al., 1989; Alabi, 2001: Peleg and Normad, 1985), that D values must be between 1.0 and 2.0 for fractal images in 2-dimensional plane. For all the countries considered, the fractal dimension of each experimental level appears the same confirming that the fractal images have a range of self similarity over any scale range as reported by Smith Jr. et al. (1989) and Thomas and Thomas (1988).

Comparing the fractal dimensions of Australia and Argentina(Figures 6 and 7, respectively), it becomes relatively easier to see that two fractal objects may appear visually different and yet have the same fractal dimension in agreement with Smith Jr. et al. (1996).

Australia has the lowest fractal dimension which may be due to its less fragmented states compared to the other countries studied which had more compacted states. It has been reported by Ogata et al. (1991) that the degree of fragmentation has an effect on the fractal dimension of fractal images. Higher values indicate higher roughness of the boundary.

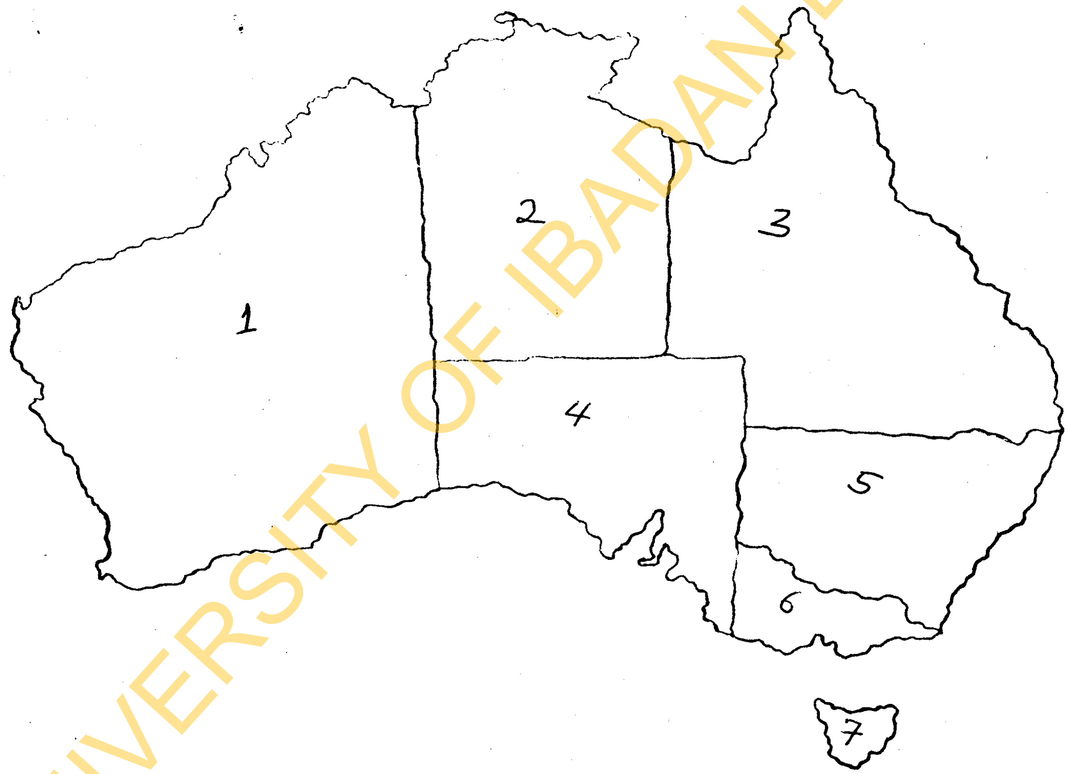

Fig. 6. Map of Australia drawn to 1cm: 500km

Fig.7. Map of Argentina drawn to scale 0.95cm: 100km

A t-Test performed on fractal dimension for the countries considered showed that at 95% confidence level there were no significant differences between the two scales of maps considered for the research.

CONCLUSION

The D values for the countries analysed lie between 1.19 and 1.55. There was no influence of number of state in a country on fractal dimension. The scale of drawing used was not a significant factor in fractal dimension D obtained for all cases considered. There was a good correlation between the experimental and predicted values with the coefficient of determination (R^2) varying from 0.90 to 0.99. A t-Test performed also showed that there was no significant variation between the two levels of map scales used in the research. This study has re-established the characterizing potentiality of fractal dimension as a measure of degree of roughness of a fractal object.

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