Mathematical Modeling of the Traffic Congestion Problem at a University Campus

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Abstract

The traffic situation at many university campuses in developing countries has been a source of concern to both the school authority and the campus populace. This may be largely due to the increasing enrolment level for university admissions since most employers are certificate-biased instead of skill rewarding. The roads are often congested during the day thereby causing inconveniences to everyone. During the "peak period" when vehicular traffic is beyond what the road can contain conveniently, traffic jams occur, which accounts for a great amount of time wastage by commuters. This paper deals with the traffic congestion problem in a university in a developing country. The approach employs mathematical modeling to solve the problem. The principles of flow in fluid mechanics are interpreted at a macro level to the flow of traffic. The model describes the traffic situation, explains the causes and periods of congestion, and proffers a solution to the problem.

Key Words : roads, traffic flow, congestion, university, Nigeria, vehicles, developing country

Introduction

Vehicles are used for easy mobility of goods and people. However, when the number of vehicles on a road is more than the number it can conveniently support, the road becomes congested and this leads to series of economic, environmental and physical problems. There are various ways to solve this problem: (1) The size of the road can be increased to allow for more vehicles. This will cause both physical and economic (monetary) inconveniences for the society; (2) The number of vehicles on the roads can be reduced. One of the ways this can be achieved is by encouraging public transport for meeting a large proportion of the mobility needs of public; (3) Alternative modes of transportation such as walking and cycling for shorter distance can be employed. This solution is not generally acceptable as mass car ownership is the trend and consists large proportion of the transportation system. Public transportation can play a strong role in solving traffic congestion problems in urban centers. Aside the waste of useful economic elements such as time, money, and labour, energy. pollution is also a very grave problem that emerges from traffic problems.

The traffic conditions at the University of Lagos campus is known to be congested at various times of the day. On university campuses, traffic congestion (free-flow and frozen-jam phases) is an embarrassing phenomenon that the stakeholders are strongly looking forward to solve (Tanaka *et al.*, 2007). The University of Lagos is an interesting situation where the high student population with automobiles contributes significantly to traffic congestion on campus. For an academic institution, wasting time in traffic jam is a serious counter-productive activity since the wasted time could have been profitably invested in research, teaching, and community development activities. Solution to the traffic congestion problem will be of great social, environmental and economic importance to the university community. The need to optimise the time utilisation of the entire stakeholders in the university system while on campus has strongly motivated this research on traffic congestion analysis at the University of Lagos.

Several accounts have been given in the literature on traffic congestion, traffic flow and related activities (Tang *et al.*, 2007). These are reviewed as a background that strengthens the need for the current study. Rao *et al.* (1991) presented an analytical framework for examining the effects of congestion on logistics performance. A numerical example is given on the calculation of costs of congestion for one component by utilizing this model and supporting data from a Chicago-area transport study. Masters *et al.* (1991) explored the influence that information technologies have on the evolving practice of corporate traffic management in the years ahead. They observed that information technologies have been the primary driver of change in the traffic function over the past ten years through a survey of over 200 traffic managers, logistics executives, and carrier executives. Dridi *et al.* (2005) dealt with real-time control of urban traffic with emphasis on public transportation systems. The problem was to find a feasible schedule for some vehicles of some line subject to certain constrains in order to design a decision support system (DSS) that detects, analyses and resolves disturbances. Tanaka *et al.* (2007) studied the traffic congestion and dispersion of vehicles occurring on a single lane highway in Hurricane evacuation. The sensitivity and speed of the leading vehicle are the strong determining factors for traffic congestion. Satirova *et al.* (2007) used a strategic transportation planning model (START) to compare marginal congestion costs computed link-by-link with measures taking into account network effects.

In-nami and Toyoki (2007) studied the dynamic phase transitions in a two-dimensional traffic flow model defined on a decorated square-lattice using numerical approach. Hamdouch *et al.* (2007) extended the toll pricing framework previously developed for vehicular traffic networks to ones with the potential to include many modes of transportation such as walking, driving, and using public conveyance (e.g. buses, subways and trains). The report by Stewart (2007) compared link-based tolling solution to achieve both the SSO (where Total Perceived Network Travel Cost (TPNTC)) is minimized and SO (where Total Network Travel Cost is minimized. Meng *et al.*'s (2007) contribution relates to the development of a model to simulate mixed traffic with motorcycles. Gao *et al.* investigated an evolution network for traffic flow. Closely related to the current paper is the documentation by Alabi *et al.* (2007). The work utilized a rescale range approach to estimate the Hurst Exponent Value (HEV) for the traffic Exponent Value (HEV) for the traffic inflow through the main entrance gate of a university as a control measure. The difference between our paper and Alabi *et al*'s work relates to focus. Alabi *et al*'s work focused on utilization of historical data to simulate the pattern of behaviour while our paper fundamentally derived the mathematics that govern the traffic flow problem based on known principles. This difference and the need to bridge the important gap of strictly monitoring traffic flow through reliable results and models that could be adapted to different situations motivated the current study.

From the above review of various papers published on the traffic congestion problem, specific cases relating to the educational institutions' environment seem poorly treated. The necessity of addressing such an important problem has motivated the current study. This paper studies the traffic situation on the University of Lagos campus and develops a mathematical model that can be used to predict the traffic conditions at various times. This will help in developing solutions to the traffic congestion problems on the campus. The following sections are methodology, case study, results, and conclusion. The methodology explores the traffic congestion problem, and related it to flow phenomena in fluid mechanics. The case study explores the practical instance that demonstrates the application of the model. The results section follows this. The concluding remarks on the research are then shown in conclusion.

Methodology

1. Assumptions

The following assumptions are made in order to make the model applicable in practical situations:

1. Vehicles are not involved in accidents during the period of study as it is a snapshot situation.
2. Sanity and normal behaviour of drivers are expected during travel.
3. No road expansion project is embarked upon during the study period as this will affect traffic flow significantly in either a positive or negative manner.
4. Traffic obstructions due to police checks is minimal and do not affect the flow of traffic.
5. Traffic flow on a particular lane is one-dimensional. That is, no crossing of lanes to avoid traffic congestion.

Assumption 1 is made so that the static nature of the population of vehicles for the particular road or routes being studies is maintained. If dynamic (i.e. reducing, increasing, or a combination of flow patterns) is allowed, then the problem involve the utilization of system dynamics which is beyond the scope of the current work. Assumption 2 is made on the premise that the psychological state of the driver follows that of a normal human being without erratic behaviour. Thus, the driver would keep to traffic safety rules and laws at all times. For assumption 3, which relates to road expansion, it is safe to assume the width of the road as well as its length do not significantly differ from the characteristics of the road when the study started. This is to avoid expansion project-induced traffic congestion problem. Traffic obstructions are usually characteristics of roads in developing countries in which plumbing works, drainages and road repair projects could influence the results of the model. As such, assumption 4 states that all these road disturbances are insignificant and would not influence the results of the model. Assumption 5 relates to drivers behaviour in traffic. It states that drivers keep to laws and would not cross to other lanes to restrict the free movement or flow of traffic on the lane to which they crossed.

2. The mathematical framework

We employ the use of a mathematical model, which will be developed based on the concept of fluid flow. We then solve model, obtain various equations and interpret the solutions to the problem in terms of the original problem. We try to obtain the number of vehicles that can conveniently pass the road without making it congested. The movement of vehicles is considered as the flow of an incompressible fluid. A uniform mass flow rate is assumed on the roads and the flow of traffic should be of uniform and steady flow. The viscosity of the flowing fluid is determined so as to be able to predict its effect on obstructions over which the flow occurs (i.e. bad roads and round about). The pavement are referred to as streamlines as there can be no flow through these solid boundaries.

The mass flow rate, type of flow, streamline pressure variation and viscosity of the fluid is considered and used to get an ideal flow situation. The shear stresses opposing the relative motion of the layers of the fluid are considered. The magnitude of these shear stresses depends on the viscosity gradient from layer to layer. The viscosity of the fluid increases with increasing pressure. Increase in temperature affects viscosity but it is irrelevant in this study. The flow of fluids will be considered in the light of Bernoulli's equation, continuity equation, momentum equation and Lagrangian approach.

The main focus of this study is the traffic flow situation at the University of Lagos Campus main road from the main gate. This road is the major entry and exit point on campus (Figure 1). The sources of vehicles movement are off-campus (school environs), main campus, and DLI/second gate. The road network under construction, the study will be considered in three time zones, which have various entry and exit gates. They are morning, afternoon and evening periods. During the morning period, the vehicles coming out of the campus are considered negligible as it is minute compared to the number of vehicles coming into campus. This therefore exkibits a forward flow. The flow pattern is shown in Figure 2. During the afternoon period, the vehicular movements are balanced. The rates and number of vehicles moving into and out campus are close to each other and are assumed equal. There is therefore an equilibrium number of vehicles on campus at this period and the number can be estimated. The flow pattern is shown in Figure 3.

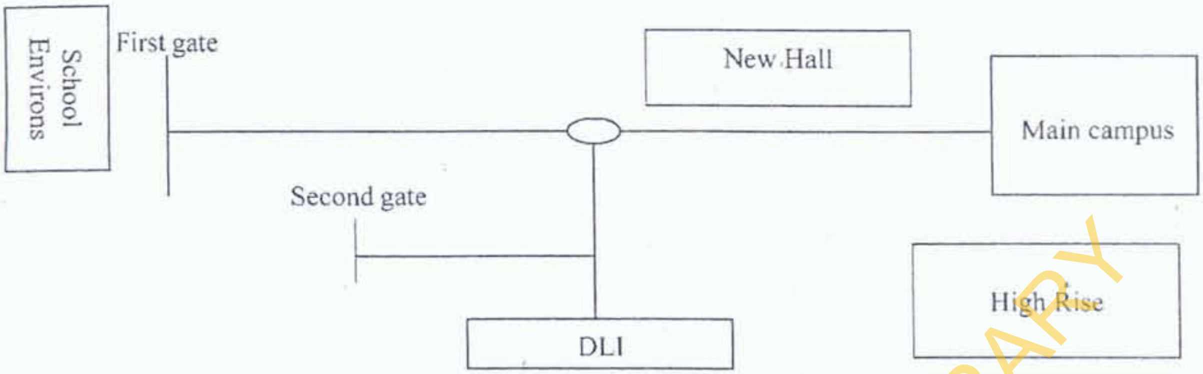

Figure 1: Pictorial view of university of Lagos transport route (intra campus)

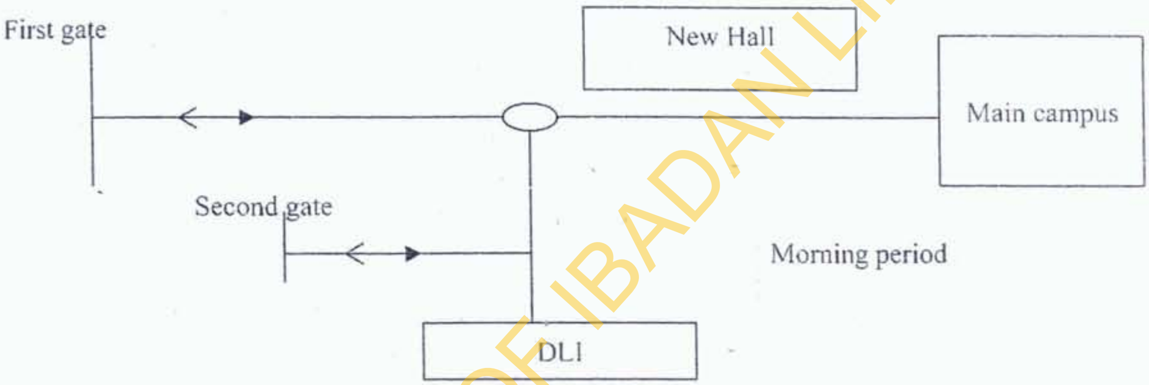

Figure 2: Flow of traffic on University of Lagos Campus (Morning period)

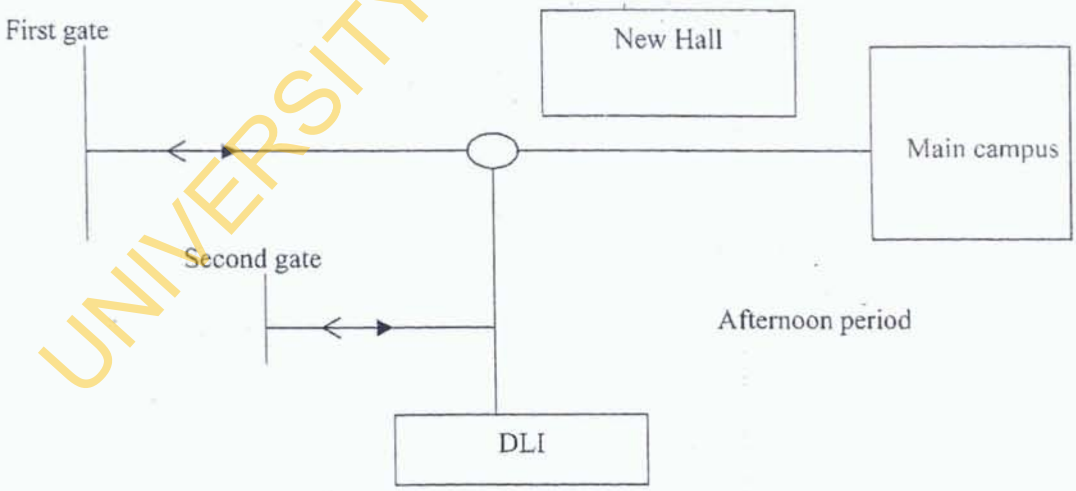

Figure 3: Afternoon period flow of traffic on University of Lagos campus

During the evening period, the vehicular movements are toward outside campus and the number of vehicles coming in the campus can be considered negligible. The flow pattern is shown in Figure 4.

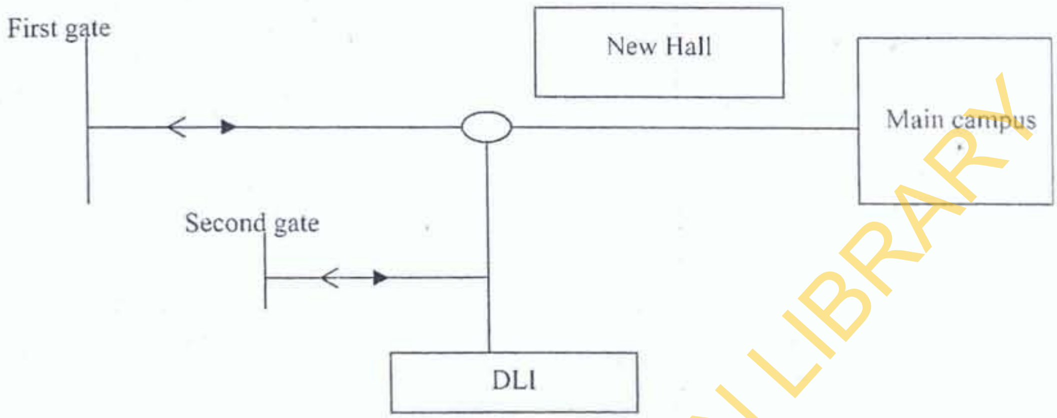

Figure 4: Vehicular movement at the University of Lagos campus (Evening period)

Newton's second law of motion can be used to relate the forces causing a vehicle to move and its resultant acceleration by acceleration in the direction of flow.

$$a = \frac{dv}{dt} = \frac{\partial v}{\partial s} \cdot \frac{ds}{dt} + \frac{\partial v}{\gamma t} \qquad (1)$$

$$a = \frac{v\partial v}{\partial s} + \frac{\partial v}{\partial t} \qquad (2)$$

where $\frac{v\partial v}{\partial s}$ is the convective acceleration and $\frac{\partial v}{\partial t}$ is the local acceleration.

For steady flow, $\partial v / \partial t = 0$. If we assume that the vehicles are moving in a curved path (as a real fluid), its velocity will be changing in direction. Generally, the motion of the vehicles will be three dimensional and its velocity and acceleration can be expressed in terms of three mutually perpendicular components. If the components of the velocity and acceleration in the x, y and z directions are V_x, V_y, and V_z and a_x, a_y, and a_z respectively. The velocity fluid can be described by

$$V_x = V_x(x, y, z) \quad (3a)$$

$$V_y = V_y(x, y, z) \quad (3b)$$

$$V_z = V_z(x, y, z,) \quad (3b)$$

Therefore the velocity at any point is given as

$$V = V_x\bar{i} + V_y\bar{j} + V_z\bar{k} \quad (4)$$

where $\bar{i}, \bar{j}$ and $\bar{k}$ are unit vectors in the x, y, and y directions.

$$\Delta V_x = \frac{\partial V_x}{\partial x}(\Delta_x) + \frac{\partial V_x}{\partial y}(\Delta_y) + \frac{\partial V_x}{\partial z}(\Delta_z) \quad (5)$$

The acceleration in the x direction is

$$a_x = \frac{\partial V_x}{\partial x}\frac{dx}{dt} + \frac{\partial V_x}{\partial y}\frac{dy}{dt} + \frac{\partial V_x}{\partial z}\frac{dz}{dt} = V_x\frac{\partial V_x}{\partial x} + V_y\frac{\partial V_x}{\partial y} + V_z\frac{\partial V_x}{\partial z} \quad (6)$$

Similarly $a_y = V_x\frac{\partial V_y}{\partial x} + V_y\frac{\partial V_y}{\partial y} + V_z\frac{\partial V_y}{\partial z}$ (7)

$$a_z = V_x\frac{\partial V_y}{\partial x} + V_y\frac{\partial V_y}{\partial y} + V_z\frac{\partial V_y}{\partial z} \quad (8)$$

If the volume flow rate of vehicles is denoted by Q (measured in m^3/s), then for an ideal fluid with fluid velocity U and cross-sectional area A, then

$Q = AU$ = volume passing,

If U is the velocity at any radius r, the flow δQ through a small portion of radius r and thickness δr will be:

$$\delta Q = 2\pi r\, \delta r\, U \quad (9)$$

$$Q = 2\pi\int_0^R Ur\,dr \quad (10)$$

If we ignore the variation of velocity over the cross-section of the road, the velocity is assumed to be constant and the mean velocity is given as:

$$\text{Mean velocity} = \frac{Q}{A} \qquad (11)$$

We will assume that no car will be made or destroyed within the campus premises.

$$\text{Therefore,} \begin{bmatrix}\text{Number of cars}\\ \text{entering per unit}\\ \text{time}\end{bmatrix} = \begin{bmatrix}\text{Number of cars}\\ \text{leaving per unit}\\ \text{time}\end{bmatrix} + \begin{bmatrix}\text{Number of cars}\\ \text{within campus}\\ \text{premises}\end{bmatrix} \qquad (12)$$

$$\text{Assuming steady flow, then} \begin{bmatrix}\text{Number of cars}\\ \text{entering per unit}\\ \text{time}\end{bmatrix} = \begin{bmatrix}\text{Number of cars}\\ \text{leaving per unit}\\ \text{time}\end{bmatrix} \qquad (13)$$

We know that continuity equation states that $\rho \delta A U = \text{constant}$ (14)

In this case, $\rho_1 A_1 \bar{U}_1 = \rho_2 A_2 \bar{U}_2 = m$ (15)

Where A = cross-sectional areas of road
ρ = No. of vehicles per unit area of road
State 1 = entry conditions
State 2 = exit conditions

Assuming an incompressible flow, $\rho_1 = \rho_2$

Therefore $A_1\bar{U}_1 = A_2\bar{U}_2 = Q$ (16)

Using the Bernoulli's equation with the addition of terms for the energy losses due to friction and separation, the pressure lost per unit volume due to friction is given as:

$$\Delta P = \frac{4fL\rho V^2}{2D} \qquad (17)$$

Assuming f = constant, $\Delta P = \frac{4L\rho V^2}{D}$ (18)

where L = length of road
ρ = no. of vehicles per cubic meter
V = rate of passage of vehicles per unit time
D = width of road
K = constant
ΔP = change in traffic congestion or pressure on roads

This equation considered the pressure a fluid causes in a pipe as similar to the congestion of the road caused by vehicles when the number is more than what the road can conveniently convey. Considering the rate at which vehicles get to their destinations rather than the rate of passage, an alternative equation can be obtained.

We have $V = \dfrac{Q}{\text{Road cross-sectional area}} = \dfrac{Q}{\pi D^2/4}$ (19)

Substituting equation (17) in (19) yields: $\Delta P = \dfrac{64 f L \rho Q^2}{2D\left(\pi D^2\right)^2}$

This can be reduced to: $\Delta P = \dfrac{3.24 f L \rho Q^2}{D^5}$ (20)

Assuming f = constant, $\Delta P = \dfrac{K_1 L \rho Q^2}{D^5}$ (21)

where K_1 = constant

The steady flow energy equation can be used to calculate the pressure at any point along the road and may be seen to represent the overall road network pressure loss relationship. If the route is wodelled as a series of roads, the pressure loss along this road is the sum of the pressure losses along each. The roads are shown in Figure 5.

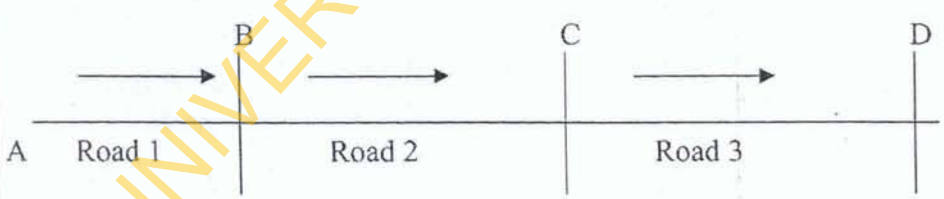

Figure 5: Road 'pressure' indicating traffic situation

The pressure loss is given as a sum of the individual pressure losses. Thus,

$$\Delta P_{A-D} = \sum \left[\frac{4\rho f (L + Le) Q^2}{2DA^2} \right] \tag{22}$$

where Le is the sum of the equivalent lengths for all separation losses in that particular route. If however the roads are built in a parallel form as shown in Figure 6:

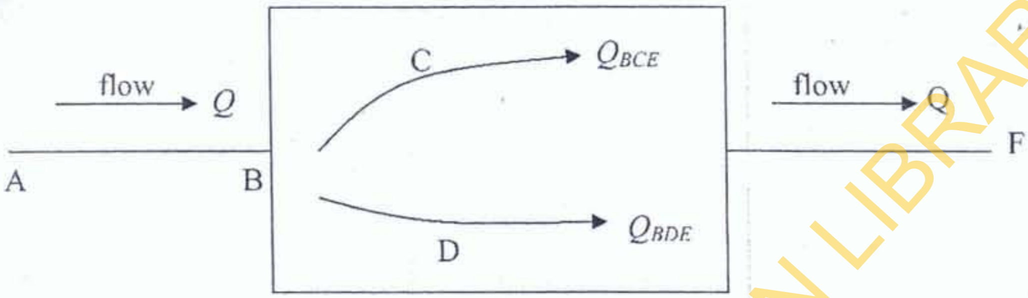

Figure 6: A pictorial view indicating traffic flow on campus

The pressure loss from A to F as identical along route BCE and BDE, but the flow will divide inversely as to the resistance of either path.

$$\Delta P_{BCE} = \left[\frac{4\rho f (L + Le) Q^2}{2\Delta A^2} \right]_{Along\ BCE\ \&\ BDE} \tag{23}$$

It shows that $\Delta P_{BCE} = \Delta P_{BDE}$. It follows that the separation losses associated with valves changes as the valve setting changes. Therefore the flow in any looped path can be determined by:

$$\frac{Q_{BCE}}{Q - Q_{BCE}} = \sqrt{\left\{ \frac{\left[\frac{4\rho f (L + Le) Q^2}{2\Delta A^2} \right]_{Route\ BDE}}{\left[\frac{4\rho f (L + Le) Q^2}{2\Delta A^2} \right]_{Route\ BCE}} \right\}} \tag{24}$$

If the road reduces in width at any particular portion, the rate of discharge of vehicles per unit width increases. If the losses are neglected, the specific energy remains constant (Figure 7).

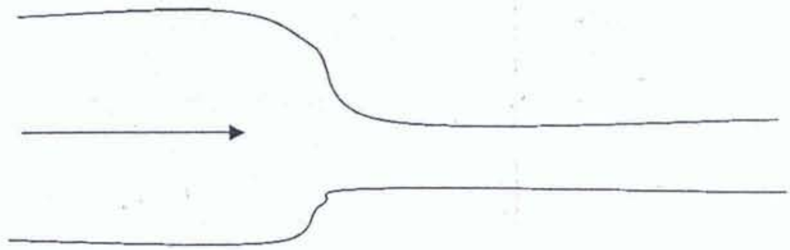

Figure 7: Road width reducing to a lower width

If the contraction is followed by an expansion, it can be used for flow measurement.

For continuity of flow

$$B_1 D_1 \bar{V}_1 = B_2 D_2 \bar{V}_2 \qquad (25)$$

Applying Bernoulli's equation to the upstream and through section,

$$D_1 + \frac{\bar{V}_1^2}{2g} = D_2 + \frac{\bar{V}_2^2}{2g} \qquad (26)$$

Substituting $\bar{V}_1$ from (25), $\frac{\bar{V}_2^2}{2g}\left(1 - \frac{B_2^2 D_2^2}{B_1^2 D_1^2}\right) = D_1 - D_2$

$$\bar{V}_2 = \sqrt{\frac{2gh}{1 - \left(\frac{B_2 D_2}{B_1 D_1}\right)^2}} \qquad (27)$$

Volume rate of flow, $Q = B_2 D_2 \bar{V}_2 = B_2 D_2 \sqrt{\frac{2gh}{1 - \left(\frac{B_2 D_2}{B_1 D_1}\right)^2}}$ (28)

If the flow pressure decreases in the direction of flow, an expression for the mean velocity in laminar flow is obtained as: $V_m = \frac{D^2}{32\mu}\left[\frac{-d}{dx}(p + \rho g z)\right]$ (29)

where dp = pressure drop over a short length dx of road
D = road width
z = the shortest distance between inlet and exit of length dx.

By the continuity equation, we have: $\frac{p_1 - p_2}{\rho g} = \frac{32\mu VmL}{\rho g D^2}$ (30)

where L = length of the road.

This can be re-written as:

$$\frac{p_1}{\rho g} = \frac{p_2}{\rho g} + h_f \quad (31)$$

Where $h_f = \frac{32\mu LVm}{\rho g D^2}$

At a point of sudden expansion of the road, we have: $\frac{p_1 - p_2}{\rho} = -\int_{p_1}^{p_2} \frac{dp}{\rho}$ (32)

At any point of sudden contraction of the road: $P_c = \frac{\rho V_2^2}{2}\left(\frac{Vc}{V_2} - 1\right)^2$ (33)

At the entrance of the road, the flow situation is similar to that of contraction of the road and pressure drops sharply to its lowest value at the entrance and increases thereafter. The pressure loss is given by:

$$\Delta P = \frac{\rho V_2}{4} \quad (34)$$

Other factors that can affect the traffic situation of the roads are the length of the road, population of vehicles, width of road, time of the day, arrival and departure rate, entry and exit points and number of vehicles from sources.

Case Study

On the University of Lagos campus, approximately 1 out of 15 people has a vehicle. For a campus population of 20,000 people, about 1,500 vehicles are used on campus. The roads can carry 2 vehicles per lane and there are 2 lanes. The length of the road considered is approximately 800m and has a width of 30m. A car is assumed to have a length of about 10m. This implies that about 80 vehicles can be on the road at the same time i.e. 160 vehicles per lane and 320 vehicles for the two lanes. The parking lots within the campus can accommodate 500 vehicles altogether. The school environs can accommodate an unlimited number of vehicles. For the morning period (8am – 11am), the vehicular movement is wostly towards on-campus. Movement off-campus is considered negligible. If at any single point of entry, we have 2 cars passing per second

and 0.1 car per m^3, then using the traffic congestion formula: $\Delta p = \frac{4f\,L\,\rho v^2}{2D}$.

The change in traffic congestion can be calculated as: $\Delta p = \frac{4\times 1.8\times 800\times 0.1\times 2^2}{2\times 30} = 38.4$,

Given that $f = 1.8$, $D = 30$m, $L = 800$m. This is the ideal value of congestion on the road. However, in the morning period, if we have 5 cars passing per second, then,

$\Delta p = \frac{4\times 1.8\times 800\times 0.1\times 5^2}{2\times 30} = 240$. At this rate, the road is congested The rate of vehicle passage is 5 cars/sec. The velocity of the car is however a function of time. Therefore, we can write that $v(t) = at^2 + bt + c$. If we are considering the rate at which vehicles get to their destination, then using the formula:

$\Delta p = \frac{3.24 F L \rho Q^2}{D^5}$. For the ideal situation, $Q=2$, $\Delta p = \frac{3.24\times 1.8\times 800\times 0.1\times 2^2}{30^5} = 7.68\text{x}10^{-5}$. In the morning period, $Q = 10$ as the vehicles arrive at their destination at the high number.

$$\Delta p = \frac{3.24\times 1.8\times 800\times 0.1\times 10^2}{30^5} = 1.92 \text{ x } 10^{-3}$$

To get a good flow of traffic, we use the formula for mean velocity to get the speed limit for vehicles, $V_m = \frac{D^2}{32\mu}\left[-\frac{d}{dx}(p+\rho gz)\right]$, where V_m = mean velocity.

The change in congestion in the morning is given by: $\Delta p = \frac{\rho v^2}{4} = \frac{0.1\times 2^2}{4} = 0.1$.

In the afternoon period (1pm – 4pm), the vehicular movement is approximately equal in both directions. It is also at a reduced rate. Therefore, if at any point of entry we have 1 car passing per second (0.1 car per m^3.), using the change in traffic congestion formula, $\Delta p = \frac{4f\,L\,\rho v^2}{2D}$, f = 1.8, and K = 400 (since we are considering 2 lanes)

$$\Delta p = \frac{4\times 1.8\times 400\times 0.1\times 1^2}{2\times 30} = 4.8$$

This is lower than the ideal value of congestion on the road (38.4). Therefore, there is free flow of traffic in the afternoon unlike the morning period which has a value of 240.

Considering the rate at which vehicles get to their destination in the afternoon, Q=5 as it is a 2-way traffic situation. Then, $\Delta p = \frac{3.24\times 1.8\times 400\times 0.1\times 5^2}{30^5} = 2.4 \times 10^{-4}$.

This value is lower than the ideal value calculated earlier indicating a free flow of traffic. For the evening period (5pm – 7pm), the traffic situation is similar to that of the morning period except that the direction of traffic flow is reversed. If we plot a graph for the traffic congestion at all periods of the day, we obtain the graph shown in Figure 8.

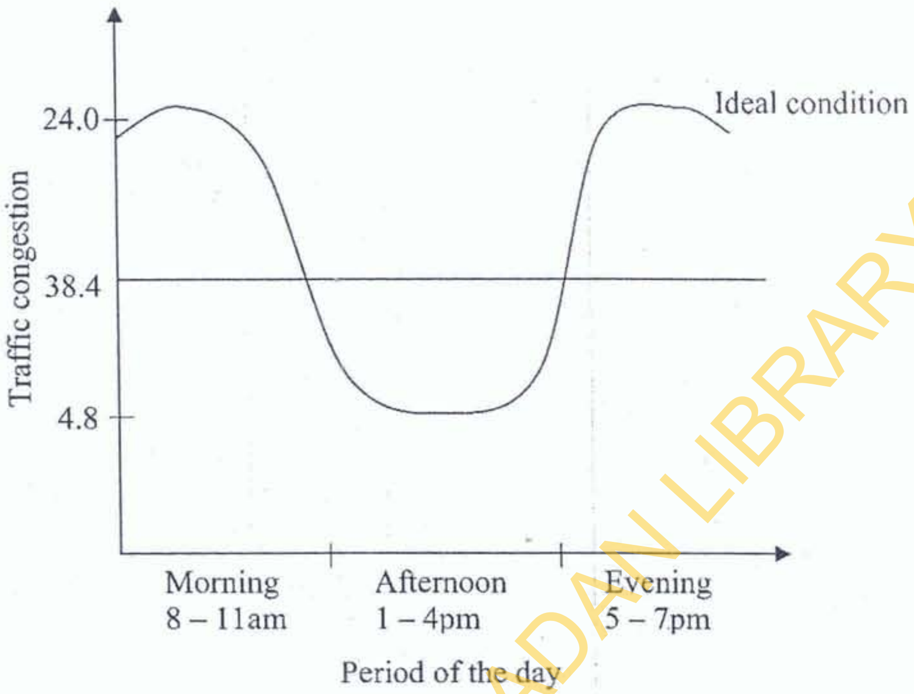

Figure 8: Traffic congestion flow diagram

Results

From the study, we can see that for a good road network, the following conditions must be satisfied: (i) continuity must be satisfied at each junction of the network, i.e. the total flow into each junction must be equal to the total flow out of the junction. (ii) The algebraic sum of pressure head drops around each circuit must be zero i.e. the pressure head drop due to forward flow must be equal to that due to backward flow. The results obtained above show that the traffic situation is more congested at the morning and evening periods when there is massive movement of vehicles towards on-campus and off-campus respectively. Also the values obtained indicate that the traffic situation is affected by the speed of the vehicles, rate of arrival and departure as well as periods of the day.

Conclusion

This work has developed a mathematical model that can be used to predict the traffic situation on the University of Lagos campus. Based on this study, it can be concluded that the traffic congestion situation on University of Lagos campus can be monitored and controlled using the equations obtained for changes in pressure provided the supporting conditions are fulfilled. This paper is just groundwork of the traffic condition on campus and it opens up a lot of possible areas for further study and work to be done. Further study can be done on the effect of obstructions such as fuel queue, on the general traffic situation. Also a study can be done to find out how a larger road can affect the traffic situation. Work can be done to show what the traffic condition will look like if another entry point like the main gate is built at the opposite end of the campus. Illegal parking at non-designated parking lots can have a great effect on the traffic conditions on campus. A further study can be carried out in this area. It was observed that alternative modes of transportation can change the traffic situation. A study on the effects of this if encouraged is worthy of study. These are just a few of the numerous areas of study that can spring forth from this work.

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