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The application of fractal box dimensions in predicting the emission characteristics of colliding sawdust particles for sustainable sawmilling

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Abstract

The prediction of emission characteristics of sawdust particles immediately after the cutting operation from the interaction of band saw's blade and plank is a growing research area. Still, a wide gap exists with respect to understanding the behaviour of sawdust particles as they collide with one another. Previous efforts have focused on non-collision states of sawdust particles. However, in real life, collision of particles must occur, with several particles colliding after the cutting operation. This paper establishes a new perspective of the fractal properties of sawdust particles in motion as a motivation to understanding how to control its toxicity of effects on sawmill workers and maintain sustainable sawmilling activities. In particular, the possibility of predicting the fractal dimension of the randomly moving sawdust particles in sawmills that is generated as fractal curves using the combination of probabilities and theoretical fractal dimensions is investigated for the first time. Cases were established on the possible representations of the theory and practice. As an example, four cases were designed around varied number of fractal pattern combinations drawn out of five and fifty different probabilities combinations, ten different random number generating seed values and maximum of four fractal curves generation iterations as driven parameters. Preliminary study of the differences between theoretical fractal box dimension recorded a maximum absolute percentage error of 7.24% for fractal curve associated with fractal pattern five (i.e. Koch 5). In all the cases studied, average absolute percentage error decreases between 3.52 ± 1.18 *and* 1.51 ± 1.14 *while the correlation coefficient (R2) decreases between* 0.9315 *and* 0.7365 *from case 1 to case 4, respectively. It is concluded that the model is a good predictor of sawdust particle emission at colliding states from cutting operation. This is reflected in the fact that the higher the number of fractal patterns (generators) in a study case, the smaller the correlation coefficient between average estimated fractal box dimension and predicted fractal dimension of the sawdust particles in motion in the sawmill.*

***Keywords**: Fractal, fractal box, fractal dimension, sawdust particle, sawmill, sustainability*

1. Introduction

For several years, sustainable development has been rigorously pursued at all levels of industrial setups by various stakeholders in the environment (Ackom *et al.*, 2010; Paivinen *et al.*, 2012; Chalikias *et al.*, 2010; Cheo *et al.*, 2011) and its growing importance is extending to small manufacturing enterprises (SMEs) such as the sawmill industries. As SMEs are becoming known as important economic drivers, innovative sustainable practices for wood processing activities in sawmills must be developed and implemented for economic growth, environmental protection, and the development of the society. For example, the term sustainability to sawmill reflects how strategically and operationally it has been able to control its hazards and protect operators against hearing losses and inhalation/ingestion of sawdust during the cutting operation. This is aimed at avoiding heavy health liabilities to sawmills and sustain the clean environment that has been set as standard for sawmill practices.

Over the years, this drive for sustainability has been coupled with the increasing demand to solve the problem of occupational hazards in workplaces, and has strengthened the urgent need for a sustainable methodology for effective practices in sawmills. Addressing the problem of occupational health hazards associated with health risks in sawmill industries will play a central role in improving total quality work culture in the workplace and should be the focus of many important research aimed at minimizing the overall safety, health and environmental cost incurred by sawmill industries as well as achieving sustainability of sawmilling. The sawmill industry, which currently exposes operators to the occupational hazards of sawdust inhalation, accidental ingestion of sawdust and associated health risks, has an implied importance in today's industrial economic scenario, particularly in developing countries, which are .focusing on agricultural development to boast their economic performance. Sawdust particles are wastes of sawn timbers, which are produced when timbers to be sawn are in contact with band saws and planks (Salau and Oke, 2010b). It is implied from the foregoing that higher levels of production outputs in agriculture would lead to increased timber processing activities and subsequently, higher levels of sawdust generation. However, the process of sawmilling must be sustainable. Thus, sawdust generation and its effects on operations should be a strong research focus towards understanding the sawdust emission process and developing scientific approaches for use in the control of its effects on operators and achieving sustainable wood processing.

Globally, the common environmental approach in controlling sawdust toxicity levels in operators is to use a collection of tests, including their blood samples, eye conditions and hearing impairment. The conclusion of these tests usually aims to decide on whether the operator is fit to continue on the work or not, and not preventive in nature. This approach is time consuming, costly, and involving. Also, very few subjects could be tested at the same time. Another approach is to utilize a set of subjective indices in evaluating the workplace. For this approach, since the results depend on the experience of judges, the challenge is the limitation of not having quality inputs from participants during the evaluation process since those assigned to represent units for inputs into the assessment forms are not always the best and most experienced operators or stakeholders in the system. The approach has also been criticised as limited. From the weaknesses of the existing approaches highlighted above, it is safe to conclude that most of the existing approaches are not sufficient to solve the problem of sawdust particle emission from the cutting equipment and its relationship with sawdust inhalation by the operator, ingestion, as well as the short-, medium-, and long-term effects of these on the operator's health. This area is scarcely investigated.

Based on these challenges mentioned above on the available approaches in this research domain, scholars have started to propose modeling approaches as solutions to the problem. Thus, there is need for a model to solve this problem, which will improve on existing studies in the area. The current work is an attempt to bridge this gap. The problem solved relates a situation in which plywood are sawn from the band saw (machine) in sawmill. During this operation, sawdust particles are emitted in large quantities and collide. Most of these emissions are concentrated around the work area where the operator carries out his duties. During the cutting operation, the eye conditions of the operator usually changes as a result of deposits of sawdust on his eyeballs. These changes in eye conditions are also accompanied with sneezes in reaction to the hostile work environment. The accumulation of this cycle of problems and reactions of the operator over a period may be harmful to the health of the operator. The prolonged exposure to these sawdust particles may cause serious health injuries and even death to the operator. The main interest in this research is to study the motion of the sawdust particles at collision and characterize it such that the information could be used for the control of sawdust motion and in evaluating possible duration of exposure of sawmill operators. However, the central focus is on the motion and the relative movement of the sawdust to the human operator and others involved in the sawmill operations, particularly, when two particles collide.

The characterizing approach is new to this field and is applied for the first time to sawdust particle motion modeling. The main thrust of this work is to model the motion of two sawdust particles colliding after emission from the band saw as a fore-running model to the complex analysis of several sawdust particles (i.e. more than two) colliding at the same time. This is achieved by determining whether or not the average box dimensions of sawdust particles in motion form a randomly generated fractal curve involving multiple generators (i.e. rules) and can be predicted using the knowledge of different appropriate probabilities and theoretical fractal dimensions (Feder, 1988; Zmeškal *et al*., 2001).

The management of sawmills require huge investments in machinery and maintenance cost, and hence must keep all these associated costs under control if the business is to be profitable. The associated cost due to accidents, accident cost itself, and compensation cost as a result of any accident that the operator may be involved in. However, to avoid these costs, stakeholders have resulted to capturing data related to health and environmental concerns, which are mainly historically treated to date. The use of the data for preventive basis has therefore caused a strong drive towards obtaining modeling and prediction information about sawdust particle movement. This is the main aim of the current work. The paper is sectioned into four parts with section 1 providing a motivation on the need for the current work. Further, the literature is surveyed to identify the research gap in section 2. Section 3 presents the model formulation and development, which provides a very strong basis for the current work. Section 4 presents simulation results. In section 5, concluding remarks are given.

2. Existing Contributions

Sawdust emission studies soon after cutting operation in the sawmill are a new class of problems with the characteristics different from the conventional particle dynamics theory as it integrates theories in particle dynamics, environment and atmospheric conditions. From a survey of the literature on emission and studies related to fractal box dimension, there is a conspicuous absence of models of sawdust particle motion (Salau and Oke, 2010a) using a fractal dimensional approach. This approach, which compares the motion behaviour of two sawdust particles that collide after being emitted from the band saw during the cutting operation, has not been documented. Although there is a rich literature on fractal box dimensions and only a limited number of sources with which we can compare the study are reported here, from the authors' knowledge and experience in working in the area no attempt has been made to solve the problem of the motion of sawdust emission from the band saw with collision among the particles considered.

In the ensuing review, literature is searched for investigations relevant to the application of fractal box dimensions to sawdust particles in collision at sawmills, which validates the need for the current study. An area in which research in the literature related to this work concentrates on, is fractal box dimensions. Bouboulis *et al.* (2006) obtained exact values for the box counting dimension of the Recurrent Bivariate Fractal Interpolation Surfaces (RBFISs) and a methodology to approximate any natural surface using RBFISs was developed. The work did not address the practical application of the solution developed to the motion of sawdust in collision. Elsewhere, Huang and Peng (2008) conducted a study, which constructed fractals defined by more than two forbidden words and also presented an efficient method of approximating their box dimensions. However, information concerning application of this model in real life case studies of sawmills is missing in the work. Yet in another study, Ruan *et al.* (2009) presented a new method to calculate the box dimension of a graph having a continuous function and obtained the box dimension formula for linear fractal interpolation functions. Li *et al.* (2009) further extended the frontier of knowledge by considering an efficient box-counting based method for the improvement of fractal dimension estimation accuracy. The authors proposed a new model to assign the smallest number of boxes to cover the entire image surfaces attached at each selected scale to yield accurate estimates. However, neither of these studies discussed any practical issues relating to the motion of sawdust particles in collision. In a study by Buczkowski *et al.* (1998), discussion centered on data scatter obtained by a modified box-counting method. The study concluded that large ε's usually characterize the embedded surface of the whole object and that small ε's approximate the dimension of the substructure for discontinuous objects. The practical application of the work did not extend to the case of sawdust, thus creating a research gap which this work attempts to fill.

A closely related set of studies in the field of fractal theory, which are related to the current paper are as follows. Peitgen *et al.* (1991) and Mandelbrot (1983) addressed the theory of fractal for the classroom and its geometric relevance to nature, respectively. There two studies provide insights into the current study but have not provided sufficient details on the practical application of the work to sawmill environments. Further theoretical developments on particle motion was proposed by Scheinerman (1996) by an exploration to the concept of dynamical system from which the current study has its source. The numerical study on dynamical system was proposed in Nusse and Yorke (1998). Also, from the chaos perspective, Alligood *et al.* (1997) studied dynamical systems. Although beneficial knowledge to the area of current study could be obtained from the works of Moon (1987), Scheinerman (1996), Alligood *et al.* (1997) as well as Nusse and Yorke (1998) and Stefanski (2000), no detailed understanding relevant to sawmill application was presented. This gap is therefore explored as its adequate knowledge would help in reducing the toxicity effects of sawdust in sawmill workers. The literature was reviewed to identify studies that address sawmill activities and in this respect similar to the present work. Such studies are expected to be implementation of sawdust particle motion dynamics in sawmills in the developed, developing or underdeveloped countries. This direction of literature review indicated that sawdust particle motions as it relates to the machine operator, atmospheric conditions, sawdust emission from the bandsaw machine and its control, is poorly implemented in sawmills. The literature was further searched to possibly identify any documentation that has reported new insights on sawdust particle motion in the scenario described above and from modelling perspectives also. The findings from the search effort revealed that two cases have been documented as papers authored by Salau and Oke (2010a, b). In one (Salau and Oke, 2010a), the focus was on the transformation of the sawdust particle trajectory, while the second paper (Salau and Oke, 2010b) emphasized fractal aggregation of sawdust particles. A critical analysis of the research reported in these two papers revealed the omission of a concrete model or methodology that studies the collision of two or more particles as they form pattern as soon as they are produced by the bandsaw in the sawmill. The reality is that several sawdust particles are emitted from the interaction of the bandsaw blade and plywood/timber being sawn. The clouds of these particles are very thick at a position very close to the blade-wood contact area, where several particles collide. The collision of these particles change the directions of individual particle and the pattern formed.

Yet another area that work has been done in close connection to sawdust and sawmill research are the studies due to Randhawa *et al.* (1994), Kersavage *et al.* (1990), Singer and Donoso (2007) and Hammed *et al.* (2000). Radhawa *et al.* (1994) presented a simulation modeling environment developed for sawmill design and analysis, which facilitates flexibility in modeling different sawdust production scenarios. Kersavage *et al.* (1990) developed mathematical models to determine volumes of the different types of materials produced in edging hardwood lumber. They concluded that the edging models developed could be useful to analysts studying sawmill fibre balances as well as lumber recovery and grade trade-offs. Neither the robust model of Randhawa et al. nor the practically-oriented work of Kersavage et al. addressed the sawdust particle motion of colliding particles in motion in the sawmill. Singer and Donoso (2007) investigated a collaborative approach and proposed a model for optimizing production and inventory planning decisions within a system of plants. The methodology models the sawmill production process as a two-transformation, two-inventory stages with supply chain decisions consisting of timber transfers among plants. They applied the model at 11 Chilean sawmills and found an opportunity to increase profit through a higher utilization of the capacity and a better assignment of production orders. Hammed *et al.* (2000) determined the airborne dust levels inside two different (ventilated and unventilated) wood working shops in Egypt and found that these levels were higher than the recommended values by the Egyptian environmental law. It was reported that the highest frequency of aerodynamic size distribution of airborne wood dust was detected at a diameter of 4.9 µm, which was recorded during a machining operation. Neither the article by Singer and Donoso (2007) nor the one by Hammed *et al.* (2000) addressed the issue of sawdust collision after emergence from the bandsaw. However, understanding this collision of sawdust is important for monitoring and control of sawmills.

Another area in which related research in the literature concentrates is the economic and experimental benefits of sawdust particles in serving as absorbents and sorption materials (Hamid and Saffle, 1965; Hamadi *et al.*, 2001; Taty-costodes *et al.*, 2003; Jadhav and Vanjara (2004); Šćiban *et al.*, 2006; Hamdaoui, 2006). Other research focus on sawdust particles include its use as base material for boilers (Akira *et al.*, 2002), catalyst in the removal of mercuric ion from aqueous solutions (Ansari and Raofie, 2006). Still, other aspects could be found in Demers *et al.* (1997), Arif *et al.* (2003), Udoeyo and Dashibil (2002), Ajayi and Owolarafe (2007); and Svedberg *et al.* (2004), the risk of childhood cancer by children of sawmill workers through their paternal exposure to sawdust (Heacock *et al.*, 2000), and prevalence of asthma due to sawdust inhalation in sawmill workers (Siracusa *et al.*, 2007). None of these studies considered the sawdust dispersion problem where detailed investigation is made on how the collision of two sawdust particles at emission from the band saw affects the motion of the particles afterwards either in a collided form or when they are separated again. Also, practical information particularly relevant to small scale sawmill setup are scarce in the studies. These practical details may entail the movement of the sawdust as they are transferred to the lungs through the inhalation system or assimilation through the skin of the operator. These missing and scanty details tend to limit the practicality of the results arising from the works of Salau and Oke (2010a,b) to sawdust particle motion, which is the subject of the present study. The foregoing are important characteristics considered in the problem of sawdust particle emission in collision.

3. Theoretical Model Equations and Method of Investigation

The fractal box dimension functions and the fractal curve concept are the models used in this study to guide our understanding of the motion of sawdust particles as they collide soon after being produced from the cutting action. The fractal box dimension and fractal curve are well established concepts in the physical and non-linear sciences, and offers a new contribution when applied to the problem of sawdust particle collision after being emitted from the cutting action of the band saw blade on planks or timbers. Advancements in applications have revealed extensive adaptation of fractal principles in the physical systems of nature and many human artifacts, which are believed to be of non-regular shapes. The motion of sawdust particles, which follows an irregular shape suggests that it could be described by non regular geometric shapes of the standard geometrics such as triangle. This concept is listed in this work.

In order to apply the fractal box dimensions and fractal curve models, the sawdust particles produced from the cutting operation is visualized as randomly floating in the air. This is described as such to capture the features of the anticipated fractal box dimensions and fractal curve. To move in different directions (x, y, z), two sawdust particles freed to overcome resistance against movement in the air. The functional force and the drag force act separately on each of the two particles. However, their action are simultaneous on the particles moving in a fluid. Their actions slow down or resist the movement of the sawdust particles. The two forces mentioned earlier are predicated on the shape of particle as well as its surface. As soon as the sawdust particles are produced from the interaction of the cutting saw with the woodpiece, they collide and form complex chains of movements, which may be computationally challenging.

However, we are limiting the study to the collusion of two sawdust particles, in the current work as further studies may investigate what interactions of more than two particles exists after collusion. The two sawdust particles are driven in a non-linear version based on the combined effects of the wind forces that blows the sawdust particles about, the emission forces from the band saw, the force of gravity, the geometry of the sawdust particles as well as the air particles. There are other factors, which impinge on the movement of the sawdust particles. These are the efficiency of the sawmill operation, which is dependent on the age of the equipment, humidity, temperature as well as the size of sawdust particle and its density. However, these factors were assumed constant in view of limited resources available to actualize the measurements; many of the equipment to utilize are unavailable locally and may cost enormous amount of money not within the reach of the researchers. For example, the wind velocity is an important factor that influences the direction of motion of sawdust particles through the action of tending to carry the particle in its direction of provision. But the prohibitive costs of acquiring equipment for measuring wind speed, humidity and temperature as well as that of collecting relevant practical data has hindered this research and led to the issue of assuming that these parameters have no dynamic effect on the sawmill cutting operation. The operator in the sawmill is assured to work under normal atmospheric conditions of temperature (room air temperature, 27^0C). The sawmill operator is assumed to be working efficiently and also follows standard acceptable practice in saw milling. Although ignoring these factors has implications on the accuracy of the predictions arising from the present work, the study serves as a basis for further investigations in an area where scanty information is available. If accurate figures of wind speeds and velocity over the years for dry, rainy and harmattan seasons were available for use, then it is possible to know the relationships between wind speeds, sawdust particle speed before and after collision of two particles studied, and the quantity of sawdust produced for different spaces of wood could be established and used to estimate the minimum and maximum quantities of sawdust inhalable by the sawmill machine operator, information from this could be useful to government in enacting laws and regulations that no operator or its co-worker in the sawmill should spend beyond certain years doing work in the sawmill environment. This is to safeguard the health of the workers and reduce safety liabilities of enterprises in an indirect manner.

The formulation of the model that establishes the number of square boxes used to cover the characteristic length of the fractal curve (x), the corresponding number of square boxes needed to cover the fractal curve at specified x (i.e y), and the fractal box dimension of the fractal curve being studied (i.e D), is guided by the fact that y varies directly as the power of x. Thus, the estimate of the fractal dimension of energized randomly generated fractal curve from studied cases using the proportional principle could be stated as:

$$y \alpha x^{D} \tag{1}$$

Where the constant of proportionality has been rightly substituted for., we have

$$y = Kx^{D} \tag{2}$$

Further, logarithmic operation could be carried out on both sides of equation (2) to obtain

$$\mathrm{Log}(y) = D\,\mathrm{Log}\,(x) + \mathrm{Log}\,(K) \tag{3}$$

This equation (3) is obviously a straight line graph in 2-dimensional Cartesian coordinate system. The slope of the best line to the collection of Log (x), and Log (y) coordinates would be an estimate of D. Recall that our interest entails observing the average distance moved by the sawdust particles away from the woodworking machine that produce it. It then means that an association could be formed between D_{ave} (estimated fractal box dimension of randomly generated fractal curve in each studied case), D_i (theoretical fractal dimension of "Koches" in each studied case), N (number of Koch patterns in each studied case), and P_i (probabilities of "Koches" in each studied case). Here, we introduce "Koch" as a generic name that describes patterns generated for fractal analysis in five different forms of Koch 1, Koch 2, Koch 3, Koch 4 and Koch5. Notice that rules that define each of the these patterns would be stated later in the work. It thus becomes necessary to define the average estimated fractal box dimension obtained from equation (1) to (3) in a way that satisfy the main study objectives as in equation (4) below:

$$\sum_{i=1}^{N} P_i D_i = D_{ave} \tag{4}$$

Notice that

$$\sum_{i=1}^{N} P_i = 1 \tag{5}$$

Having defined the guiding equations for the model, it may be important to state the rules for implementing the model.

3.1 Rules for Pattern Implementation

Considering the model discussed in this work, there are basically five rules that guide their implementation, and are referred to as Koch 1, Koch 2, Koch 3, Koch 4, and Koch 5. The basic difference among these rules is the number of lines with which any straight line would be replaced with. This ranges from 4 to 10 in the cases considered. The principle behind this is to consider the motion of two sawdust particles relative to each other after collision with themselves. Collision of sawdust particles occur after being produced by the iteration of the cutting blade of the band saw and the plank/timber to be sawn. Schematic diagrams of the motion of two sawdust particles under the five rules are shown in Figures 1 to 5. From Figure 1 (koch 1), the two sawdust particles at collision are at point A. Initially, point C coincided with point A but the sawdust particle was thrown off from the collision state to a point C, where it starts moving in the pattern CD to DE to EF to FG. The inclined angle of movement is θ. We are considering the two sawdust particles in which sawdust particle 1 is assumed to be static at the time

Koch1

Replace any straight line encountered with the pattern indicated below (Figure 1). The four (4) equal lines segment are one third of the length of the straight line encountered. Angle θ is arbitrary and it indicated the orientation of the straight line encountered relative to the horizontal direction.

Koch1 Pattern with equal 4-lines segment. The angle between the middle 2-lines segment is 60°

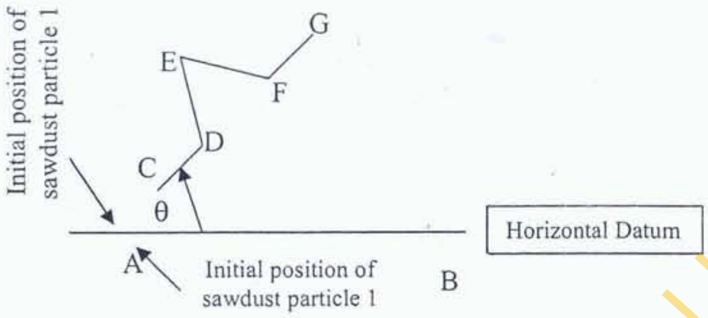

Figure 1. Pattern for Koch1

Koch2

Replace any straight line encountered with the pattern indicated below (Figure 2). The five (5) equal lines segment are one third of the length of the straight line encountered. Angle θ is arbitrary and it indicated the orientation of the straight line encountered relative to the horizontal direction.

Koch2 Pattern with 5-equal lines segment. The angle between the lines segment being 90°

Figure 2. Pattern for Koch2

Koch3

Replace any straight line encountered with the pattern indicated below (Figure 3). The ten (10) equal lines segment are one fourth of the length of the straight line encountered. Angle θ is arbitrary and it indicated the orientation of the straight line encountered relative to the horizontal direction.

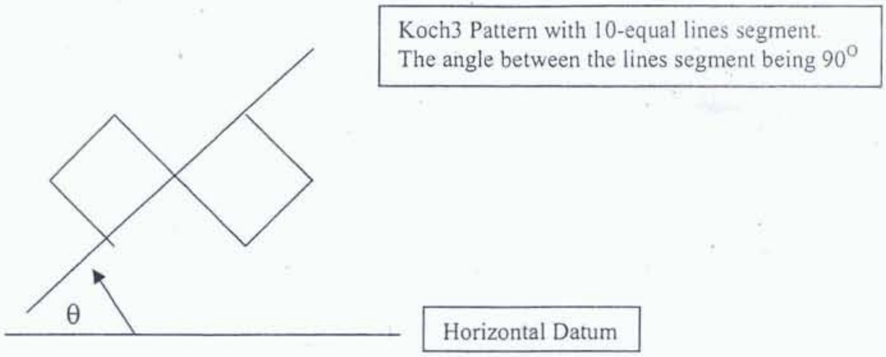

Figure 3. Pattern for Koch3

Koch4

Replace any straight line encountered with the pattern indicated below (Figure 4). The eight (8) equal lines segment are one fourth of the length of the straight line encountered. Angle θ is arbitrary and it indicated the orientation of the straight line encountered relative to the horizontal direction.

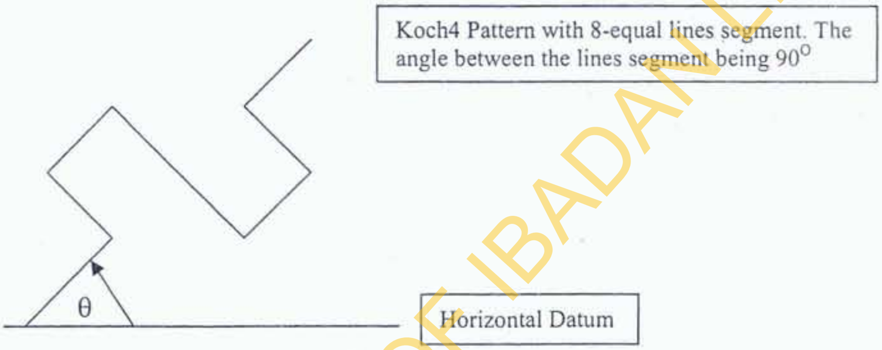

Figure 4. Pattern for Koch4

Koch5

Replace any straight line encountered with the pattern indicated below (Figure 5). The six (6) equal lines segment are one third of the length of the straight line encountered. Angle θ is arbitrary and it indicated the orientation of the straight line encountered relative to the horizontal direction.

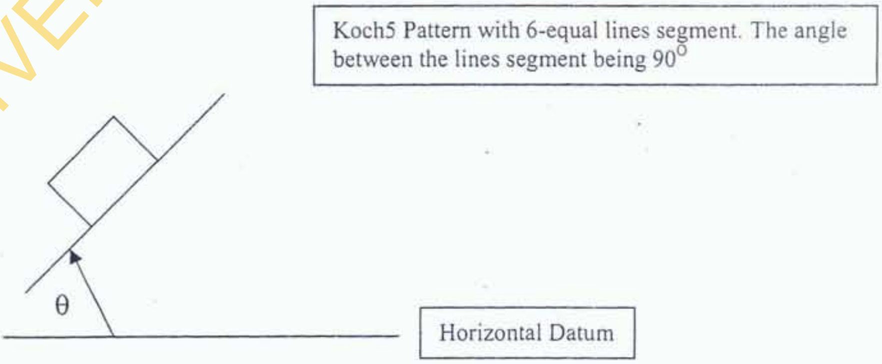

Figure 5. Pattern for Koch5

of measurement while sawdust particle 2 moves in the path from C to G. Although the pattern of movement in the remaining koch 2 to koch 5 are different from that of koch 1, the principle behind the movement in which sawdust particle 1 is static momentarily while the sawdust particle 2 moves in the specified pattern is the same.

The Koch patterns (Koch1, Koch2, Koch3, Koch4 and Koch5) were prepared using Auto-shapes Microsoft facility.

Also, we established four cases to study and are differentiated below as cases 1 to 4.

3.2 Cases Studied

Case1	Randomly Generated fractal curve using varied probabilities of Koch1 and Koch2.
Case2	Randomly Generated fractal curve using varied probabilities of Koch1, Koch2 and Koch3.
Case3	Randomly Generated fractal curve using varied probabilities of Koch1, Koch2, Koch3 and Koch4.
Case4	Randomly Generated fractal curve using varied probabilities of Koch1, Koch2, Koch3, Koch4 and Koch5.

4. Results and Discussion

The proposed method was tested using simulation process and the results have been obtained to evaluate its effectiveness. The flowchart, which indicates the algorithm for this method (Figure 6) was programmed in Fortran environment and run on a Intel (P) Pentium (R) Dual CPU T3400 @ 216GHz 217GHz Personal Computer with 2.00GB RAM. For the simulation process, it is conventional to utilize seed values to initiate the simulation. Hence, for the simulation of the sawdust particle movement in the sawmill environment, ten randomly generated numbers used as seed values are as stated: The ten different random number generating seed values used are 9876, 6789, 5679, 4567, 7634, 3468, 5829, 2378, 9729 and 6396 respectively.

In applying the model developed in equations (1) to (5) and running programs in Fortran, the following results of the study are presented using table (Table 1). Table 1 reflects the fractal dimension curves associated with Koch patterns. The different terms utilized in Table 1 are as follows. The term "Koch" refers to multiple generators which describes the characteristics of sawdust particles in motion, described as fractal patterns in five terms: Koch 1, Koch 2, Koch 3, Koch 4 and Koch 5, for patterns 1 to 5, respectively. The term "theoretical" represents the dimension of fractal curves with no experimentation. The Koch values seem to be increasing from Koch1 to Koch 5 except for Koch 4, which suddenly drops and then Koch 3 under the theoretical value estimation. The difference between the theoretical values and the estimated average fractal box dimension gives the absolute relative percentage error, which measures the degree of accuracy of the theoretical prediction from the estimation. There seems to be a relatively low level of error for the comparative values of theoretical and prediction for Koch 3 (i.e. 2.66%) while the highest error is recorded for Koch 5 (i.e. 7.24%). It is obvious from Table 1 that the maximum absolute relative percentage error is 7.24% (i.e. Koch 5) and it is anticipated that all average fractal box dimensions in the study will suffer a maximum error of 7.24%. Figure 7 shows the fractal curve generated using fractal pattern three (Koch 3). Most of the patterns fall below 0.0 (i.e to the negative side of the y-coordinates). This may be attributed to the absolute negative percentage error of 0.51% being the lowest in the series. The pattern is also noticed to be symmetrical along both the x- and y-coordinates.

Figure 8 represents the fractal curve generated using fractal pattern five (Koch 5). It is observed that the pattern has the highest peak along the y-coordinates. This is anticipated from the absolute relative percentage error of 7.24% being the highest.

Figure 9 represents randomly generated fractal curve patterns (generators) involving Koch 1, Koch 2, Koch 3, Koch 4, and Koch 5 according to the specified probabilities. It is also observed that the pattern is non-symmetrical unlike what was obtained in Figures 7 and 8. Sample fractal curves in Figures 7, 8 and 9 are presented for validation of performance of Fortran platforms used to generate fractal curves in this study. The sample was drawn out of a total of five self-similar fractal curves and two thousand (2000) randomly generated fractal curves.

Table 2 shows sample results of ten rows and five columns of random number generating seed values out of 50 rows and 10 columns generated. Row one represents probabilities of Koch patterns (i.e. Koch 1 and Koch 2) which are fraction patterns. Row two is a term representing the average estimated fractal box dimension. Row four describes the predicted dimension based on equation (5). Row five is a term representing the absolute percentage errors. Rows six to ten represent the dimension estimated base on seed values. The ten (10) different random number generating seed values used are 9876, 6789, 5679, 4567, 7634, 3468, 5829, 2378, 9729 and 6396, respectively. Figure 4 is a description of the plot of the predicted fractal dimension using equation (5) versus average estimated fractal box dimension. It is observed that intersection points clustered around the straight line equation of $y = 0.7026x + 0.4109$ towards one side of the line, thus giving correlation coefficient of fitness as $R^2 = 0.8361$. Table 6 gives a summary of estimated absolute percentage errors in studied cases. The errors are calculated based on correlation coefficient (R) for case 1, case 2, case 3

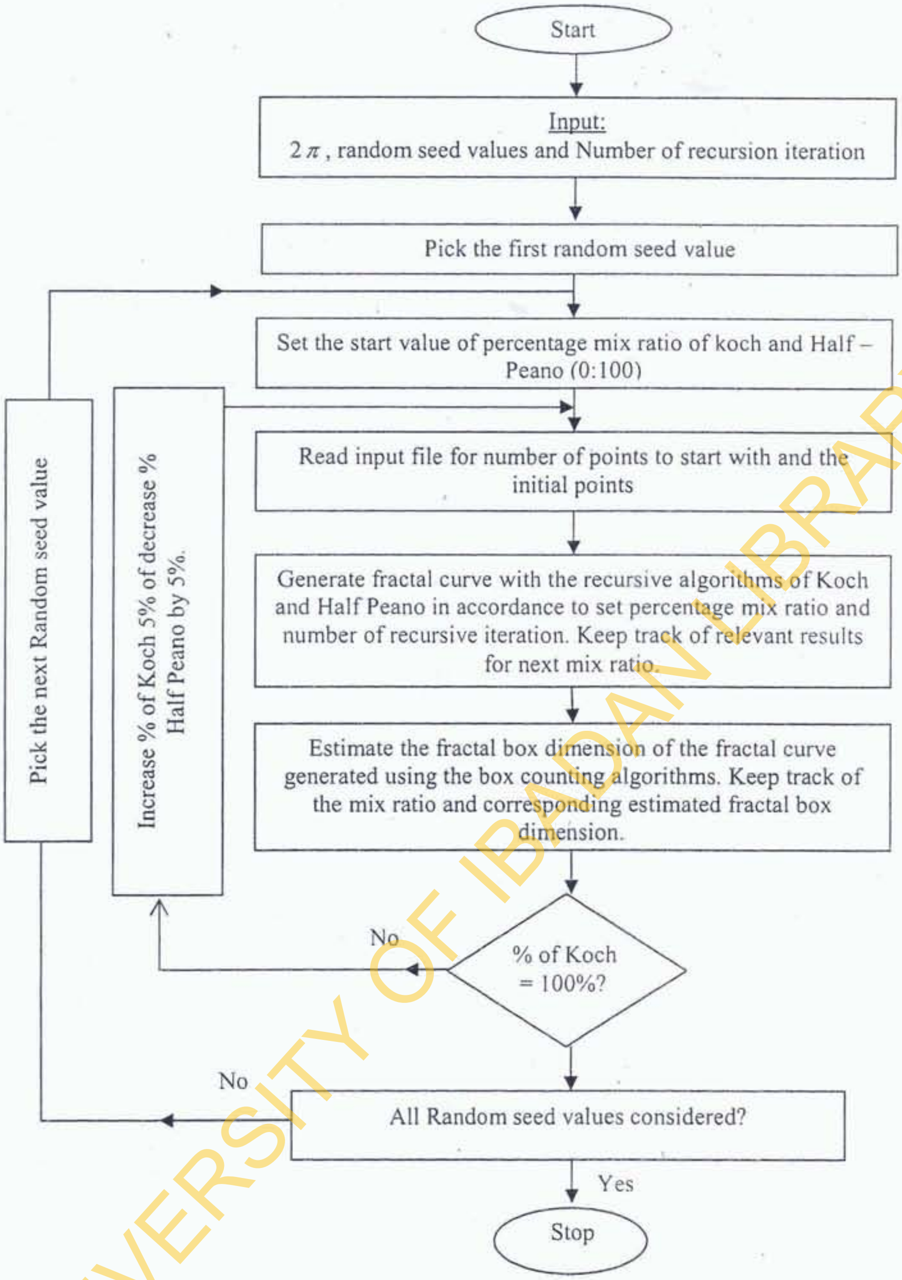

Figure 6. Flowchart for the fractal box dimension of modeling two colliding sawdust particles.

Table 1. Fractal dimension of curves associated with Koch patterns.

Fractal patterns	**Fractal curves dimension**		**Absolute relative percentage error**
	Theoretical	**Estimated average fractal box dimension using 4-iterations**	
Koch1	1.2619	1.2283	2.66
Koch2	1.4650	1.3743	6.19
Koch3	1.6610	1.6694	0.51
Koch4	1.5000	1.5633	4.22
Koch5	1.6309	1.5128	7.24

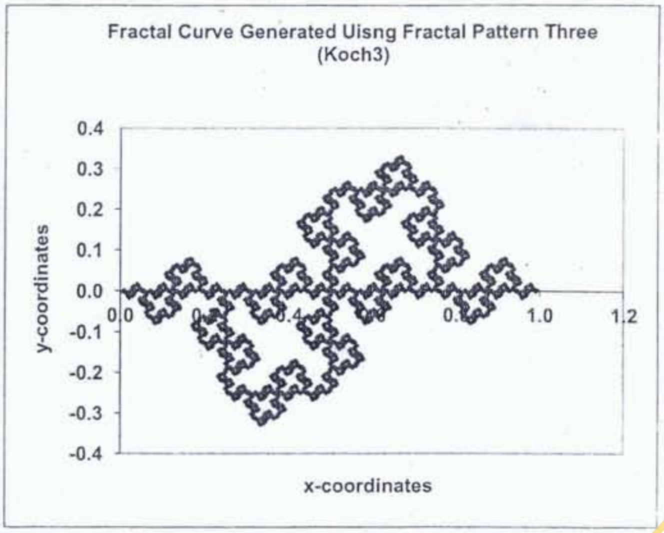

Figure 7. Fractal curve generated using fractal pattern three (Koch3).

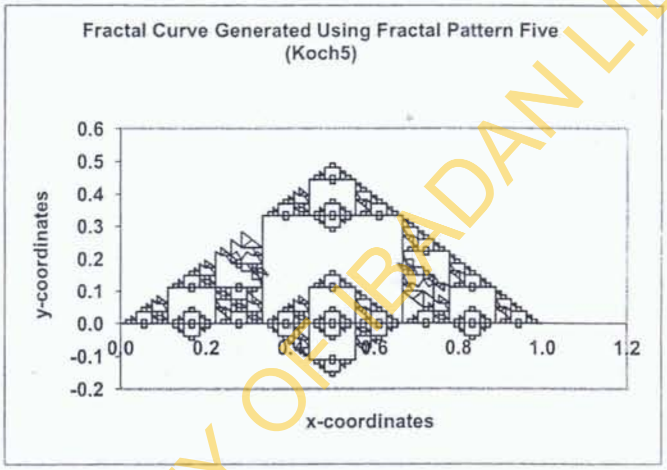

Figure 8. Fractal curve generated using fractal pattern five (Koch5).

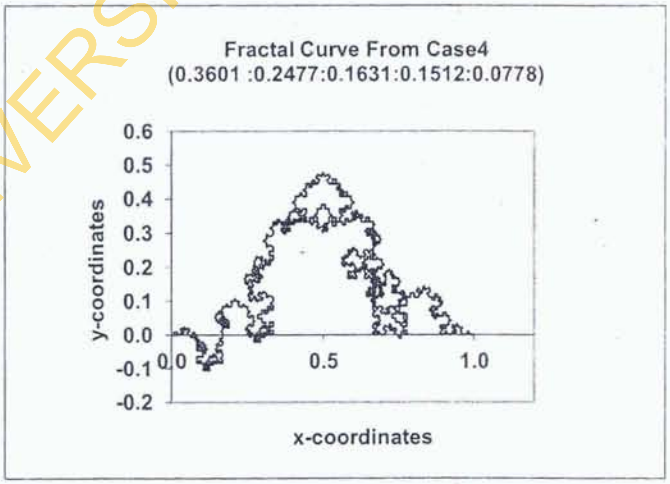

Figure 9. Randomly Generated Fractal Curve: Fractal Patterns (generators) involved are Koch1, Koch2, Koch3, Koch4 and Koch5 according to the specified Probabilities.

Table 2. Sample results for Case 1.

Probabilities of Koch Patterns		Average Estimated Fractal Box Dimension	Predicted Dimension Based on Equation (5)	Absolute Percentage Errors	Dimension Estimated Base on Seed Values				
Koch1	**Koch2**				**9876**	**6789**	**5679**	**4567**	**7634**
0.2208	0.7792	1.34	1.42	6.32	1.37	1.33	1.33	1.35	1.33
0.6344	0.3656	1.32	1.34	1.39	1.33	1.32	1.31	1.27	1.31
0.5850	0.4150	1.31	1.35	2.98	1.28	1.31	1.27	1.30	1.34
0.5524	0.4476	1.31	1.35	3.07	1.31	1.35	1.30	1.30	1.30
0.3319	0.6681	1.35	1.40	3.56	1.35	1.35	1.33	1.35	1.35
0.4764	0.5236	1.33	1.37	3.13	1.32	1.33	1.34	1.31	1.31
0.2632	0.7368	1.35	1.41	4.22	1.37	1.37	1.35	1.31	1.37
0.5397	0.4603	1.32	1.36	2.92	1.31	1.34	1.33	1.35	1.28
0.5835	0.4165	1.31	1.35	2.45	1.33	1.30	1.31	1.34	1.29
0.2094	0.7906	1.35	1.42	5.33	1.37	1.35	1.37	1.32	1.33

and case 4. Maximum and minimum errors for each of the cases were given in addition to mean/average and standard deviation. Referring to Table 6, the average absolute percentage error decreases between 3.52 ±1.18 and 1.51± 1.14 while the correlation coefficient (R^2) decreases between 0.9315 and 0.7365 from case 1 to case 4, respectively. See also Figures 10 and 11.

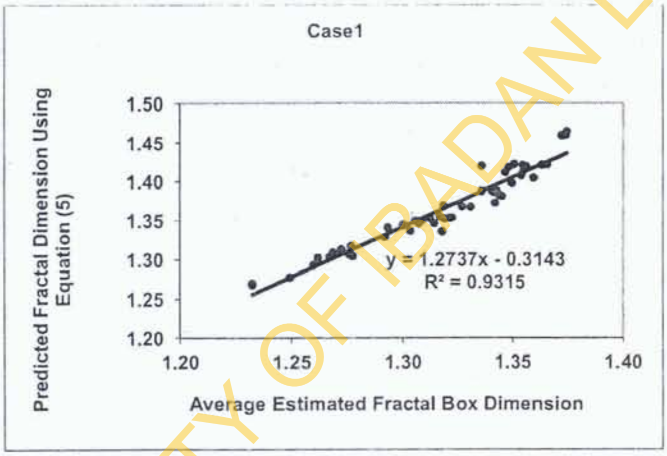

Figure 10. Correlation for Case 1.

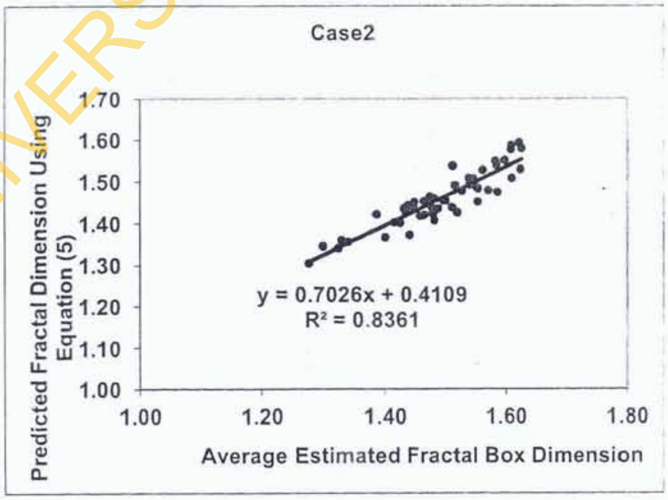

Figure 11. Correlation for Case 2.

Table 3. Sample results for Case 2.

Probabilities of Koch Patterns			Average Estimated Fractal Box Dimension	Predicted Dimension Based on Equation (5)	Absolute Percentage Errors	Dimension Estimated Base on Seed Values			
Koch1	**Koch2**	**Koch3**				**9876**	**6789**	**5679**	**4567**
0.1128	0.3982	0.4890	1.51	1.54	1.76	1.56	1.53	1.49	1.50
0.3786	0.3635	0.2579	1.51	1.44	4.75	1.57	1.61	1.44	1.45
0.4307	0.3489	0.2204	1.46	1.42	3.01	1.58	1.54	1.41	1.58
0.5424	0.2180	0.2396	1.42	1.40	1.60	1.38	1.55	1.31	1.40
0.0855	0.2395	0.6749	1.63	1.58	2.84	1.65	1.66	1.59	1.64
0.3817	0.3607	0.2575	1.45	1.44	0.63	1.46	1.45	1.38	1.37
0.1335	0.5040	0.3625	1.54	1.51	1.86	1.62	1.64	1.52	1.55
0.5986	0.3882	0.0133	1.30	1.35	3.66	1.28	1.29	1.33	1.32
0.4071	0.0737	0.5192	1.55	1.48	4.48	1.43	1.63	1.63	1.61
0.2114	0.4368	0.3518	1.52	1.49	1.61	1.58	1.45	1.61	1.50

Table 4 describes the sample results for case 3. Here, probabilities of Koch patterns were represented using fractal patterns one, two, three and four using Koch 1, Koch 2, Koch 3 and Koch 4, respectively. The corresponding values of these fractal patterns were described in rows one to four of Table 4. Row five describes the average estimated fractal box dimension. Figure 12 describes the correlation diagram for case 3 where predicted fractal dimension using equation (5) is plot against average estimated fractal box dimension. The equation of straight line used is $y = 0.731x + 0.3681$. It is observed that intersection points of x- and y-coordinates scattered around the straight line yielding correlation coefficient of fitness of $R^2 = 0.8176$. Table 5 is sample results of ten (10) rows and two (2) columns of random number generating seed values out of 50 rows and 10 columns generated.

Table 4. Sample results for Case 3.

Probabilities of Koch Patterns				Average Estimated Fractal Box Dimension	Predicted Dimension Based on Equation (5)	Absolute Percentage Errors	Dimension Estimated Base on Seed Values		
Koch1	**Koch2**	**Koch3**	**Koch4**				**9876**	**6789**	**5679**
0.0880	0.3106	0.3815	0.2198	1.52	1.53	0.61	1.55	1.53	1.50
0.2089	0.1482	0.3551	0.2877	1.56	1.50	3.93	1.54	1.62	1.56
0.2122	0.4273	0.1717	0.1888	1.51	1.46	3.21	1.47	1.52	1.60
0.0543	0.1520	0.4283	0.3654	1.60	1.55	3.13	1.50	1.56	1.63
0.4511	0.3220	0.0475	0.1794	1.40	1.39	0.71	1.37	1.40	1.37
0.1386	0.5156	0.3343	0.0114	1.53	1.50	1.73	1.61	1.48	1.62
0.3003	0.0544	0.3830	0.2622	1.52	1.49	2.36	1.48	1.55	1.55
0.4594	0.3699	0.1573	0.0134	1.43	1.40	2.18	1.35	1.56	1.43
0.2018	0.0520	0.1675	0.5787	1.55	1.48	4.97	1.54	1.56	1.54
0.3838	0.0952	0.2455	0.2755	1.48	1.44	2.33	1.50	1.48	1.49

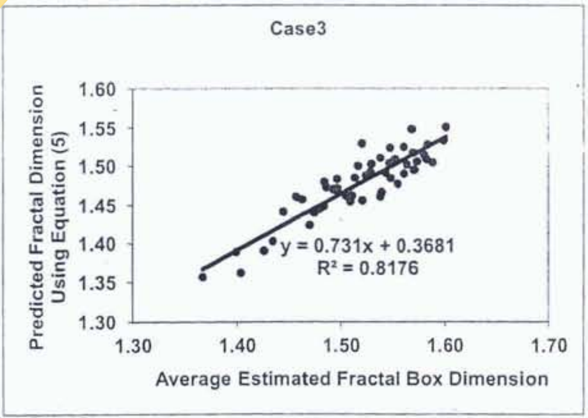

Figure 12. Correlation for Case 3.

Table 5. Sample results for Case 4.

Probabilities of Koch Patterns					Average Estimated Fractal Box Dimension	Predicted Dimension Based on Equation (5)	Absolute Percentage Errors	Dimension Estimated Base on Seed Values	
Koch1	**Koch2**	**Koch3**	**Koch4**	**Koch5**				**9876**	**6789**
0.0727	0.2565	0.3150	0.1815	0.1743	1.52	1.55	1.90	1.56	1.53
0.1107	0.2653	0.2150	0.1357	0.2733	1.54	1.53	0.27	1.51	1.51
0.1911	0.2100	0.0512	0.1434	0.4042	1.48	1.51	1.63	1.50	1.51
0.3231	0.3053	0.2180	0.0322	0.1215	1.47	1.46	0.16	1.48	1.57
0.0976	0.3630	0.2354	0.0080	0.2960	1.55	1.54	0.93	1.59	1.61
0.0324	0.2283	0.1563	0.3230	0.2601	1.56	1.54	1.12	1.54	1.63
0.3298	0.0280	0.3075	0.0793	0.2553	1.56	1.50	3.77	1.57	1.54
0.2672	0.2813	0.0697	0.1799	0.2019	1.49	1.46	1.57	1.42	1.38
0.1007	0.3711	0.0556	0.1876	0.2850	1.50	1.51	0.61	1.45	1.54
0.1847	0.2143	0.2339	0.1030	0.2641	1.55	1.52	1.71	1.54	1.61

Referring to Figure 13 the correlation coefficient of fitness is $R^2 = 0.7365$. The summaries presented in Table 6 are drawn from calculation based on full results of Tables 2 to 5 and reading of equation of best line of fit in Figures 4 to 7 respectively. Referring to Table 6 the average absolute percentage error decreases between 3.52±1.18 and 1.51±1.14 while the correlation coefficient (R^2) decreases between 0.9315 and 0.7365 from Case1 to Case4, respectively.

5. Conclusions

A method for characterizing the possible pattern of movement of two sawdust particles colliding after emission from the band saw has been described. The method is based on fractal theory which determines the possibility of the average box dimensions of sawdust particles in motion forming a randomly generated random curve and could be predicted with the knowledge of different appropriate probabilities

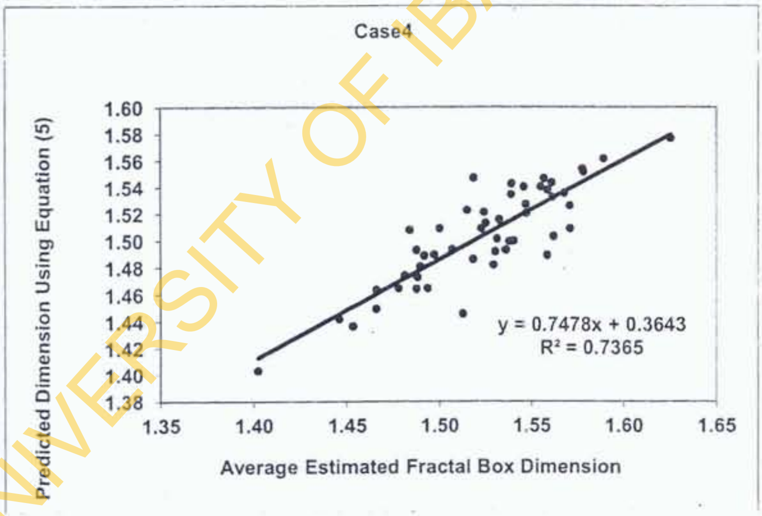

Figure 13. Correlation for Case 4.

Table 6. Estimated Absolute Percentage Errors In Studied Cases.

Errors/R^2	**Case1**	**Case2**	**Case3**	**Case4**
Maximum	6.35	6.94	5.25	4.45
Minimum	1.39	0.22	0.23	0.08
Mean/Average	3.52	2.77	2.64	1.51
Standard Deviation	1.18	1.75	1.44	1.14
Correlation Coefficient (R^2)	0.9315	0.8361	0.8176	0.7365

and theoretical fractal dimensions. An iterative procedure was utilized, developed to reveal the pattern of movement of the sawdust particle just as they leave the production source to various positions in the atmosphere in the environment that the saw mail operator works. The validity of the model was confirmed by simulation experimental results with four cases drawn on fractal pattern with fair cases drawn on fractal pattern combinations drawn out of five and fifty different probability combinations, ten random numbers and four fractal curves generation iterations. It is observed that a maximum absolute percentage error was 7.24% for fractal curve associated with pattern five. It is also observed that the smaller correlation coefficient between average estimated fractal loose dimension and predicated fractal dimension of the sawdust exists. This study shows that fractal curve generated at the end of four iterations by repeated replacement of straight line segments with fractal pattern (generator) satisfactorily represented the corresponding theoretical fractal curve. Furthermore this study has shown that the correlation between average estimated fractal box dimension and predicted dimension is high. The case study presented, based on simulated data sets from sawmill, aims the development of knowledge related to sawdust environment in and around the real life sawmill environment. The nature of the data (i.e. multi-parameter datasets) makes the employment of fractal analysis a powerful tool for analysis. The tool is extremely effective for the detection of patterns that can assist in concluding on the possible deposit patterns of sawdust after being released from the cutting source. The aforementioned pattern formed may be dynamic in nature (the knowledge is updated with every new dataset added to the analysis). The study reinforces previous work on statistical and analytical modeling of the distribution pattern of sawdust in air (Salau and Oke, 2010a, b). Based on a strong case made in the introduction, the study determined whether or not the average fractal box dimensions of sawdust particles in motion in sawmills in randomly generated fractal curve involving multiple generations (i.e rules) can be predicted with the knowledge of different appropriate probabilities and theoretical fractal dimensions. Future investigations could study the chaotic dynamic behaviour of the sawdust particle movement in a sawmill with consideration of the forced duffing's system approach. Here, defined parameters space of the forced dulfing's dynamic system could be statistically investigated for the probability of total parameter points to understand if it exhibits chaotic behaviour or not.

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