EFFECT OF CONCRETE-REPRESENTATIONAL-ABSTRACT AND EXPLICIT INSTRUCTIONAL STRATEGIES ON SENIOR SECONDARY SCHOOL STUDENTS' ACHIEVEMENT IN AND ATTITUDE TO MATHEMATICS

BY

Sabainah Oyebola AKINOSO
(130111)
B. Ed. (Hons.) Educational Management/Mathematics, M. Ed. (Mathematics) University of Ibadan

A Thesis in the Department of Teacher Education, Submitted to the Faculty of Education, in partial fulfillment of the requirements for the award of the degree of

DOCTOR OF PHILOSOPHY (Ph. D) Mathematics Education
UNIVERSITY OF IBADAN

APRIL, 2013

ABSTRACT

Mathematics is viewed as the basis for science and technology as well as a tool for achieving scientific and technological development. Despite the importance to human activities and development, students generally view Mathematics as being an abstract subject. This has resulted in poor performance in the subject arising from the poor instructional strategies adopted in its teaching. Many studies attempting to find a solution to this problem but only a few had focused on such strategies that involve active participation of students in learning through cutting and modeling of the concepts and mastery at every step. This study, therefore, examined the effect of Concrete-Representational-Abstract Instructional Strategy (CRAIS) and Explicit Instructional Strategy (EIS) on students' achievement in and attitude to Mathematics. It also investigated the moderating effects of Mathematics learning difficulty and gender on dependent variables.

The study adopted the pretest-posttest, control group, quasi experimental design with a 3x3x2 factorial matrix. Two hundred and seventy-nine senior secondary II students from six public schools purposively selected from three local government areas in Ibadan municipality were randomly grouped into two experimental groups and a control group. The experimental groups were exposed to CRAIS and EIS while the control group was exposed to Modified Conventional Teaching strategy for six weeks. Instruments used were: Test on Students Mathematics Learning Difficulties (r = 0.85); Students Mathematics Achievement Test (r = 0.83) and Students Mathematics Attitude Questionnaire (r = 0.79). Three instructional guides on Concrete-Representational–Abstract Instructional Strategy, Explicit Instructional Strategy and Modified Conventional Teaching Strategy were also used. Seven hypotheses were tested at 0.05 significance level. Data were subjected to Analysis of Covariance and Scheffe Post hoc test.

Treatment had significant main effect on students achievement in Mathematics ($F_{(2, 260)}$ = 86.4; p<.05). Students taught with the CRAIS had higher achievement ($\bar{x}$ = 25.1) than those in the control group ($\bar{x}$ = 19.8) and EIS group ($\bar{x}$ = 18.4). There was a significant effect of treatment on students' attitude to Mathematics ($F_{(2, 260)}$ = 11.6; p<.05). The CRAIS group had higher attitude ($\bar{x}$ = 99.0) than the EIS group ($\bar{x}$ = 96.6)

and control group ($\bar{x}$ = 93.5). This shows that CRAIS was effective in enhancing the achievement while CRAIS and EIS were found to be more effective at improving students attitude towards Mathematics. Mathematics Learning Difficulty (MLD) has significant effect on students' achievement in Mathematics.
($F_{(2, 260)}$ = 139.1; $p<.05$). Students with low MLD had higher achievement ($\bar{x}$ = 25.8) than their moderate ($\bar{x}$ = 17.8) and high MLD ($\bar{x}$ = 14.3) counterparts. Also, there was a significant effect of MLD on attitudes to Mathematics ($F_{(2, 260)}$ = 20.2; $p<.05$). Students with low MLD had higher attitude ($\bar{x}$ = 99.5) than their moderate ($\bar{x}$ = 95.2) and the high MLD group ($\bar{x}$ = 89.3). Gender has no significant effect on students' achievement and attitude to Mathematics. Further, there was significant interaction effect of treatment and MLD on achievement in Mathematics ($F_{(4, 260)}$ = 9.3; $p<.05$). Among students in the CRAIS group, those with low MLD had higher achievement ($\bar{x}$ = 31.9) than the moderate ($\bar{x}$ = 21.3) and the high MLD ($\bar{x}$ = 12.4) respectively.

Concrete-Representational-Abstract Instructional Strategy enhanced students' achievement in Mathematics whereas both CRAIS and EIS improved their attitude to the subject. Therefore, CRAIS and EIS should be adopted for the teaching of Mathematics. Senior secondary school students should be screened for Mathematics learning difficulties while training programmes on the use of CRAIS and EIS should be organised for Mathematics teachers.

Key words: Concrete-Representational-Abstract Instruction, Explicit Instruction, Students' achievement and attitude to Mathematics, Mathematics learning difficulty.

Word Count: 498

CERTIFICATION

I certify that this research work was carried out by Sabainah Oyebola AKINOSO of Mathematics unit in the Department of Teacher Education, Faculty of Education, University of Ibadan.

………………………………………

SUPERVISOR
Dr B. O. Ogunleye
NCE (Lagos), B. Ed. (Hons.),
M. Ed., Ph. D (Ibadan).
Lecturer in Science Education and Statistical Research Methods,
Department of Teacher Education,
University of Ibadan, Ibadan.

Date: ……………………

DEDICATION

This work is dedicated to

God Almighty

My Husband, Mikail Aderemi Ishola Akinoso

For all his contributions from the beginning to the completion of this programme and for all the denials he endured while this work lasted

and to

My Children

Zainab Bolaji Ayobami, Aminat Adepeju Ayomide, Halimah Oyebola Ayomikun and Naimah Modupeoluwa Ayokunumi,

for the endurance throughout the period of this work.

ACKNOWLEDGEMENTS

To God is the glory for great things He has done. He is the author and finisher of our faith. He who designed those things that are not as though they were. He gave the life, the grace, the strength and the opportunity with which this research work started and reached completion. Glory be unto Him forever and ever.

My supervisor, Dr B. O. Ogunleye, I really appreciate you sir, for giving all it required to ensure that this work progressed and ultimately successful. He has taught me how to work without wasting a single minute. His style of prompt attention while moving or standing and the speed with which he went through the write-ups at every stage of the study is directly proportional to the high motivation I had towards the work. Indeed, he has contributed immensely to the success of this programme, his attitude to work is quite worthy of emulation. You will move from grace to grace and you will eat the fruit of your labour. I specially thank Professor V. F. Payne who chaired my post-field seminar, for his interest in this study right from inception. His contributions have helped this work a lot.

The useful contributions of Prof. F. A. Adesoji, the current Head of the Department of Teacher Education who chaired my departmental presentation are highly appreciated. His advice during TEE 801/802 class yielded immeasurable result in this work. Also, I acknowledge the valuable contributions of Prof. Oluremi Ayodele-Bamisaye who showed special interest in this work during my departmental presentation. My sincere appreciation goes to the Dean, Faculty of Education, Prof. C. O. O. Kolawole for his contributions. May you continue to wax stronger. I also appreciate the valuable contributions of Dr M. K. Akinsola, his words of encouragement, his fatherly support, advice and useful suggestions. Also, I need to express my profound gratitude to Dr J. O. Ajiboye who also contributed a lot toward the success of this work. Thank you very much sir. The contributions of Dr Alice. M. Olagunju who chaired my unit proposal seminar presentation are highly appreciated. Dr O. A. Adegbesan, the Sub-Dean (Postgraduate), Faculty of Education is highly appreciated for his contributions during the process of registering the title of this study.. I also appreciate the contributions of Dr Tunde Yara of the Department of Computer Science. I cannot but express my profound gratitude to Dr S. A. Babarinde, Dr Temisan. A. Ige, Dr Ayotola Aremu, Dr F. O. Ezeokoli, Dr A. Abimbade, Dr Esther Oduolowu, Dr P. A. Amosun, Dr G. O Adedoja, Dr Ajitoni, Dr A. A. Adeyinka, Dr J. G. Adewale, Dr S. O. Popoola Prof. M. F. Fapojuwo and a host of others for their encouragement at different times during the course of this programme, I appreciate you all.

I also appreciate Dr D. O. Fakeye for editing this work and for the word of encouragement towards the success of this work, may you eat the fruit of your labour. Dr R. O. Akinbote and Dr I. N. Ohia, you are highly appreciated, for your contributions to this work especially during the post-field presentation. Aunt Morenike Odunsi, former Permanent Secretary (PS) TESCOM, my former principal, I really appreciate your contribution in terms of permission and words of encouragement. I also thank my research assistants, Mr. Adewale, Mr. Olatubosun, Mr. Adeyanju, Mr. Allen, Mr. Fajoogun, Mr. Ajakaye, Mrs. Omisoore, Mrs Adeloye, the principals and the students of the schools used, without their support, this work could not have been a great success, I appreciate you all. Mr. J. A. Adeniji, Elder Adesiyan, Alhaji Jamiu Ayorinde thanks for your prayer towards the success of this programme. Alhaji Akinoso Jamiu and Alhaji Subair, thank you very much.

My friends, mentors and advisers who encouraged me to start this programme, Miss. Funmi Fakeye and Mrs. Oyediji, you can never be forgotten in the history of my life. Also Mrs. Gbenla, Mrs. Adefioye, Mrs. Adegbiji Idayat, Mr. Ajagbe Olaoluwa, Alhaji Asipa, Mr. T. M. Asiru, Mr. Tunde Dada, Mrs. Akinsola, Mrs. Olarewaju, Mr. I. A. Olasunkanmi and Mrs. Adediran-Omowon Oluwatoyin, I appreciate you all. I need to show my appreciation to my senior colleagues and fellow doctoral students, who offered helpful comments and gave useful advice at different stages of this study. Dr Sangodoyin, Mrs. Aminat Aderonke Agoro, Mrs. Folake Amao, Mr. Ige, Mr. Etukudo, I appreciate you all. My typists, Mr. Omoniyi Niyi and Mrs. Abigeal Adekiitan, thank you so much. I cannot but remember my late father, Mr. Akande Olaleke Daniel. May his gentle soul rest in perfect peace. My mother, Mrs. Akande Abigael, thank you very much for your encouragement and advice. My siblings, Mr. Akande Philip, Mrs. Olayinka Folasade, Mr. Akande Biodun and Mrs. Awelewa Olukemi, I say thanks.

Specially, I acknowledge the effort and support of my caring husband and my loving brother, Mikail Aderemi Ishola Akinoso, his contribution, financially and morally as well as his understanding, endurance, words of encouragement and prayers towards the success of this study have given me assurance that he is really my man, thanks so much. Also I want to thank my children, Bolaji, Ayomide, Oyebola and Modupeoluwa Akinoso, you have endured till the end of this programme. I thank you very much.

TABLE OF CONTENTS

APPENDICES

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

CRAIS	Concrete –Representational –Abstract Instructional Strategy
EIS	Explicit Instructional Strategy
MCTS	Modified Conventional Teaching Strategy
STM	Science, Technology and Mathematics
NPE	National Policy on Education
NMC	National Mathematical Centre
WASSCE	West African Senior Secondary School Certificate
LD	Learning Difficulty
NCTM	National Council of Teacher of Mathematics
IDEA	Individual with Disabilities Education Act (IDEA)
ADD	Attention Deficit Disorder
UNESCO	United Nation Education Scientific and Culture Organization
AAAS	American Association for the Advancement of Science
ICT	Information Communication Technology
NCSALL	National Center for the Study of Adult Learning and Literacy
NDCCD	National Dissemination Centre for Children with Disabilities
NICHCY	National Dissemination Centre for Children with Disability
TOSMALD	Test on Students' Mathematics Learning Difficulties
SMAT	Students' Mathematics Achievement Test
SMAQ	Students' Attitude Questionnaire
CPA	Concrete Pictorial and Abstract
ANCOVA	Analysis of Covariance
MCA	Multiple Classification Analysis

NMAP	National Mathematics Advisory Panel
MAN	Mathematics Association of Nigeria
STAN	Science Teachers Association of Nigeria
SS II	Senior Secondary School II
AAUW	Americans Association of University Women
MSEBNRC	Mathematical Sciences Education Board and National Research Council
ISO	Instructional Strategies Online
MDL	Mathematics Difficulty Level

CHAPTER 1
INTRODUCTION

1.1 Background to the Study

The importance of science and technology in contemporary society is demonstrated by its conscious and unconscious use in our daily lives. Indeed, transportation, electrical devices, medicine and food are benefits of science and technology. Also, modern societies are literally built on Science and Technology. Opara (2004) states that science and technology have long been recognized as the *instrument* pa*r excellence* for nation building. This is why greater emphasis is being placed on industrial and tec-hnological development not only in Nigeria but the world over. Science, Technology and Mathematics (STM) education plays a dominant role in the development of nations (Jegede, 2001). Obioma (2006) in an address delivered at one-day consultative forum on the review of the Nigerian National Policy on Education (NPE) in Kano stresses that it is a known fact globally that STM education is the bedrock of civilization and development and no nation can afford to ignore its impact on the modern world.

Mathematics is viewed as the basis for science and technology and it is the tool for achieving scientific and technological development. Mathematics is essentially the analytical tool for the rationalization and development of science and technology (Ezeani & Ebere, 1999). Mathematics plays important roles in the expression of scientific models while the extensive use of its method is required in observing, collecting information, measuring, hypothesizing and predicting result of scientific investigation. Furthermore, Olusi and Anolu (2010) identify the importance of Mathematics in the relationship among Mathematics, Science and Technology and concluded that without Mathematics, there is no science. Cangiano (2008) and Gauss (2010) also reiterate the fact that without modern technology there is no society; and that Mathematics is the precursor and queen of science and technology as well as an indispensable single element in modern societal development. Mathematics provides powerful intellectual tools that have led to tremendous advancement in modern science and technology. Above all, Mathematics is indispensable as it is the prerequisite for advanced training and lifetime career choice. In the same vein, it is useful for analyzing the cost of a business, for data entry using the computer, for decision making and interest rates in company (Mathematics Worksheet Center, 2011). Salawu (2001) avers that Mathematics is indispensable because it has substantial application in all school subjects particularly in science and technology. In his own view, Eze (2007) asserts that science evolved with the use of mathematical principles and Mathematics is a necessary tool needed by individuals to be able to function effectively in the present technological age.

Perhaps, this is why the Federal Government of Nigeria (FGN, 2004) presents Mathematics as relevant to the individual's day to day living and ascribes to it an important role in the scientific and technological advancement of the nation. Mathematics is the underpinning technology for modern society. Without Mathematics, there would be no computers, aeroplanes, space programme, weather prediction and scientific prediction (National Mathematical Centre NMC, 2011). Without Mathematics, there would be no surgery, possibly the use of some prescribed drugs would be uncontrolled or dangerous and the financial system would collapse. Mathematics is used all the time and for everything even without realizing it (Mahadevan, 2009). It is a desirable tool in virtually all spheres of human endeavour, be it science, engineering, industry, technology and even the arts (Oyedeji, 2000). As a basic element in industrial and technological advancement, any nation that desires to become developed must put emphasis on the teaching and learning of Mathematics for computing or calculating (Oluwaniyi & Ibiyemi, 2007).

Mathematics is a fundamental science which is necessary for the understanding of most other fields (Jain, 2010). Without Mathematics we are lost in the dark. It seems to be the glue that binds scientific and artistic cultures and its language and symmetry are spoken everywhere (Marcus, 2009). It is a compulsory subject in the curriculum of the primary and secondary school levels in the Nigeria educational system (FGN, 2004) and a prerequisite to the study of science courses in higher institutions. That may be the reason why Mahadevan (2009) observes that Mathematics is a structured language and it is the language of the sciences. He noted that just like a poet uses structured language to express an idea, Mathematics is also used to communicate abstract ideas.

Furthermore, Mathematics has often been referred to as the language of science, as everything involves Mathematics from the use of formulae to model the world, to the use of trials and measurements to test and apply models. It cuts across all other subjects apart from Science and Engineering. Mathematics is heavily used in business, economics, social studies and communication. In reality, it is the basic language of the world, for everyone uses it in almost everything without realizing it (Devlin, 2009; Smith, 2009). Mathematics provides a powerful universal language and intellectual toolkit for abstraction, generalization and synthesis. It enables us to probe the natural universe and to develop new technologies that have helped us to control and master our environment, changing societal expectations and standards of living. Mathematical training disciplines the mind, develops logical and critical reasoning, and develops analytical and problem-solving skills to a high degree (Smith, 2004).

As important as Mathematics is to human activities and development, one would expect that students' learning outcomes would be good. The opposite is however the case. Students view Mathematics particularly in more abstract form, as an abstruse and pointless subject to study (Otung, 2001). Mathematics educators (Udousoro, 2000; Akinsola, 2001; Akinsola & Ogunleye, 2003; Ifamuyiwa, 2006; Adekoya, 2008; Ugboduma, 2008; Afolabi, 2010) carried out researches on methods and ways of improving the teaching and learning of Mathematics at primary and secondary school levels. In spite of these numerous researches as well as efforts at the instructional level over the years, the performance of students at the West African Senior Secondary School Certificate Examinations (WASSCE) is yet to improve significantly as students are still performing poorly in the subject.

Research reports indicate that the performance of students in Mathematics is below expectation (Afolabi, 2010). Table 1.1 illustrates the summary of students' achievement in Mathematics over a period of thirteen years from 1999 to 2011.

Table 1.1: Statistics of Entries and Results in Mathematics 'O' Level at the Senior Secondary School Certificate Examination May/June for Nigeria (1999-2011)

Year	Number of candidates	A1-C6		D7-E8		F9	
		N	%	N	%	N	%
1999	756,680	138098	18.3	212514	28.0	381029	50.4
2000	634,604	208244	32.8	196080	30.9	230280	36.3
2001	1,023,102	373955	36.6	334907	32.7	314240	30.7
2002	908,235	309409	34.1	308369	34.0	290457	32.0
2003	926,212	341928	36.9	331348	35.1	229878	24.8
2004	832,689	287484	34.5	245071	29.4	300134	36.0
2005	1,054,853	402982	38.2	267600	25.4	363055	34.4
2006	1,149,277	472674	41.1	357325	31.1	286,826	25.0
2007	1,249,028	584024	46.3	333,844	26.7	302774	24.2
2008	1,268,213	726398	57.3	302266	23.8	218618	17.2
2009	1,348,528	634382	47.0	344635	25.6	315738	23.4
2010	1,306,535	548065	42.0	363920	27.9	355382	27.2
2011	1,508,965	608866	40.4	474664	31.5	421412	27.9

West African Examination Council Statistic Department Lagos.

Table 1.1 shows that it is only in 2008 that students recorded fairly good results with about 57.3% obtaining credit pass and above in the subject. The percentage pass was as low as 18.3% in 1999 while in years 2000-2005, it revolves around 35% and in more recent years 2006, 2007, 2009, 2010 and 2011, it revolves around 45%. This trend is poor for a very important subject like Mathematics and raises questions on the effectiveness of classroom teaching and instructional strategies adopted. Figure 1.1 shows the picture of the data presented on Table 1.1 more clearly.

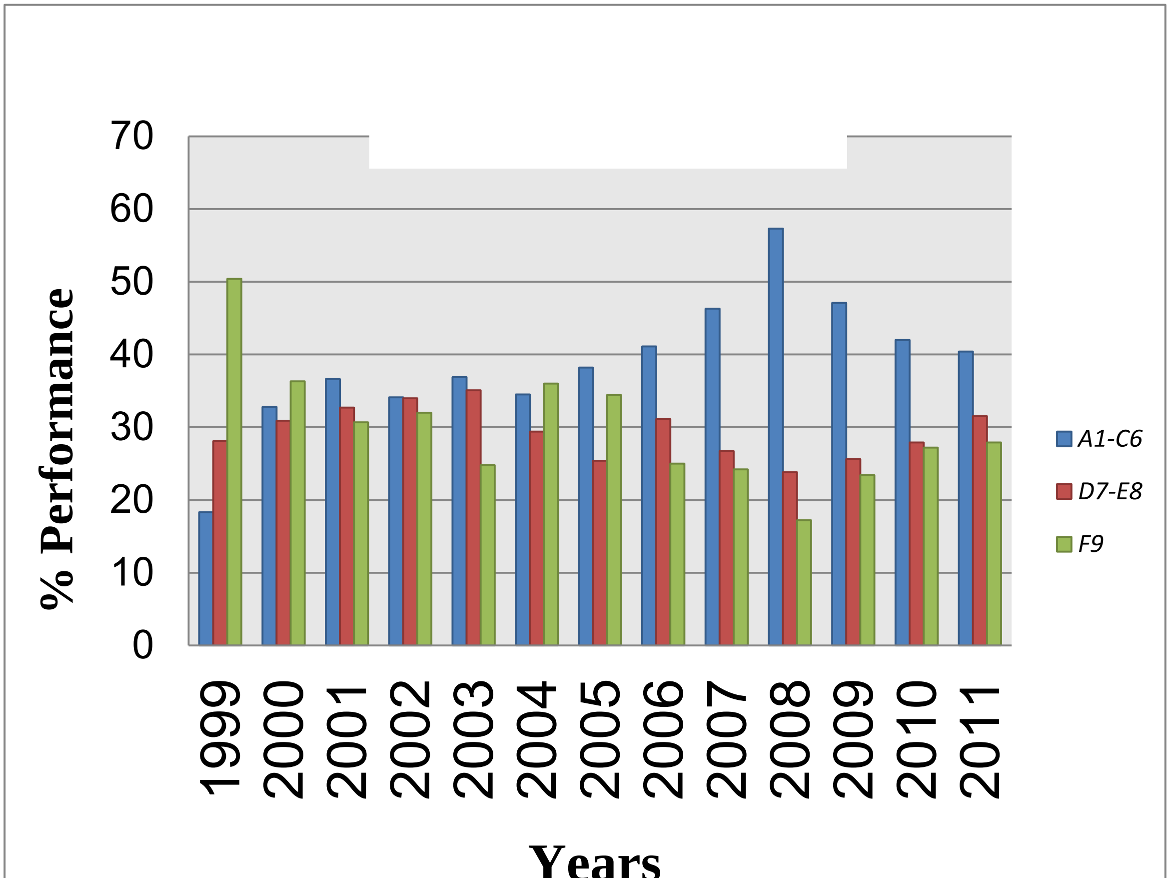


Figure 1.1: Bar Chart of Students' Performance in Mathematics (1999-2011)

The percentage pass in 1999 is the lowest compared with other years, the bars increased from 2005-2008 but decreased from 2009-2011. The failure percentage in 1999 is the highest, though the percentage failure reduced for other years, the percentage pass (A1-C6) bars are not high enough. A cursory look at the bars shows that, the highest bar for (A1-C6) in 2008 is not up to 60%. This trend is too low for the subject that is a prerequisite to other subjects.

An investigation into the problems of poor students' achievement in the subject revealed that learning problems in Mathematics may be caused by intellectual, physical, social and emotional factors (Mahesh, 1999). Hence many reasons are usually adduced for students poor achievement in Mathematics. These include student factors, government factors, language problems and teacher factors. Teacher factors already identified in corresponding researches include poor teacher preparation, shortage of qualified

Mathematics teachers, lack of devotion to teaching, incessant transfer of available teachers, teachers poor knowledge of the subject matter, poor method of teaching and negative attitudes towards Mathematics (Adetunji, 2000; Adegoke, 2003; Akinoso, 2011). Other factors include difficulty with language processing, visual reasoning skill, central nervous system dysfunction, Attention Deficit Disorder, biological factors such as brain injury, errors in brain development, brain abnormality due to birth trauma, nutritional deprivation, exposure to toxic substances, child environment, home environment lack of motivation, poor cognitive processing skills and lack of proper instruction (Sheldon, 2013).

Students' attitude to Mathematics could also be considered very important. Attitudes are defined as positive or negative emotional dispositions (Aiken, 2002). However, according to Akinsola and Olowojaiye (2008), the exact definition of attitudes toward Mathematics varies, but it is generally believed that students attitude towards a subject determines their success in that subject. In other words, favourable attitude results in good achievement in Mathematics. On the other hand, constant failure in that subject can bring about discouragement towards the subject. The attitude towards Mathematics could therefore be described as either a positive or negative emotional disposition towards Mathematics (Zan & Martino, 2007).

Graham and Fennel (2001) find that students' confidence in and disposition towards Mathematics are influenced by the learning environment. Heidmann and Humphrey (2002) also note that investigation into students' Mathematics attitudes and perspectives not only inform teachers, parents and administrators about students needs but also serve as a catalyst for reform in Mathematics education. Attitudes indeed influence success and persistence in the study of Mathematics (Webb, Lubinski & Benbow, 2002). Moreover, instructional strategies may also reinforce students needs in order to increase their achievement. Bottge (2001) finds that when Mathematics is taught indeed, in interesting and engaging approach, students with learning difficulties could solve problems that emphasize higher level thinking skills.

It has been observed that without interest and personal effort in learning Mathematics by the students, they can hardly perform well in the subject (Yara, 2009). Bolaji (2005) in a study of the influence of students' attitude towards Mathematics finds that the teachers method of Mathematics teaching and his personality greatly accounted for the students positive attitude towards the subject. Also, studies related to instructional practices and academic achievements have suggested that the quality of teachers' instructional strategies affects students' task involvement and subsequent learning in Mathematics (Butty, 2001). It is, therefore, imperative for teachers to appreciate and inculcate in students positive attitude towards Mathematics by using improved and appropriate instructional strategy (Akinsola & Olowojaiye, 2008).

Poor instructional strategies and techniques for teaching Mathematics are critical problems in Mathematics learning. This is attested to by Onabanjo (2000), Odogwu (2002) and Ifamuyiwa (2006). Language and communication problems also limit students' performance in Mathematics (Akinsola, 2000; Etukudo, 2002). School and society-related factors, inadequate instructional materials, textual material and governmental factors were also identified by Oyedeji (2000), Udousoro (2000) and Afolabi (2010) as part of the problems while the socio-economic status of parents, parental attitude, physical amenities at home (Wilson, 2010) account for students' difficulty in learning Mathematics. Brew (2011) further traces poor performance in Mathematics to lack of concrete teaching materials. It is, therefore, obvious that with these myriad of problems, student cannot but encounter difficulties in Mathematics.

Several instructional strategies have been recommended for the teaching and learning of Mathematics to students especially those with Mathematics learning difficulties. These include use of Mnemonics, Peer tutoring, Computer and Text-Assisted instruction (Udousoro, 2000), Explicit instruction (Access Center, 2004), Personalization approach (Heng-Yuku & Howard, 2000) and Concrete-Representational-Abstract instructional strategy (Allsopp & Minskoff, 2003). Though these strategies were reported to have improved the performance of students in Mathematics, Nigerian schools still adopt the lecture method in Mathematics not minding whether a student has learning difficulty or not and this has not been yielding the desired results (Ojo, 2003). There is, therefore, the need to search for alternative strategies of teaching Mathematics especially to students with learning difficulties. The basic ingredient that has been identified as necessary for effective learning is the active participation of the learner in the learning situation (Akinsola, 1994). It is in this regard that Concrete-Representational-Abstract and explicit Instructional Strategies which permit the active participation of learners become relevant.

Concrete-Representational-Abstract Instructional Strategy (CRAIS) is one of the interventions in Mathematics instruction that research suggests could enhance the Mathematics performance of students with Learning Difficulties (LD) (Access Center, 2004). The Concrete-Representational-Abstract Instructional Strategy provides an organizational structure within which lessons can be designed to effectively help students reach an abstract level of thinking around difficult concepts and content. CRAIS is a three-part instructional strategy with each part building on the previous instruction to promote student learning and retention, and addressing conceptual knowledge of students.

Concrete: At this stage, the teacher begins instruction by modelling each mathematical concept with concrete materials. The students when modelling the concrete materials work together with teacher's guidance, student interactions, repeated teacher demonstrations and explanations and many opportunities for students to practice and demonstrate mastery of concepts.

Representational: At this stage, the Mathematics concept is modelled at the semi-concrete level which may involve drawing pictures that represent concrete objects.

Abstract: At this stage, the Mathematics concept is modelled at the abstract or symbolic level using only numbers, notation and mathematical symbols.

CRAIS ensures that students develop a tangible understanding of the Mathematics concepts. When students are supported to first develop a concrete level of understanding in any Mathematics concept, this foundation can later be used to link their conceptual understanding to abstract Mathematics learning activities. When students represent the concrete understanding by drawing simple pictures that replicate the use of concrete materials, it provides student with supported process for transferring their concrete understanding to the abstract level. The strategy is probably the most common example of Mathematics instruction incorporating visual representations. This strategy has its root in Dale's Cone of Experience that learners retain more information by what they "do" as opposed to what is "heard", "read" or "observed" (Dale, 1969). The author asserts that, people generally remember 10% of what they read, 20% of what they hear, 30% of what they see, 50% of what they see and hear, 70% of what they say and write and 90% of what they do as they perform a task. This implies that action learning techniques result in up to 90% retention. Dale (1969) also emphasizes that instructors should design instructional activities that build upon more real-life experiences. Real-life experiences make use of more of human senses and the more senses used, the greater to learn and remember an event (Dale, 1969).

Figure 1.2: Dale's Cone of Experience (Dale, 1969)

Concrete-Representational-Abstract instructional strategy actually refers to a simple concept of teaching Mathematics to students with learning difficulties (Butler, Miller, Crehan, Babbitt & Pierce, 2003). As the Access Center (2004) points out the strategy works well with individual students, in small groups, and with the entire class.

Concrete-Representational-Abstract Instructional Strategy is also appropriate at both the elementary and secondary levels. The National Council of Teachers of Mathematics (NCTM) recommends that, when using the instructional Strategy, teachers should make sure that students understand what has been taught at each step before moving instruction to the next stage (Berkas & Pattison, 2007). This would be ensured by giving them questions on the topic as the teaching goes on.

The Concrete-Representational-Abstract Instructional Strategy (CRAIS) makes learning real by changing the abstract concepts of Mathematics to real objects that can be visualized. Another purpose of teaching through a concrete-to-representation-to-abstract sequence of instruction is to ensure that students truly have a thorough understanding of the Mathematics skills they are learning. Akinoso (2011) in the study of factors affecting students' achievement discovered that availability of concrete objects as instructional materials can affect achievement in Mathematics. Students with learning difficulties often have difficulties in symbolic concepts and reasoning. Such students need extra assistance through hands-on manipulative and pictorial representation of mathematical concepts. Concrete knowledge involves knowing how to manipulate concrete objects or their representation to solve a problem (Slavin, 2009).

The Explicit Instructional Strategy (EIS) involves directing students' attention toward specific learning in a highly structured environment (Instructional Strategies Online, 2011). It is a strategy that focuses on producing specific learning outcomes. It is also an approach that encompasses the goal of improving learning for all students especially for low performing students (Hall, 2002). The EIS is a scripted programme that is very systematic with a step-by-step format requiring student mastery at each step. Also, students respond to instruction and receive immediate feedback (Access Center, 2004). According to Steedly, Dragoo, Arefeh, and Luke (2008), explicit instruction refers to an instructional practice that carefully constructs interactions between students and the teacher, teachers clearly state a teaching objective and follow a defined instructional sequence. They assess how much students already know on the subject and tailor subsequent instruction, based upon that initial evaluation of students' skills. Students move through the curriculum, both individually and in groups, repeatedly practising skills at a pace determined by the teacher's understanding of student needs and progress

(Swanson, 2001). Explicit instruction has been found to be successful when a child has problems with a specific skill (Kroessbergen & Van, 2003). In Explicit instruction, topics are taught individually, it involves explanation, demonstration and practice. Topics are taught in a logical order and directed by the teacher which involves modelling skills and behaviours as well as modelling thinking. The teacher thinks out loud when working through problems and demonstrating processes for students.

There is a school of thought that presumes all students can learn if they are provided with the appropriate learning conditions (Jung & Guskey, 2007). Mastery learning refers to the idea that teacher should organize learning through ordered steps. In order to move to the next step, students have to master the prerequisite step which is in harmony with explicit instruction. Explicit instruction refers to a relatively scripted version of mastery learning, meaning that it not only organizes the curriculum into small modules but it also dictates how teachers should teach, including the words they should speak while teaching (Magliaro, Lockee & Burton, 2005). Explicit instruction programs share one of challenges of mastery learning programs because they hold all students to the same high standard of achievement, they must deal with differences in how long students require to reach the standard. But explicit instruction has another challenge of its own. Such programs often rely on small-group interaction more heavily than do other mastery learning programs and use self-guiding materials comparatively less. Also explicit is effective in teaching basic skills.

Explicit instruction is useful for introducing topics and specific skills and it provides guided instruction in the basic understanding of required skills which students can then build on through practice, collaboration, repetition, hands-on activities and developmental play (Instructional Strategies Online, 2011). With explicit instruction, teachers follow a sequence of events, generally stating the objective, revisiting skills necessary for new information, presenting new information, questioning students, providing group instruction and independent practice, assessing performance, and giving more practice (Swanson, 2001). Swanson identified 12 criteria associated with explicit instruction. When any four of these indicators are presented, explicit instruction is occurring. Breaking down a task into small steps, administrating probes, administering feedback repeatedly, providing a pictorial presentation, allowing independent practice and individually paced instruction, breaking the instruction down into simpler phases, instructing in a small group, teacher modelling a skill, providing set materials at a rapid pace, providing individual child instruction, teacher asking questions, teacher presentation of the new materials (Swanson, 2001).

According to Instructional Strategies Online (2011), explicit instruction is a sequence of supports:

- setting a purpose for learning
- telling students what to do.
- showing them how to do it.
- guiding their hands-on application of the new learning.

From the foregoing, Explicit instruction begins with setting the stage for learning, followed by a clear explanation of what to do (telling), followed by modelling of the process (showing), followed by multiple opportunities for practice (guiding) until independence is attained. Explicit instruction moves systematically from extensive teacher input and little student responsibility initially to total student responsibility and minimal teacher involvement at the conclusion of the learning cycle (Instructional Strategies Online, 2011).

In using any strategy for improving students' achievement in Mathematics, some other factors have been identified to be of utmost importance, these include students' learning difficulties and gender. Individuals with Disabilities Education Act (IDEA) (2011) describes students with learning difficulties as students who have serious learning problems in schools but do not fall under other categories of handicap. Learning difficulty is a disorder in which a person needs much effort to learn effectively, caused by an unknown factor or factors, that affect the brain's ability to receive and process information. It can disturb a person to learn as quickly as someone who is not affected by a learning difficulty, but is not indicative of intelligence level. Students with learning difficulties are not dumb or lazy, they have average or above average intelligence, their brains process information differently (IDEA, 2011). If a students' cognitive ability is much higher than his or her academic performance, the student is diagnosed with a learning difficulty.

For a student to have learning difficulties, three criteria must be met (Pandit, 2000).

(a) Students with learning difficulties must have significant discrepancy between their potential and or ability and actual achievement.

(b) Students with learning difficulties must have learning problems that cannot be attributed to other handicapping condition, such as blindness or mental retardation.

(c) Students with learning difficulties must need special educational services to succeed, services that are not needed by their non-handicapped peers.

Pandit (2000) then defines students with learning difficulties in Mathematics as those who have problems in Mathematics but their intellectual functioning as measured by general mental ability test is average or above average, and they have normal hearing and visual acuity, no history of chronic disease, regular attendance in the class and there is a significant discrepancy between their potential and actual achievement. A learning difficulty can cause a person to have trouble learning and using certain skills. The skills most often affected are: reading, writing, listening, speaking, reasoning and doing Mathematics (IDEA, 2011).

A brief review of past studies reveals that environmental factors contribute to learning difficulties such as poor quality of teaching that is poor instruction and poor teachers with inadequate techniques, poor nutrition, health and safety can also enhance learning difficulties. Pandit (2000), points out that socio-cultural factor that do not reinforce values for education, regular school attendance, work and study habits, and other supportive skills may create more difficulty for learning difficulties person.

Deficit in any area of information processing can manifest in a variety of specific learning difficulties and it is possible for an individual to have more than one of these difficulties. The different types of learning difficulties include dyslexia which is reading difficulty; dysgraphia which is difficulties with handwriting; dyscalculia which is Mathematics difficulties or poor number sense; nonverbal learning difficulty often manifest in motor clumsiness, poor visual-spatial skills, problematic social relationships, difficulty with Mathematics and poor organizational skills; dyspraxial refers to a variety of difficulties with motor skills; disorders of speaking and listening; auditory processing disorder which is difficulties processing auditory information and comprehending more than one task at a time (Gersten, Beckmann, Clarke, Foegen, Marsh, Star & Witzel, 2009).

Dyscalculia is a specific learning difficulty involving in learning Mathematics which includes confusion about Mathematics symbols and inability to understand, remember, and manipulate number facts, that is inability to conceptualize numbers as abstract concepts of comparative quantities (Cusimano, 2001). Dyscalculia affects a total of about 6% of all students (Mills, 2011). Several factors contribute to learning difficulties including poor instructional strategies, trying to teach a subject before the student is developmentally ready, negative socio-economic factors, parent and teacher's attitude (Berch & Mazzocco, 2007; Mills, 2011). Mills (2011) explains further that Mathematics learning difficulties affect 30% of all students; causes include poor preparation, a specific Mathematics learning disability (Dyscalculia), Mathematics anxiety, poor instructional strategy, dyslexia and Attention Deficit Disorder (ADD). A small number of students is identified as having a specific mathematical learning difficulty (dyscalculia) but there is a divergence of views about causes (Hannell, 2005). It has been suggested by Carnellor (2004) that very few students actually have a mathematical learning difficulty but Booker, Bond, Sparrow and Swan (2004) suggest that most students have a learning difficulty. Adesoji (2008) shows that method of instruction can influence the performances of students with low achievement in Mathematics. Sherman, Richardson and Yard (2005) emphasize that Mathematics instruction must provide many opportunities for concept building, relevant challenging questions, problem solving reasoning, and connections within the curriculum and real-world situations. Hence, students' Mathematics learning difficulty as it influences their achievement was investigated in this study.

According to the New Analysis of International Research (Science Daily, 2010), girls around the world are not worse at Mathematics than boys, though boys are more confident in their Mathematics abilities and girls from countries where gender equity is more prevalent are more likely to perform better on Mathematics assessment tests. Gender inequality in education has remained a perennial problem of global scope (Bordo, 2001; United Nations Educational Scientific and Cultural Organization UNESCO, 2003).

The poor Mathematics Performance of Students is worsened by gender imbalance leading to the problem which now constitutes a major research focus across the globe (UNESCO, 2003).

Akinsola and Tijani (1999) point out that Mathematics is not a male dominated subject as claimed by some people, but rather for both sexes provided they are subjected to the same teaching and learning conditions. Plato disagreed with the assertion that there is any systematic difference between men and women with respect to the abilities relevant to guardianship- the capacity to understand reality and make reasonable judgment (Republic, 2001). Thus Plato advocates that prospective guardians, both male and female, should receive the same education and be assigned to the same vital functions within the society. In Nigeria, gender-achievement studies include Abiam and Odok (2006) who find no significant relationship between gender and achievement in number and numeration, algebraic process and statistics. They, however, find the existence of a weak significant relationship in geometry and trigonometry. Opolot-Okurot (2005) also found differences in students' attitude towards Mathematics based on gender. Due to this inconsistency, contradictions and lack of finding clear trend in gender as it influences students' achievement in Mathematics, more investigation has become necessary. The variable is therefore investigated in terms of its influence on students' achievement in Mathematics.

1.2 Statement of the Problem

Mathematics plays a significant role in this modern age of science and technology, yet, students are not performing well in the subject. Available evidence also shows that students poor performance is due to their learning difficulties in the subject which could be ameliorated using strategies which include concrete-representational-abstract instructional strategy which allow active participation of students in learning through cutting and modelling of the concepts and explicit instructional strategy which allow mastery at every step. However, the teaching of Mathematics at the senior secondary school level in Nigeria has not explored these strategies, also their effects on achievement in Mathematics of students with Mathematics learning difficulties have not received much research attention. This study therefore, measured the effect of Concrete-

Representational-Abstract and Explicit Instructional Strategies on students' achievement in Mathematics and their attitude to Mathematics. It also investigated the moderating effects of Mathematics learning difficulty and gender on the dependent variables.

1.3 Hypotheses

The following null hypotheses were formulated for the study.

$H0_1$: There is no significant main effect of treatment on students'

(a) achievement in Mathematics and

(b) attitude to Mathematics.

$H0_2$: There is no significant main effect of Mathematics learning difficulty on students'

(a) achievement in Mathematics and

(b) attitude to Mathematics.

$H0_3$: There is no significant main effect of gender on students'

(a) achievement in Mathematics and

(b) attitude to Mathematics.

$H0_4$: There is no significant interaction effect of treatment and Mathematics learning difficulty on students'

(a) achievement in Mathematics and

(b) attitude to Mathematics.

$H0_5$: There is no significant interaction effect of treatment and gender on students'

(a) achievement in Mathematics and

(b) attitude to Mathematics.

$H0_6$: There is no significant interaction effect of Mathematics learning difficulty and gender on students'

(a) achievement in Mathematics and

(b) attitude to Mathematics.

$H0_7$: There is no significant interaction effect of treatment on Mathematics learning difficulty and gender on students'

(a) achievement in Mathematics and

(b) attitude to Mathematics.

1.4 Scope of the Study

This study covered SS II Mathematics students drawn from six selected senior secondary schools in Oyo State, Nigeria. The study determined the effects of Concrete-Representational-Abstract and Explicit Instructional Strategies on achievement in Mathematics and attitude of the students to Mathematics. It also found the moderating effects of Mathematics learning difficulty and gender on their achievement in and attitude to Mathematics. The contents selected for this study were circles, volume of solids and Angles of elevation and depression. These concepts were listed in WASSCE Chief Examiners' reports of 2004, 2005, 2007 and 2009 as the areas where candidates performed poorly in the senior secondary school Mathematics examinations. It was also stated in the 2009 report that majority of the candidates could not apply the basic concepts and theorems correctly in some areas of the syllabus like mensuration of three dimensional shapes, circle theorems, trigonometry and geometrical construction.

1.5 Significance of the Study

It is expected that findings of this study would help students to develop a tangible understanding of Mathematics concepts. When students are supported to first develop a concrete level of understanding for any Mathematics concepts, this foundation can be used to later link their conceptual understanding to abstract Mathematics learning activities. More importantly, students with Mathematics learning difficulties would be able to acquire the same level of knowledge with their counterparts without the problem and this would give them confidence in coping with everyday life problems.

It is also anticipated that findings from this study would help teachers of Mathematics in making the learning of different concepts in Mathematics real and reduce the teacher's constraints in teaching and would further improve students' achievement in Mathematics and attitude towards Mathematics. Parents would also benefit from this study in the sense that they would be relieved of the financial burden arising from persistent poor performance of their children and wards in Mathematics all the time and the attendant re-enrolment of such students for the examinations. Hence, their resources would be conserved when students do not have to re-register for examinations especially in Mathematics. The study is significant to the society in the advancement of science and technology and overall development which cannot be achieved without a sound knowledge of Mathematics. Also, the study would provide useful information to Mathematics educators, curriculum developers in Mathematics and government agencies on the introduction of remediation into the programme of Mathematics teaching and learning as a subject specifically and any other school subject posing learning difficulties to students.

1.6 Operational Definition of Terms

Achievement in Mathematics: This is students' level of knowledge in Mathematics. In this study, it is represented by the scores obtained by students on the test on selected Mathematics concepts.

Attitude towards Mathematics: Students' attitude towards Mathematics is represented by their emotional disposition towards Mathematics. It is represented by the scores obtained by students on Students Mathematics Attitude Questionnaire.

Concrete-Representational-Abstract Instructional Strategy: This is a strategy in which Mathematics concept is modelled with real-life materials, using pictures or graphs and symbols. It occurs when students have opportunities to manipulate concrete objects to solve problems.

Conventional Teaching Strategy: This refers to the prevalent methods used by Mathematics teachers in the teaching of Mathematics in Nigerian secondary schools.

Explicit Instructional Strategy: Explicit instructional strategy is a sequence of supports which involves setting the stage for learning, followed by a clear explanation of what to do, then modelling of the process, followed by multiple opportunities for practice until independence is attained.

Mathematics Learning Difficulty: Mathematics learning difficulty is a disorder in which a student has serious Mathematics learning problems in schools but do not fall under other categories of handicap. This problem will not prompt student to learn mathematical concepts as quickly as someone who is not affected by learning difficulty.

CHAPTER 2

LITERATURE REVIEW

Literature for the study is reviewed around major headings and relevant sub-headings as follows.

2.1.1 Bruner's Theory of Instruction (Constructivist Theory)

Constructivism is a learning theory that draws on students' existing knowledge, beliefs and skills. Bruner's theory on constructivism encompasses the idea of learning as an active process in which learners construct new ideas based upon their existing knowledge. Bruner (1960) emphasized that the learner will take pieces of past knowledge and experiences and organize them to make sense of what they know and solve additional problems based upon a combination of what they already processed and what they think should be processed next.

Bruner's theory emphasized the significance of categorization in learning. To perceive is to categorize, to learn is to form categories, to make decisions is to categorize. Also information interpretation and experiences by similarities and differences is a key concept of Bruner's theories. He emphasized the role of structure in learning and how it may be made central in teaching. The approach taken should be a practical one. The teaching and learning of structure, rather than the mastery of facts and techniques, is at the center of the classic problem of transfer, it must do so by providing a general picture in terms of which the relations between things encountered earlier and later are made as clear as possible.

Bruner introduced the ideas of readiness for learning. He believed that any subject could be taught at any stage of development in a way that fit the students' cognitive abilities. He believed that intuitive and analytical thinking should both be encouraged and rewarded. The learner selects and transforms information, construct hypotheses, and makes decisions, relying on a cognitive structure that is schema, or mental models which provides meaning and organization to experiences and allows the individual to go beyond the information given. As far as instruction is concerned, the instructor should try and encourage students to discover principles by themselves. The instructor and student should engage in an active dialogue. The task of the instructor is to translate information to be learned into a format appropriate to the learner's current state of understanding. Curriculum should be organized in a spiral manner so that the student continually builds upon what they have already learned.

Burner (1966) stated that a theory of instruction should address four major aspects:

(1) **Predisposition towards learning:** This feature specifically stated the experiences which move the learner toward a love of learning in general or of learning something in particular. Motivational, cultural, and personal factors contributed to both general and specific learning. Bruner emphasized social factors and he believed learning and problem solving emerged out of exploration and explained further that part of the task of a teacher is to maintain and direct a student' spontaneous explorations.

(2) **Structure of knowledge:** This is the ways in which a body of knowledge can be structured so that it can be most readily grasped by the learner.

(3) **Modes of representation:** This is the most effective sequences in which to present material, which should be visual, words and symbols.

(4) **The nature and pacing of rewards and punishments:** This is the form in which rewards and punishments are been paced. Reward and punishment should be selected and paced appropriately.

Bruner's constructivist theory is a general framework for instruction based upon the study of cognition. The ideas outlined in Bruner (1960) originated from a conference focused on science and Mathematics learning. Bruner illustrated his theory in the context of Mathematics and social science programmes for young children (Bruner, 1973). Bruner postulated three stages of intellectual development. The first stage is "Enactive" that is when a person learns about the world through actions on physical objects. The second stage was called "Iconic" where learning can be obtained using models and pictures. The final stage was "Symbolic" in which the learner develops the capacity to think in abstract terms. Bruner's theory has to do with representations of mathematical concepts according to different levels of students' thinking. The representation based on the concrete, pictorial, and abstract (CPA) (Bruner, 1960). Bruner's idea was to emphasize concrete representation, which is in harmony with some children's ability to understand mathematical concepts at the early stages. According to Kheong (2009), research showed that students could not depend too much on concrete representation as they need to move on to the next level so that they could conceptualize abstract situations using pictorial representation such as the 'model' approach.

2.1.2 Kolb's Experiential Learning

Experiential learning theory defined learning as the process whereby knowledge is created through the transformation of experience. Knowledge results from the combination of grasping and transforming experience.

- Setting a positive climate for learning
- Clarifying the purpose of the learner(s)
- Organizing and making available learning resources.
- Balancing intellectual and emotional components or learning and

- Sharing feelings and thoughts with learners but not dominating.

Kolb proposed a four-stage learning process with a model referred to, experiential learning (McGill & Beaty, 1995; Kolb, 2006). Kolb experiential learning is a four-stage cyclical theory of learning. It is a holistic perspective that combines experience, perception, cognition and behaviour. Kolb theory was built upon earlier work of John Dewey and Kurt Levin. Kolb believes that learning is the process whereby knowledge is created through the transformation of experience. The theory presents a cyclical model of learning, consisting four stages below, one may begin at any stage but follow the sequence.

- concrete experience (or "do")
- reflective observation (or 'observe')
- abstract conceptualization (or 'think")
- active experimentation (or "plant")

Kolb's four-stage learning cycle shows how experience is translated through reflection into concepts, which in turn used as guides for active experimentation and the choice of new experiences.

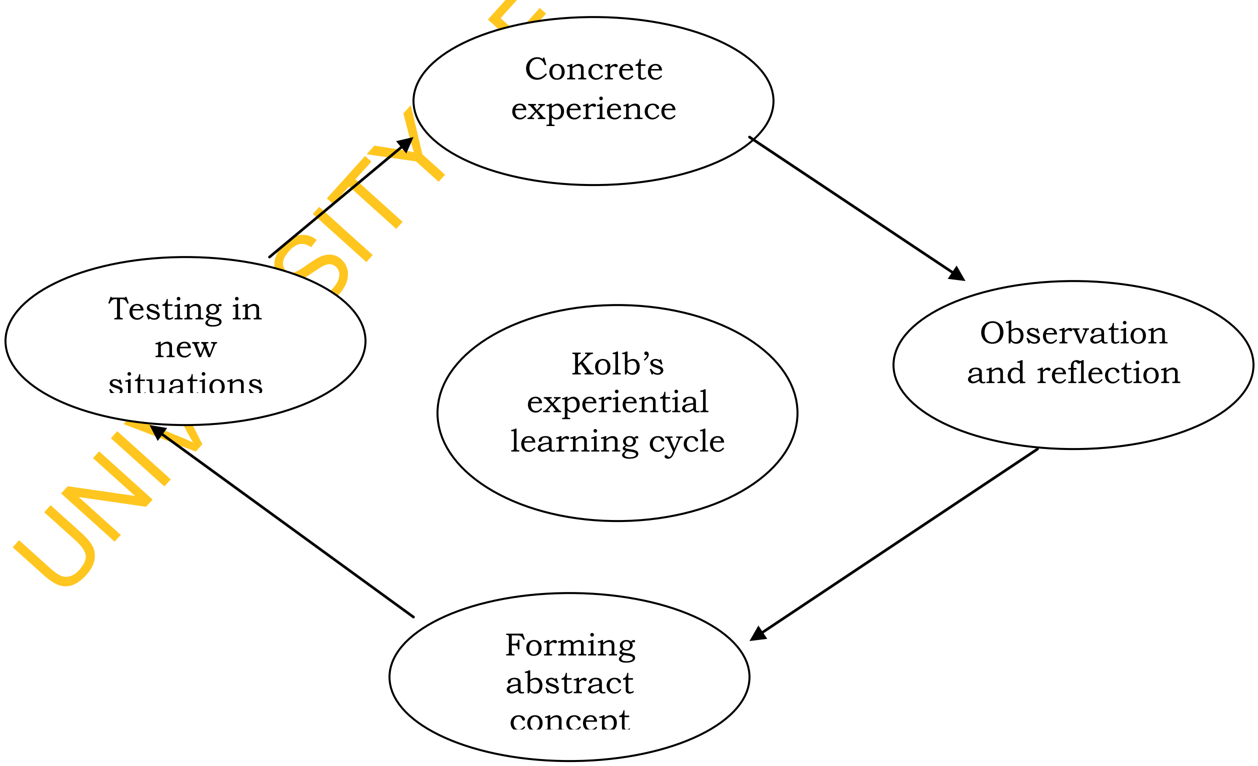


Figure 2.1: Kolb's Experiential Learning Cycle (McGill & Beaty, 1995; Kolb, 2006).

Kolb included this cycle of learning as a central principle of experiential learning theory, typically expressed as four-stage cycle of learning, in which concrete experiences' provided a basis for 'observations and reflections'. These observations and reflections' are assimilated and distilled into 'abstract concepts' producing new implications for action which can be actively tested in turn creating new experiences.

Kolb's model therefore works on two levels (Kolb, 2006) – a four-stage cycle:

1. Concrete Experience (CE)
2. Reflective Observation (RO)
3. Abstract Conceptualization (AC)
4. Active Experimentation (AE)

and a four-type definition of learning styles each representing the combination of two preferred styles, illustrated as.

1. Diverging (CE/RO)
2. Assimilating (AC/RO)
3. Converging (AC/AE)
4. Accommodating (CE/AE)

(Kolb Diagrams 2006)

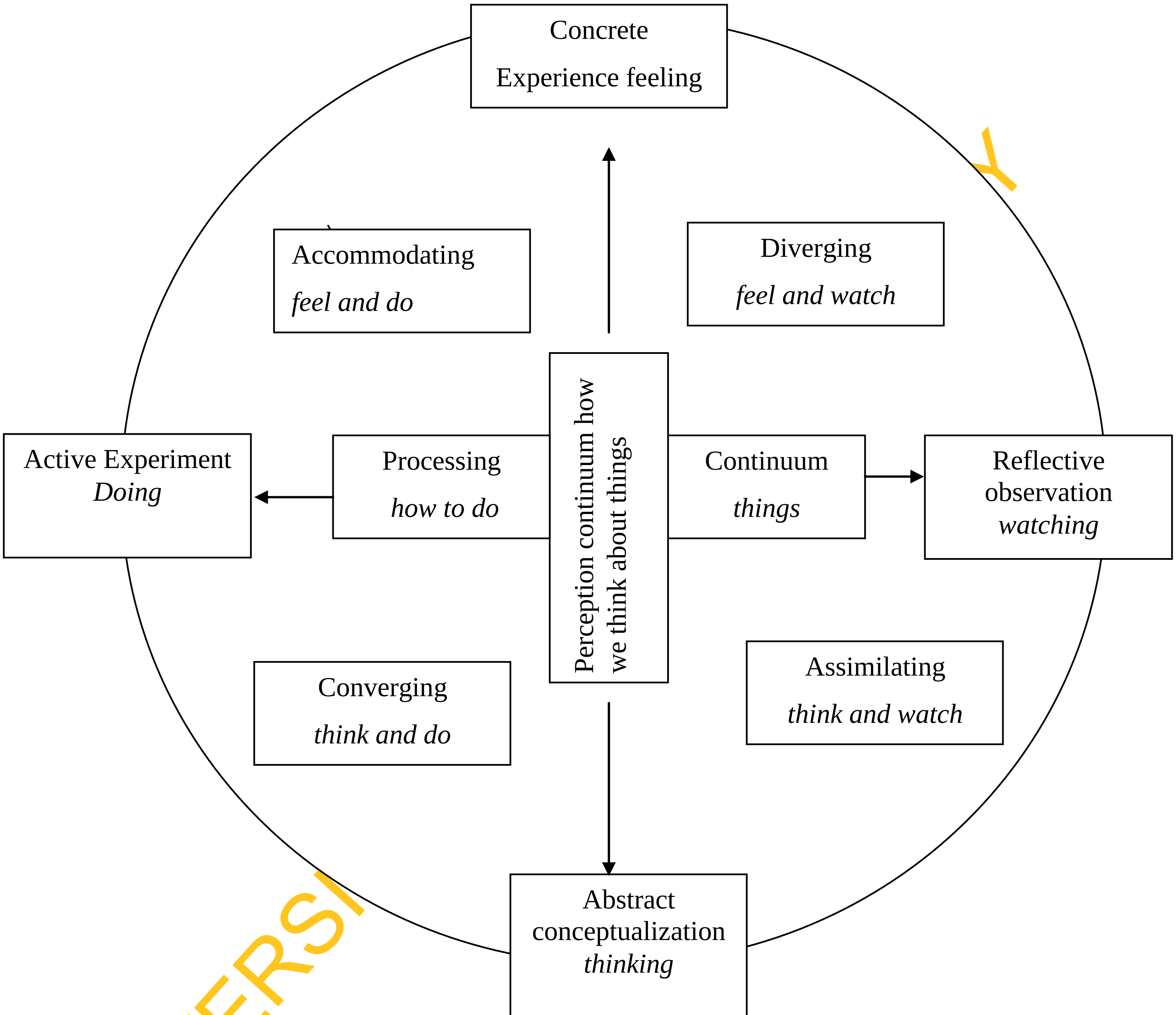


Figure 2.2: Kolb's Learning Styles

Kolb explained that different people naturally prefer a certain single different learning style. Various factors influence a person's preferred style, notably in his experiential learning theory model, Kolb defined three stages of a person's development, and suggested that our propensity to reconcile and successfully integrate the four different learning styles improves as we mature through our development stages. The development stages that Kolb identified are:

1. **Acquisition**: birth to adolescence-development of basic abilities and 'cognitive structure'.
2. **Specialization:** schooling, early work and personal experiences of adulthood.
3. **Integration:** mid-career through to later life-expression of non-dominant learning style in work and personal life.

Whatever influences the choice of style, the learning style is actually the product of two pairs of variables which Kolb presented as lines of axis.

Concrete Experience- CE (feeling) ……… V ……….. Abstract Conceptualization- AC (thinking)

Active Experimentation- AE (doing) ………. V ………… Reflective Observation-RO watching.

A typical presentation of Kolb's two continuums is that the east-west axis which is called the processing continuum (how we approach a task), and the north-south axis called the perception continuum (our emotional response or how we think).

These styles are the combination of two lines of axis each formed between what Kolb called 'dialectically related modes' of grasping experience, doing or watching and 'transforming experience', feeling or thinking.

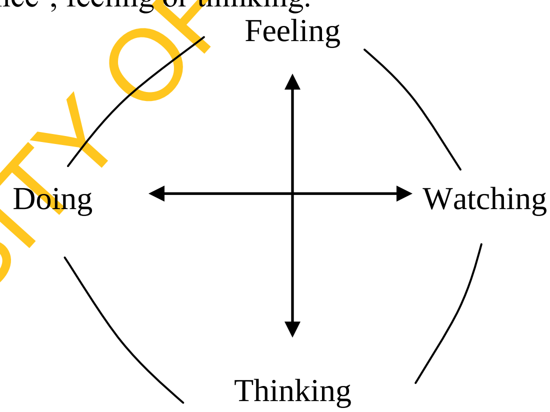


Figure 2.3: Kolb's learning styles

2.1.3 Relevance of the theories to the Study

Bruner's theory is relevant to this study in that the ideas outlined in Bruner (1960) focused on science and Mathematics as well as mode of representation referred to as Concrete, Pictorial and Abstract which is synonymous with Concrete-Representational-Abstract Instructional Strategy in the study. Students who have difficulties with Mathematics can benefit significantly from lessons that include multiple models that approach a concept at different cognitive levels.

According to David (2007), Mathematics educators have recognized a substantial body of research showing that the optional presentation sequence for new mathematical content is Concrete-Pictorial-Abstract (CPA). This approach has also been referred to as Concrete-Representational-Abstract (CRA). Regardless of the name, the instruction approach is similar and originally based on the work of Bruner in the 1960s (Bruner, 1960). Concrete components include manipulative, measuring tools, and other objects the students can handle during the lesson. Pictorial representations include drawings, diagrams, charts and graphs that are drawn by the students or are provided for the students to read and interpret. Abstract refers to symbolic representations such as numbers and letters that the student writes or interprets to demonstrate understanding of a task. This CPA approach benefits all students but has been shown to be particularly effective with students who have Mathematics difficulties, mainly because it moves gradually from actual objects through pictures and then to symbols (Jordan, Miller & Mercer, 1998). The theory is also relevant to explicit instruction by considering students existing knowledge, beliefs and skills.

Experiential learning requires qualities such as self-initiative and self-evaluation. For experiential learning to be timely effective, it should employ the whole learning wheel, from goal setting to experimenting and observing, to reviewing and finally to action planning. This complete process allows students to learn new skills, new attitudes and new ways of thinking. Learning through fun helps the learner to retain information for a longer period and students are encourage to directly involvement in the experience, in order that they gain a better understanding of the new knowledge and retain the information for a longer time. Kolb experiential theory which involves four-stages of learning, "do, observe, think and plant" helps students to participate actively in learning.

2.2 Empirical Literature

2.2.1 Nature and Importance of Mathematics

Mathematics relies on both logic and creativity and it is pursued both for a variety of practical purposes and for its intrinsic interest. According to American Association for the Advancement of Science (AAAS) (1989) for some people, and not only professional Mathematics, the essence of Mathematics lies in its beauty and its intellectual challenge. For others, including many scientists and engineers, the chief value of Mathematics is how it applies to their own work. Mathematics plays a central role in modern culture, and some basic understanding of the nature of Mathematics is requisite for scientific literacy. To achieve this, students need to perceive Mathematics as part of the scientific endeavour, comprehend the nature of mathematical thinking, and become familiar with key mathematical ideas and skills.

American Association for the Advancement of Science (AAAS) (1990) also explained further that as a theoretical discipline, Mathematics explores the possible relationship among abstractions without concern for whether those abstractions have counterparts in the real world. The abstractions can be anything from strings of numbers to geometric figures to sets of equations. Dossey (1992) argued that different conceptions of Mathematics influence the ways in which society views Mathematics. This can influence the teaching of Mathematics, and communicate subtle messages to students about the nature of Mathematics that “affect the way they grow to view Mathematics and its roles in their world”. Similarly, Presmeg (2002) has argued that beliefs about the nature of Mathematics either enable or constrain “the bridging process between everyday practices and school Mathematics. Numeracy emphasizes the practical or everyday uses of Mathematics in contexts such as homes, workplaces, and communities (Stroessiger, 2002). Writers who argued that Mathematics is valuable for its own sake often wrote about the beauty and aesthetic of Mathematics, and the sheer enjoyment of doing Mathematics (Holton, 1993; Winter, 2001). Grootenboer (2003) investigated the views and feelings of students on the nature and purpose of Mathematics and how they saw themselves as learners of Mathematics. The students’ responses indicated a rather narrow conception of Mathematics, limited mostly to number concepts and arithmetic. Smith (2009) explained that Mathematics is of central importance to modern society by providing the vital underpinning of the knowledge economy and is essential in the physical sciences, technology, business, financial services and many areas of ICT. Also, it forms the basis of most scientific and industrial research and development, many complex systems and structures in the modern world can only be understood using Mathematics and much of the design and control of high-technology systems depends on mathematical inputs and outputs.

Knowledge of Mathematics is very essential in workplace, major employers in the engineering, construction, pharmaceutical, financial and retail sectors have all made clear need for people with appropriate mathematical skills, even advanced economies need an increasing number of people with more than minimum qualifications in Mathematics to stay ahead in international competitiveness and to effectively exploit advances in technology. An adequate supply of young people with mastery of appropriate mathematical skills at all levels is vital to the future prosperity of every nation (Smith, 2009). Requirements for mathematical skills in the workplace have been examined in detail in a recent report, mathematical skills in the workplace (Hoyles, Wolf, Molyneux-Hodgson & Kent, 2002; Smith, 2009). A key finding of the study was that although with the ubiquitous use of information technology all sectors has changed the nature of the mathematical skills required, it has not reduced the need for Mathematics. The report concluded that there is an increasing need for workers at all levels of organizations to possess an appropriate level of mathematical literacy.

Mathematics is universal in a sense that other field of human thought is not because of its abstractness. It finds useful application in business, industry, music, historical scholarship, politics, sports, medicine, agriculture, engineering, and other social and natural sciences, the relationship between Mathematics and other field of basic and applied science is very strong (AAAS, 1990). Science provides Mathematics with interesting problems to investigate, and use in analyzing scientific data ideas and the symbolic language valuable for expressing scientific ideas unambiguously it is the chief language of science (AAAS, 1989; 1990).

2.2.2 Students' Learning Difficulties in Mathematics

Mathematics literacy is the ability to apply skills and concepts, reason through, communicate about, and solve mathematical-problems (NCTM, 1989). Mathematics instruction involves the pedagogical strategies, curricular materials, and assessments that help all students master the skills and concepts relevant to the development of mathematical literacy. From the earliest grades and throughout their school experiences, students should feel the importance of success in solving problems, figuring things out, and making sense of Mathematics.

Although every student is affected by the increasing demands and expectations in Mathematics, students with difficulties are placed at an even greater disadvantage because of the difficulties they tend to experience in acquiring and retaining knowledge (Miller & Mercer, 1997). Many students with mild difficulties experience difficulty with Mathematics due to characteristics that impede their performance, especially in problem solving and computation (Maccini & Gagnon, 2000). Deficits in Mathematics performance may be as serious a problem for these students as the reading deficit commonly attributed to characteristics of learning disabled students (Mastropieri, Scruggs, & Shiah, 1991). Learning difficulties according to National Center for the Study of Adult Learning and Literacy (NCSALL, 2000) is a generic term that refers to a heterogeneous group of disorders manifested by significant difficulties in acquisition and use of reasoning, or mathematical abilities or social skills. It may occur along with socio-environmental influences such as cultural differences, insufficient or inappropriate instruction, or psychogenic factors, or with attention deficit disorder, all of which may cause learning problems.

Garnett (1992) reiterated that the difficulty is manifested in conceptual understanding, counting sequences, written number symbol systems, the language of Mathematics, basic number facts, procedural steps of computation, application of arithmetic skills and problem-solving. Several research studies have described students with learning difficulties who exhibited deficits in both Mathematics computation and problem-solving (Cawley, Miller & School, 1987; Englert, Culatta & Horn, 1987; Geary, 2004), as well as the execution of specific Mathematics strategies (Swanson & Rhine 1985). Cawley, Parmar, Yan and Miller (1996) found that while typically mainstream students learn mathematical concepts at a steadily increasing pace, students with learning difficulties acquire skills in a broken sequence and have lower retention rates than their peers without difficulties. These retention problems increase the concepts become more difficulty. Specifically, Miles and Forcht (1995) reported that many students with LD demonstrated problems when they first encountered algebraic concepts because of the symbolic reasoning involved.

Baroody and Hume (1991) reported that most students with learning difficulty are not intellectually impaired but require instruction that is developmentally appropriate to the ways students think and learn. Instruction should focus on understanding, learning that is active and purposeful, linking formal instruction to informal knowledge and encouraging reflection and discussion. Baroody and Hume (1991) reiterated further that more specifically, Mathematics instruction for all students including those with LD, should promote a broad range of mathematical concepts that go beyond computation and include geometry and fractions, actively involve students in doing Mathematics that have a purpose, encourage and build on students' strengths and their informal knowledge and finally encourage students to justify, discuss, and compare ideas and strategies.

All students need to have the ability to solve problems, make connections with Mathematics and with other disciplines, and represent Mathematics in different forms visually and abstractly. The NCTM process standards encourage instruction in which students' access Mathematics through an understanding of mathematical concepts. For students with LD, the process standards become even more important to the development of the process skills with the strategies designed to assist LD students in bridging the gap between "doing" Mathematics and "knowing."

One of the themes of NCTM (1988) is for students to value Mathematics as a connection with the real world. The principle in the NCTM framework is the equity principle that opens the door for all students to engage in mathematical content and processes. All students, regardless of their personal characteristics backgrounds, or physical challenges, must have opportunities to study and support to learn Mathematics (NCTM, 2000). Equity does not mean that every student should receive identical instruction, instead, it demands that reasonable and appropriate accommodations be made as needed to promote access to, and attainment of, Mathematics for students with difficulties. This equity promotes an approach through the process standards as a foundation to build upon the understanding of mathematical content in accessing the general Mathematics curriculum and encourages students to become independent learners and thinkers of Mathematics.

Many experts attributed learning difficulties in Mathematics to deficits in one or more of five different skill types (Paula, 2012). These deficits can exist independently of one another or occur in combination which can impact a student ability to progress in Mathematics.

Incomplete Mastery of Number Facts: Number facts are the basic computations students required to know in the earliest grades of elementary school. Recalling these facts efficiently is critical because it allows a student to approach more advanced mathematical thinking.

Computational Weakness: Despite a good understanding of mathematical concepts, many students are inconsistent at computing. They make errors because they misread signs, carry numbers incorrectly, may not write numerals clearly enough or in the correct column.

Difficulty Transferring Knowledge: Inability to easily connect the abstract of Mathematics with reality is one fairly common difficulty experienced by students.

Making Connections: Some students have difficulty making meaningful connections within and across mathematical experiences. For instance, a student may not readily comprehend the relation between numbers and the quantities they represent.

Incomplete Understanding of the Language of Mathematics: For some students, a difficulty in Mathematics is driven by problems with language. These students may also experience difficulty with reading, writing, and speaking, this is confounded by the inherently difficulty terminology, some of which the students hear nowhere outside of the Mathematics classroom.

Also information from the Ball State University stated that students with learning difficulty have their own patterns of strengths and weaknesses. By definition, these individuals have average or above average intelligence. However, there is still a significant discrepancy between their potential to learn and their achievement in certain areas. The manifestations resulting from this difficulty may lead to in skill deficits in one or more of the following areas: reading, understanding what is read, listening effectively because there is difficulty distinguishing among similar sounds, symbols, or objects, comprehending mathematical concepts, retaining information, expressing thoughts through writing, expressing thoughts through speaking, spelling correctly, maintaining a level of academic performance relative to cognitive evaluations, abstract and general reasoning, executive functioning (planning and time management), visual spatial skills, and memory (long-term, short-term).

2.2.3 Causes of Mathematics Learning Difficulties

According to Mills (2011), more than 30% of students in school today have significant difficulties learning Mathematics in spite of normal intelligence. There has been a wide range of problems observed within this group, leading educators initially to propose that there were a number of different types of Mathematics difficulties. However, a careful review of the recent literature suggested that most symptoms can be ascribed to one of four main causes, each having its characteristic cluster of symptoms.

- Inadequate preparation: The largest fraction of the students having Mathematics learning difficulties suffer from inadequate preparation and ineffective early education in the underlying basic Mathematics operations that are required for Mathematics studies at their current level.
- A specific Mathematics learning difficulties of genetic origin called dyscalculia affects 5% -7% of the population.
- Mathematics anxiety, including any negative emotional reaction toward Mathematics, can be considered a primary cause if significant enough by itself to limit educational opportunities.
- Dyslexia not only affects reading but contributes to significant problems learning Mathematics. It affects the same number of people as dyscalculia, but causes a different cluster of Mathematics learning difficulties.
- The category of poor preparation which include causes like poor teaching techniques, poor school and home environments among others.

2.2.4 Accommodating Students with Mathematics Learning Difficulty in Classroom Teaching

The National Center for the study of Adult Learning and Literacy (NCSALL), (2000) defined accommodation as "any change to a classroom environment or task that permits a qualified individual with difficulty to participate in the classroom process, to perform the essential tasks of the class and enjoy benefits and privileges of classroom participation equal to those enjoyed by learners without difficulties.

An accommodation is a legally mandated change that creates an equitable opportunity for task completion. They can be strategies and modifications that are used by the classroom teacher; there are also accommodations such as specialized equipment, assistive technology, and variations in the methods and materials of testing for students with learning difficulties. Accommodations help persons with difficulties to have a fair and equal chance to work, learn and have access to physical facilities such as buildings and parks. They are based on individualized, documented needs. In order to work within curriculum guidelines while accommodating the diversity of students in their classrooms, educators need to be realistic and systematic in the way they structure their Mathematics programme. The use of an engaging, and age appropriate, theme is the way into developing conceptual knowledge and skills. Westwood (2000) and Carnellor (2004) highlighted the importance of educators using a judicious blend of constructivist and explicit teaching. Teachers consider the characteristics of each student and may select appropriate accommodations and instructional modifications.

Having a learning difficulty does not mean being unable to learn, a learning difficulty exists when information is absorbed through the senses but inaccurately transmitted to the brain or inappropriately expressed. Special accommodations will need to be individually tailored because learning difficulty students will vary depending on their types and degrees of learning difficulty. The following classroom accommodation was suggested by Ball State University and Simmons College Offices of Disabilities.

- Clearly defining course requirements such as assignments and their deadlines. Be sure to provide advance notice of any schedule changes.
- Being open to students' tape recording lectures.

- Encouraging the use of word processor that will help learning difficulty students compose, edit and spell more accurately.
- Using the chalkboard, handouts, videos, group discussions, role playing and overhead projectors.
- Preparing handouts and review technical terms used in your class.
- Pointing out the organizational items in textbooks, e.g., chapter summaries, sub-headings, graphic design, charts and maps.
- Giving all assignments and course expectations in written and oral form.
- Incorporating "hands on" and laboratory experiences when they are appropriate.
- Seating students with learning difficulties in the front row.
- Giving students a clear syllabus, listing tests and assignments with due dates noted.
- Using demonstrations and hands-on experiences.
- Using overhead projectors or PowerPoint presentations. Breaking down difficult concepts into steps or parts.
- Giving assignments verbally and in writing.
- Outlining the day's lecture on the chalkboard, overhead, or PowerPoint.
- Giving a brief review of the material presented and emphasizes key points.
- Including a time for questions and answers.
- Giving students study questions for examinations that demonstrate the format as well as the content of the test and an explanation of what constitutes a good answer and why.
- Encouraging all students to take advantage of the Academic Success Center and Help Laboratory tutoring services.
- Extending the time allowed to complete assignments when appropriate.
- Making alternative assignments in some cases.
- When talking, being mindful of the speed and audibility of your lecture. Using consistent pauses or voice inflections can be effective in emphasizing important points.
- Course substitution, when appropriate.
- Providing Information about classroom and study strategies and tutoring available through the Academic Success Center.

2.2.5 Strategies for Remediating Students' Learning Difficulties in Mathematics

Educators already know there is a wide range of student abilities within a year level and with this come significant planning and programming issues. Commenting on Mathematics teaching, Elkins (2005) noted that there has been a move away from the transmission model of content delivery for all, that is, what has been referred to as *'you watch what I do, and then you do it'* to a focus on conceptual understanding that is supported by constructivist teaching approaches. Research has highlighted fundamental principles that maximize Mathematics learning may be different for students and amount of learning may be different for students with learning difficulties. The National Council of Teachers of Mathematics (NCTM, 2006) stated, effective Mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.

A small number of students is identified as having a specific mathematical learning difficulty (dyscalculia) but there is a divergence of views about causes and identification (Hannel, 2005). It has been suggested that very few students actually have a mathematical difficulty (Carnellor, 2004; Westwood, 2000). But Booker, Bond, Sparrow and Swan (2004) suggested that most students have a learned difficulty. Regardless of the reason for students, mathematical difficulties, either intrinsic or extrinsic factors, educators still need to get on and teach. Students with learning difficulties may already have an eternal locus of control that is they believe they can't improve their mathematical capacities, it is when they feel confident, make mistake and ask question, that engagement and achievement will occur.

Mathematics instruction must provide many opportunities for concept building, relevant challenging questions, problem solving reasoning, and connections within the curriculum and real-world situations. Sherman, Richardson and Yard (2005) believed that students with learning difficulties are given tedious and boring activities to develop the basics. To overcome the problem of learning, educators must programme-up and have ambitious but achievable goals. Ajiboye (1996) asserted that knowing the intelligence level of learners will, to a large extent, determine how much the learner will achieve from a learning process or skill programme. In this case, it becomes necessary to screen learners so as to know their levels of Mathematics learning difficulties.

Gersten, Russell, David, Jayanthi and Baker (2006) found out that for low-achieving students, the use of structured peer-assisted learning activities, along with systemic and explicit instruction furnished both the teacher and the students. For instance with learning difficulties, explicit and systematic instructions that involve extensive use of visual representations appear to be crucial. Teaching Mathematics concepts to students with learning difficulties has been an area relatively neglected by both researchers and curriculum developers (Mercer & Mercer, 1998). Researchers have recently begun to identify effective instruction for students' with learning difficulties and other learning problems (Brigham, Wilson, Jones, and Miosio, 1996; Kelly & Carnine, 1996).

Explicit instructional strategy provided opportunities for students to respond to learning tasks (David, 1999). Three effective instructional strategies in combination can effectively help students with learning problems (David, 1997). The instructional strategies are: Explicit instruction; Specific learning strategies, Mathematics problem-solving through a concrete-to-representational-to-abstract (CRA) sequence of instruction. Researchers have strongly suggested the importance of CRAIS to promote understanding of Mathematics concepts and skills (Mastropieri, Scruggs & Shiah 1991; Mercer, Jordan & Miller, 1996; Miller & Mercer, 1997). When teaching students' with Mathematics learning difficulties, Concrete-Representational-Abstract Instructional Strategy can be used effectively to promote student understanding from a concrete level of understanding to a representational level of understanding and then to an abstract level of understanding (Mercer, Jordan & Miller, 1996; Miller & Mercer, 1997).

The above information has shown the importance of CRAIS which makes the approach relevant for teaching students' with Mathematics learning difficulties. In explicit instruction, consistent communication between teacher and students creates the foundation for the instructional process; the strategy involves pacing a lesson appropriately, allowing adequate processing and feedback time, encouraging frequent student responses, and listening and monitoring throughout a lesson according to National Dissemination Center for Children with Disabilities (NDCCD, 2009). Adesoji (1995; 1997) also suggested that problem-solving strategies were effective in teaching students of different ability levels.

According to (Steedly, Dragoo, Arafeh & Luke, 2008), four methods of instruction show the most promise in assisting students with learning difficulties in Mathematics, they are:

- Systematic and explicit instruction, a detailed instructional approach in which teachers guide students through a defined instructional sequence. Within systematic and explicit instruction, students learn to regularly apply strategies that effective learners use as a fundamental part of mastering concepts.
- Self-instruction, through which students learn to manage their own learning with specific prompting or solution-oriented questions.
- Peer tutoring, an approach that involves pairing students together to learn or practice an academic task.
- Visual presentation, this has to do with the use of concrete materials that can be visualized; which uses manipulative pictures, number lines, and graphs of functions and relationships to teach Mathematics. This in line with Concrete-Representational-Abstract Instructional Strategy. Therefore the two strategies that were used in this study are relevant for they are included in the methods of instruction which can assist students with Mathematics learning difficulties.

To remediate the problem of learning difficulties, the following is a collection of key teaching issues in no particular order but all worthy of reflection.

- Use brief, mini-lessons for specific skills with the whole class (Peterson, Hittie & Tamor, 2002).
- Provide opportunities to work alone as well as together (Westwood, 2000).
- Use problem solving with divergent questions (Booker et al, 2004).
- Use concrete materials (Westwood, 2000; Booker et al, 2004; Carnellor, 2004).
- Confirm student understanding of mathematical language (Sherman, Richardson &Yard, 2005).
- Play games (Booker, 2000).
- Use technology (NCTM, 2006).
- Educator must have pedagogical knowledge and positive attitude (Carnellor, 2004).
- Choose published materials carefully (Pincott, 2004).

- NCTM (2006) reiterated that assessment should support the learning of Mathematics.

2.2.6 Concrete-Representational-Abstract Instructional Strategy (CRAIS) and Students' Achievement in Mathematics and Attitude towards Mathematics

Concrete representation of mathematical ideas is hardly a new idea. Bowen (1972) suggested that as early as 3000BC the Mesopotamians were using a one-to-one correspondence to perform counting, the use of tally, sticks, also the abacus are all familiar. So, the use of concrete materials in teaching Mathematics has been supported for decades in Mathematics education literature, the work of Piaget on concrete operational thinking and Dienes' writings on the structure of Mathematics to support the use of concrete materials. Moyer (2001) stated that it is necessary to use concrete materials to teach abstract mathematical concepts because students' abstract thinking is closely anchored in their concrete perceptions of the world and actively manipulating these materials allows learners to develop a repertoire of images that can be used in mental manipulation of abstract concepts.

According to American Association for the Advancement of Science (AAAS, 1990), young people can learn most readily about things that are tangible and directly accessible to their senses– visual, auditory, tactile and kinesthetic. They grow in their ability to understand abstract concepts, manipulate symbols, reason logically, and generalize. These skills develop slowly, and the dependence of most people on concrete examples of new ideas persists throughout life. Concrete experiences are most effective in learning when occur in the context of some relevant conceptual structure. The difficulties many students have in grasping abstractions are often masked by their ability to remember and recite technical terms that they do not understand, as a result, teachers sometimes overestimate the ability of students to handle abstractions and take the students use of the right word as evidence of understanding. The Nuffield Mathematics Project's Slogan (Hall, 1991) stated.

I hear, and I forget

I see, and I remember

I do, and I understand.

Concrete knowledge involves knowing how to manipulate concrete object to solve a problem (Slavin, 2009). Use of a Concrete-Representational-Abstract Instructional Strategy (CRAIS) sequence insures that the student understands the fact by first using manipulative, then drawing representations of the problem, and finally solving the problem with actual numbers. CRAIS may help improve the students' calculation skills, also help the students' reasoning skills and promoting understanding of mathematical concepts. There are three main steps in Concrete-Representational-Abstract Instruction (CRAIS), the first (concrete) requires that concrete objects be used to model each concept in the problem. The second step (representational) requires that the concept is modelled at the semi-concrete level through drawings. The third step (abstract) requires that the concept is modelled using only numbers, notations and mathematical symbols.

Concrete-Representational-Abstract Instructional Strategy (CRAIS) supports understanding underlying mathematical concepts before learning rules. Research-based studies show that students who use concrete materials develop more precise and more comprehensive mental representations, often show more motivation and on-task behaviour, understand mathematical ideas, and better apply these ideas to life situation (Suydam & Higgins, 1977; Harrison & Harrison, 1986).

Mathematical thinking often begins with the process of abstraction, which is, noticing of similarity between two or more objects. The common aspects, whether concrete or hypothetical, can be represented by symbols such as numbers, letters, other marks, diagrams, geometrical constructions, even words. Whole numbers are abstractions that represent the size of sets of things or things within a set. The circle as a concept is an abstraction derived from human faces, flowers, wheels, or spreading ripples. Abstractions are made not only from concrete objects or processes, can also be made from other abstractions, such as kinds of numbers like even numbers such abstraction enables mathematicians to concentrate on some features of things and relieves them of the need to keep other features continually in mind.

When using CRAIS, the sequencing of activities is critical. Activities with concrete materials should come first to impress on students that mathematical operations can be used to solve real-world problems. Pictured relationships show visual representations of the concrete manipulative and help students visualize mathematical operations during problem solving. It is important that the teacher explain how the pictorial examples relate to the concrete examples. Finally, formal work with symbols is used to demonstrate how symbols provide a shorter and efficient way to reach that final abstract level by using symbols proficiency with many of the mathematical skills they master. However, the meaning of those symbols must be firmly rooted in experiences with real objects. Otherwise, their performance of the symbolic operations will simply be note repetitions of meaningless memorized procedures.

Students often get frustrated when teachers present Mathematics problem only in the abstract. Mathematics teachers need to organized content into concepts and provide instruction that allows students to process the new language in meaningful and efficient ways. When solving for algebraic variables, students who used the CRAIS sequence of instruction performed fewer procedural errors (Witzel, Mercer & Miller, 2003). Concrete and pictorial representations should be used at all grade levels. By using cognitive strategies such as CRAIS, teachers provide students a technique for tackling Mathematics problems rather than just searching for an answer.

Students with learning difficulties often have difficulty with symbolic concepts and reasoning. These students may need extra assistance through hands-on manipulative and pictorial representations of mathematical concepts. Hand-on experiences allow students to understand how numerical symbols and abstract equations operate at a concrete level, making the information more accessible to all students (Devlin, 2000; Maccini & Gagnon, 2000). Visual representations of mathematical relationships are consistency recommended in the literature on Mathematics instruction (Witzel, Mercer & Miller, 2003). Mathematics requires representations, which serve as tools for mathematical communication, thought, and calculation, allowing personal mathematical ideas to be externalized, shared and preserved.

When introducing new concepts to students, describe and model the concept using concrete materials. The concrete level of understanding is the most basic level of cognitive skill. When students have mastered this level of concrete understanding by modelling and describing the concept concretely themselves, instruction moves to the next step which is representational level. When students have mastered the concrete level, describe and model the concept by drawing and using other pictures that represent the concrete objects. When students have mastered the representational level, describe and model the concept in an abstract manner, using the typical form in which problems are presented. The ability to work abstractly is the most advanced cognitive level. Wolfe (2001) pointed out that learning is the process of building neural networks. The three levels of learning are concrete, representational and abstract and it is the interconnection between the three types of learning that strengthens neural networks.

Ross, Hogaboam-Gray and McDougall (2002) stressed that Mathematics education reform needs to focus on teachers helping students to develop self-confidence in Mathematics, which is as important as achievement. Therefore, students need to have favourable experiences with Mathematics in order to create memories and feelings that lead to positive emotional responses. Teachers must make Mathematics learning environments inviting, engaging and safe for the learners to demystify Mathematics content by making Mathematics lessons real, true to life, and hence meaningful through the use of readily available real life concrete objects. Grouws and Cebulla (2000) stated that, long-term use of concrete materials is positively related to increases in student achievement and improved attitudes towards Mathematics ideas.

Wisniewski and Smith (2002) reiterated that the use of concrete materials can improve the achievement of students that struggle with learning and also effective for all learners. Cass, Cates, Jackson and Smith (2002) found that concrete materials resulted in a fairly rapid acquisition and maintenance of basic perimeter and area problem-solving skills. Butler, Miller, Crehan, Babbitt and Pierce (2003) during the instruction on equivalent fractions, separated students into two treatment groups, concrete-representational-abstract (CRA) and representational-abstract (RA), the findings revealed that students in the CRA group had better knowledge and understanding of equivalent fractions than the RA group.

2.2.7 Explicit Instructional Strategy and Students' Learning Outcomes in Mathematics

A good deal of the special education literature in Mathematics has called for instruction to be explicit and systematic (Gersten, Baker, Phgah, Scanhon & Chard, 2001;

Fuchs & Fuchs, 2003). Explicit instruction possessed the following three specific components:

- The teacher demonstrated a step-by-step strategy for solving the problem.
- This step-by-step plan needed to be specific for a set of problems,
- Students were asked to use the same procedure demonstrated by the teacher to solve the problem.

Explicit instruction is a teacher-centered approach that is most effective for teaching basic skills (Kroesberger & Van, 2003). It is generally fast-paced instruction and often used with a small group of students. Students respond to instruction and receive immediate feedback. Explicit instruction includes continuous modelling by teachers, followed by more limited teacher involvement as students begin to master the material (Maccini & Gagnon, 2000). Teaching basic skills to students through explicit instruction and retrieve the information will ensure a successful educational experience for all students. For students with difficulties the strategy is crucial for the retention of new skills. It is an instruction that carefully constructs interactions between students and their teacher. It always includes continuous checking for understanding to verify that students are learning during lesson. Explicit instruction is an approach that encompasses goal of improving learning for all students and especially for low-performing students.

According to Hall (2002), explicit instruction is a systemic instructional approach that includes set of delivery and design procedures. Well designed explicit instruction consists of two essential components: (a) visible delivery features are group instruction with a high level of teacher and student interactions and (b) the less observable instructional design principles and assumptions that make up the content and strategies to be taught. The Centre for Applied Special Technology (CAST) offers a helpful snapshot of an explicit instructional episode (Hall, 2002), shown in figure below. Instructional episodes involve pacing a lesson appropriately, allowing adequate processing and feedback time, encouraging frequent student responses, and listening and monitoring throughout a lesson.

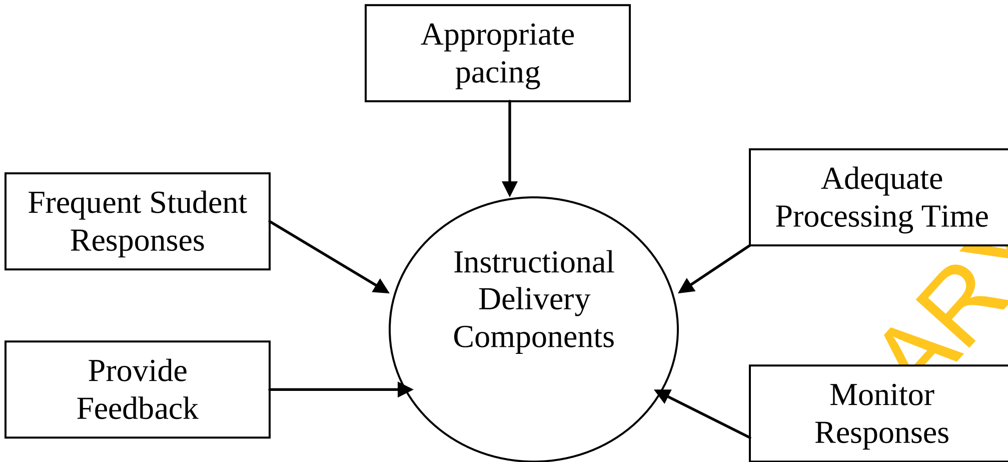


Figure 2.4: Explicit Instructional Episode

Montague (2007) suggested that "the instructional method underlying cognitive strategy instruction is explicit instruction." Explicit instruction, according to Rosenshine (1986) "is a systematic method for presenting learning material in small steps, pausing to check for student understanding and eliciting active and successful particular from all students." It always includes continuous checking for understanding to verify that students are learning during the lesson. Explicit modes of instruction are well grounded in findings from evidence-based research in cognitive science and give little attention to the causes of under-achievement, learning difficulties, or to students underlying abilities (Casey & Tucker, 1994).

Explicit instruction is based on both the theory and evidence that learning can be greatly accelerated if instructional presentations are clear, minimize misinterpretations and facilitate generalizations (Northwest Regional Education Laboratory, 2005). The principles upon which explicit instructions are based include:

- all students can learn, regardless of their intrinsic and context characteristics.
- the teaching of basic skills and their application in higher-order skill is essential to intelligent behaviour and should be the main focus of any instructional programme.

The National Mathematics Advisory Panel Report (2008) found that explicit instruction was primarily effective for computation of basic Mathematics operations, but not as effective for higher order problem solving. While Swanson (2001) asserted that breaking down instruction into steps, working in small groups, questioning students directly, and promoting ongoing practice and feedback seem to have greater impact on students' achievement and attitudes toward the subject. According to Freebody, Ludwig and Gunn (1995); Edwards-Groves (1998) a large body of research shows effective interaction leads to successful learning when it is explicit and student-centered, student have opportunity to invest in their own learning in a meaningful way and this will improve both achievement and attitude toward the subject.

2.2.8 Students' Learning Difficulties and Achievement in Mathematics

The National Dissemination Centre for Children with Disabilities (NICHCY, 2009) stated that several specific areas of difficulties are clearly connected to Mathematics learning difficulties. Visual processing, visual memory, and visual-spatial relationships all impact Mathematics proficiency by threading in the fabric of conceptual understanding and procedural fluency (Kilpatrick, Swafford & Findell, 2001). Learning difficulties affect Students' ability to formulate, represent and solve Mathematics problems.

Learning Difficulties affects Mathematics learning and performance, with a well-documented impact on the learning 5% to 10% of Students in secondary school (Geary, 2001, 2004; Fuchs & Fuchs 2002; Mazzocco & Thompson, 2005). It is not all the students with learning difficulties that have Mathematics troubles. If a student's cognitive ability is much higher than his or her performance, the student has learning difficulty. Mathematics learning difficulty is referred to as dyscalculia which can cause difficulties as learning Mathematics concepts such as quantity, place value, and time, also cause difficulty memorizing Mathematics facts, organizing numbers, and understanding how problems are organized on the page (Cusimano, 2001). Generally, dyscalculics are often referred to as having poor number sense and these will definitely affect achievement in Mathematics. Cawley, Parmar, Yan and Miller (1996) found out that students' with learning difficulties have low retention rates. But Ma (1999) and Woodard (2004) revealed a negative relationship between Mathematics learning difficulty and achievement in Mathematics.

2.2.9 Students' Learning Difficulties and Attitudes toward Mathematics

Attitude is concern with an individual way of thinking, acting and behaving. It has very serious implications for the learner, the teacher, the immediate social group with which the individual learner relates and the entire school system (Yara, 2009). Attitude is formed as a result of some kind of learning experiences. They may also be learned simply by following the example or opinion of parent, teacher, or friend. Teachers are, invariably, role models whose behaviours are easily copied by students; in this case, teachers' attitudes towards their students in school must be favourable enough to carry students along. Burstein (1992), in comparative study of factors influencing Mathematics achievement, found out that there is a direct link between students' attitudes toward Mathematics and students' achievement. Also, Akinoso (2011) found that students' attitude was positively related to achievement in Mathematics. In this case, Students need to develop the attitudes and habits that are considered for meaningful work in Mathematics. Some students have anxiety that is specific to Mathematics, teachers can benefit from the awareness that Mathematics anxiety can have negative influences on Students' academic achievements, and attitudes to Mathematics can influence achievement in Mathematics (Ashcraft, Krause, & Hopko, 2007)

2.2.10 Gender and Students' Achievement in Mathematics

Much attention has been devoted to the role of gender differences in achievement in Mathematics, such as reports of the alleged male superiority in Mathematics and the under-representation of females in mathematically oriented careers (Ogunleye & Ogunsanwo, 2001; Lachance & Mazzocco, 2006). In spite of the fact that gender differences in educational outcomes are quite many in literature, recent research reports on gender influence on students' learning outcomes in Mathematics revealed yet conflicting results (Ifamuyiwa, 2006). While some reported significant main effects of gender on students' achievement (Onabanjo, 2000; Odogwu, 2002, Ojo, 2003), others found no significant gender difference on students' learning outcomes (Popoola, 2002; Adegoke, 2003; Akinola, 2003,). The findings of Abimbade (1997) on the use of programmed texts for Mathematics instruction of boys and girls of secondary school age show that in achievement, the boys performed significantly better than the girls. Odogwu (2002) reported that female students exhibited negative attitude towards Mathematics, which consequently lead to their poor achievement and low participation in the subject.

Fennema (2000) in her study showed that gender differences existed in learning complex mathematical tasks in middle and secondary schools. Home (2003) found that although achievement tests showed no significant difference in achievement outcomes, there have been differences shown in thinking. More recently, Fryer and Levitt (2009) found that the gender gap in Mathematics not only existed in the early elementary school years but had also grown in each grade. Vale (2009) found that many studies conducted between 2000 and 2004 showed no significant differences in achievement in Mathematics between males and females, though males were more likely to obtain higher mean scores.

An interesting body of international literature suggested that the gap in achievement in Mathematics needs to be re-examined, as female students perform better than male students (Hydea & Mertzb, 2009). Studies done by Arnot, David and Weiner (1999) noted that female students' achievement in Mathematics increasingly showed that the gender gap was closing. Female students were viewed as doing very well in Mathematics classes.

Gender-achievement studies include Abiam and Odok (2006) who found no significant relationship between gender and achievement in number and numeration, algebraic processes and statistics. These researchers, however, found the existence of a weak significant relationship in geometry and trigonometry. Bassey, Joshua and Asim (2007) in their study in a rural secondary found a significant gender gap in favour of rural males as well as among low socio-economic students.

Many studies have focused on factors related to differences in the performance of boys and girls in Mathematics (Mahlomaholo & Sematle, 2005; Opolot-Okurot, 2005; Abiam, & Odok, 2006; Zhu, 2007). Waiden and Walkerdine (1985) found and suggested that perceptions of teachers are that girls' performance in Mathematics are dependent on rote learning, hard work and perseverance rather than natural talent, flexibility and risk taking which are the learning styles of boys. Teachers are also of the view that girls "learn" Mathematics whilst boys "know" Mathematics. Opolot-Okurot (2005) found that for all the attitudinal variables (anxiety, confidence and motivation), males had higher means scores than female. Mutemeri and Mygweni (2005) argue that the idea that Mathematics is for boys may result in low motivation in girls and could widen the gender gap of achievement in Mathematics in favour of boys.

According to Fennema and Leder (1990), gender differences in Mathematics teaching, learning and achievement have been explained on the basis of gender differences in cognition and brain lateralization. Koehler (1990) cited in Fennema and Leder (1990) reported that teachers treat male and female students differently. Other studies have shown that many teachers, especially males, view Mathematics as a male domain and convey this attitude both directly and indirectly to students (Waiden & Walkerdine, 1985). Girls valued experiences that allowed them to think and develop their own ideas their aim was to gain understanding. Boys, on the other hand, emphasized speed and accuracy and saw these as indicators of success. Duncan (1989) and Marope (1992) cited in Kaino (2001) indicated that cultural expectations of society could result in differences in performance between girls and boys in certain school subjects such as Mathematics. Society fixes gender roles and conditions males to engage in intellectually and physically more challenging.

2.2.11 Gender and Students' Attitudes towards Mathematics

Attitude towards Mathematics has been considered an important factor in influencing participation and success in Mathematics. One of the areas of life which have been affected by gender differences is the field of education and precisely that of academic performance of students (Adeosun, 2008). The nature and extent of gender differences in mathematical performance remain a controversial topic for there are many confounding variables and a variety of variables are used as measures of performance (Malone & Miller, 1993 Leeson, 1995). Many researchers have found that male students performed better than their female counterparts (Adesoji & Fisuyi, 2001; Mboto, 2001; Kolawole, 2002; Oginni, 2007). Some found no significant difference in academic performance in Science and Mathematics between male and female Students (Owolabi, 2002; Ogunkola, 2007). Poor attitude towards Mathematics has often been cited as one factor that has contributed to lower participation of girls in Mathematics courses

(Fullarton, 1993; Willis, 1995). Tymms (2001) suggested that gender was weakly associated with attitudes. Koller, Baumert, and Schnabel (2001) studied gender differences in achievement in Mathematics and favoured males achievement interest, and placement in advanced Mathematics courses. Saha (2007) conducted a study Gender, Attitude to Mathematics, Cognitive style and Achievement in Mathematics and found out that the three contributes to statistically significant difference in achievement in Mathematics.

Due to the fact that females are often discouraged from mathematical work at lower level which provides base for higher studies in Mathematics, fewer women are employed in industry in post needing mathematical ability (Arnot, David & Weiner, 1999). Still research indicates that this difference prevails in some areas of complex mathematical tasks. Mullis, Martin, Gonzalez, Conner, Chrostowski, Gregory, Garden and Smith (2001) emphasized that attitude is based on value and belief, as well as varying degree of factual knowledge. The consequences about the gender differences in Mathematics are not conclusive because there are so many other factors which contribute towards achievement (Leeson, 1995; Malone & Miller, 1993). Poor attitude towards Mathematics has often been cited as one factor that has contributed to lower participation and success of female in Mathematics (Fullarton, 1993; Willis, 1995). Interest and attitude in the subject are the special predictors for the students' participation and success in the subject.

Costello (1991) reported that males are more inclined towards Mathematics than females being the male dominated domain. Also Farooq and Shah (2008) stated that it was found that at secondary school level most of the females don't actively participate in Mathematics classes due to their poor perceptions about Mathematics. Also Ethington (1990) stated that females are negatively influenced by their sex-role stereotypes. Farooq and Shah (2008) emphasized that poor mathematical skills in females deprived them from a larger number of professions because in some countries mathematical background knowledge is the prerequisite for entrance in any profession. In this era of science and technology, every field of life has been tremendously changed so it is necessary to compare the attitudes and achievements in Mathematics on gender basis. This is necessary for designing programmes and strategies for females' participation in Mathematics at higher level.

Several measures and intervention programmes have been designed for improving females' attitude towards Mathematics (Americans Association of University Women, 1992). Observations and researches indicate that females are clustered in the life sciences with far fewer in physical sciences, Mathematics and engineering and computer science (Gavin, 1997). Teaching learning process of Mathematics depends upon the positive attitude towards Mathematics (Farooq & Shah, 2008). Having positive attitude towards Mathematics means generally enjoying working with Mathematics and having confidence in one's own ability (Robson, 1996). Attitude of student contributes a lot towards his perception about Mathematics and develops the adaptability and applicability in the learners (Schiefelee & Csikszentmihalyi, 1995). Students should be encouraged and prepared for accepting the challenges of day to day life (Mathematical Sciences Education Board and National Research Council MSEBNRC, 1989).

Farooq and Shah (2008) concluded that male and female have same type of attitude towards Mathematics as a result of research carried out on students' attitude towards Mathematics which indicates that gender differential has no impact on the attitude of students towards Mathematics. There are many different factors that intervene in and affect students' Mathematics learning, including beliefs and conceptions (Andrews & Hatch, 2000), motivation (Middleton & Spanias, 1999), cognitive variables (Schiefele & Csikzentmihalyi, 1995), emotions (McLeod, 1992; Hannula, 2002).

Gomez-chacon (2000) suggested that attitudes might act as cognitive guides that inhibit learning. Meanwhile, Manassero, Vazquez, and Acevedo (2001) consider attitudes to be a determinant in the acceptance or rejection of Mathematics. Some studies show that a positive attitude tends to correlate positively with an increased effort to learn and with achievement (Minato, 1983; Minato & Yanase, 1984; Kloosterman, 1990) and that self-confidence is a good predictor of success in Mathematics (Randhawa, Beamer & Lundberg, 1993), while others, such as Ma and Kishor (1997), have found no significant correlation between attitude and achievement. In studies of grade 9 students, Campos (2006) and Mercado (2007) found a positive correlation between attitude and the perceived utility of Mathematics. Stanic and Hart (1995) stressed that a causal link between attitude and achievement cannot be easily established, but Auzmendi (1992) has pointed out that this relationship might emerge more clearly if specific groups were studied.

Ruffell, Mason and Allen (1998) argued that attitudes are not stable constructs and may change easily from negative to positive depending on a variety of social factors. In particular, attitude towards Mathematics can be affected when technology is used to support learning in this area. McLeod (1992) emphasized the need to investigate how beliefs and attitude towards Mathematics are affected by the introduction of technology. Mason and Allen (1998) and Martino and Zan (2001) stressed that attitudes is an ambiguous construct. Nevertheless, there seems to be agreement on at least two aspects: that attitude is created and modified by events and the way these are perceived, and that attitudes have affective, cognitive, and behavioural component (Ursine & Sanchez, 2008).

2.3 Appraisal of the Literature Reviewed

Looking through available research reports on different teaching strategies in senior secondary school Mathematics, many of the studies reviewed did not move from concrete to representational before abstract level of thinking. Also some of the studies did not consider students' level of Mathematics learning difficulties, therefore, it is necessary to use the strategy that will assist student to move from simple to complex and complex to abstract and to use the strategy that will assist students' with Mathematics learning difficulties in order to improve their performance so that they can have the same opportunity with their counterparts without learning difficulty. Different teaching strategies have been used by researchers but the areas of Concrete-Representational-Abstract and Explicit Instructional Strategies (CRAIS and EIS) have not been examined fully, especially in Nigeria. The present study therefore used the Concrete-Representational-Abstract and Explicit Instructional strategies learning procedure with teachers' involvements to remediate achievement in Mathematics of students with Mathematics learning difficulties. Also, of the existing studies, conflicting results have been reported on the issue of gender and achievement in Mathematics that is why the variable was considered for investigation in this study to see whether it has a significant effect on achievement in Mathematics and attitude.

CHAPTER 3

METHODOLOGY

This section presents the research design, variables of study, selection of participants, instruments, research procedure and method of data analysis.

3.1 Research Design

The study adopted the pretest- posttest control group quasi-experimental design. This design is schematically represented as:

E_1:	0_1	X_1	0_2
E_2:	0_1	X_2	0_2
C:	0_1	X_3	0_2

Where 0_1 represents pretest for the experimental groups and control group.

0_2 represents posttest for the experimental groups and control group.

X_1 represents the Concrete-Representational-Abstract Instructional Strategy (CRAIS).

X_2 represents the Explicit Instructional Strategy (EIS).

X_3 represents the control treatment of Modified Conventional Teaching strategy (MCTS).

This design also employed the 3 X 3 X 2 factorial matrix. The 3 X 3 X 2 factorial matrix is presented on Table 3.1.

Table 3.1- The 3 X 3 X 2 Factorial Matrix

Treatment	**Gender**	**Students Mathematical Learning Difficulty Level**			**Total**
		Low	Moderate	High	
1.Concrete-Representational-Abstract Instructional Strategy	Male	24	10	8	42
	Female	29	5	6	40
2.Explicit Instructional Strategy	Male	16	18	7	41
	Female	24	14	9	47
3.Modified Conventional Teaching Strategy	Male	14	16	9	39
	Female	27	26	17	70
Total		**134**	**89**	**56**	**279**

3.2 Variables in the Study

The variables in the study are:

A. **Independent Variable:** This is the instructional strategy which was manipulated at three levels:

(i) Concrete-Representational-Abstract Instructional Strategy (CRAIS)

(ii) Explicit Instructional Strategy (EIS)

(iii) Modified Conventional Teaching Strategy (MCTS)

B. **Moderator Variables**

There are two moderator variables. These are:

(i) Students Mathematical Learning Difficulty at three levels:

(a) Low

(b) Moderate

(c) High

(ii) Gender at two levels:

(a) Male

(b) Female

C. **Dependent Variables**

There are two dependent variables viz:

(i) Students achievement in Mathematics

(ii) Students attitude to Mathematics

3.3 Selection of Participants

Three local government areas were selected randomly from the list of the local government areas in Ibadan. Two senior secondary schools were purposively selected from each of the three selected local government areas, making six schools based on the following criteria:

1. The school must be public co-educational.
2. The SS II students in the schools must have completed SS I Mathematics curriculum at the time of the study.
3. The school must have at least one graduate Mathematics teacher.

4. The school must have been graduating students in public examinations for at least ten years.

Each local government area selected was randomly assigned to treatment such that the two schools in the local government area were for the same treatment group. To this end, two schools were assigned Concrete-Representational-Abstract instructional strategy, two for explicit instructional strategy while the remaining two were for control. One intact class of SS II students was randomly selected from each of the schools.

3.4 Instruments

Six instruments were used for this study. The instruments are:

1. Instructional Guide on Concrete-Representational-Abstract Instructional Strategy (CRAIS)
2. Instructional Guide on Explicit Instructional Strategy (EIS)
3. Instructional Guide on Modified Conventional Teaching Strategy (MCTS)
4. Test on Students' Mathematics Learning Difficulties (TOSMALD)
5. Students' Mathematics Achievement Test (SMAT)
6. Students' Mathematics Attitude Questionnaire (SMAQ)

3.4.1 Instructional Guide on Concrete-Representational-Abstract Instructional Strategy (CRAIS)

This is a self-designed guide to teach the students in first experimental group for 18 periods, 40 minutes per period and 12 hours for six weeks. It was based on the steps included in Concrete–Representational–Abstract Instructional Strategy as listed by the Access Centre (2004). The main features of the guide are: general information which consists of subject, topic, class and sex. Also, it consists of the information about the guide, the teacher, general objectives, the students, contents to be taught for six weeks and specific treatment package for each week. Concrete–Representational–Abstract instructional guide was given to experienced Mathematics teachers in senior secondary school and university lecturers in Teacher Education Department Mathematics and Science unit to examine its content and face validity. The recommendations given were used to reconstruct some of the guide.

3.4.2 Instructional Guide on Explicit Instructional Strategy (EIS)

This is a self-designed guide to teach the students in second experimental group for 18 periods, 40 minutes per period and 12 hours for six weeks. The lessons were based on the steps as listed by Swanson (2001). The main features of the guide are; general information which includes subjects, topic, class and sex. Also it consists of the procedure, the teacher, general objectives, the students, contents for the six weeks and specific treatment package for each week. Explicit Instructional guide was given to two university lecturers in Teacher Education Department for review, to examine its face and content validity. Their comments, criticism and suggestions were used to reconstruct some of the guide.

3.4.3 Instructional Guide on Modified Conventional Teaching Strategy (MCTS)

The instructional guide on modified conventional teaching strategy is a self-designed guide to teach students in control group for 18 periods, 40 minutes per period and 12 hours for six weeks. The lessons were based on normal way of writing lesson note. The main features of the guide are: general information which consists of subjects, topic, the procedure, the teacher, general objectives, contents for each week and specific treatment package for each week. The instructional guide was given to two senior secondary school Mathematics teachers for review and all their suggestions were incorporated in the guide.

3.4.4 Test on Students Mathematics Learning Difficulties (TOSMALD)

Test on Students' Mathematics Learning Difficulties is a self-designed instrument which consists of three stages with ten questions each on basic Mathematics skills. The skills were appropriate computational skills such as addition, subtraction, multiplication and division, as well as basic skills such as decimals and simple fractions, number and numeration, numbers and factors, fractions, rate, ratio and proportion, percentage, roots and indices (Audiblox, 2000; Onaolapo, 2001). Other skills in TOSMALD include problem-solving skills and measurement skills in angles, distance and simple areas (Nakonia 2005). This test was used to identify students with Mathematics learning difficulties. The test was scored manually. The steps involved in the workings of the questions were considered to identify the area of basic skills lacked by the students. Students' who obtained 0-33% were classified as having high Mathematics learning difficulty, those ranging between 34-66% as moderate and 67-100% as low.

The test items were subjected to two university lecturers' review and corrections. Their suggestions were effected on the items. Copies of TOSMALD were then administered to thirty SS II students that were not part of the main study and Split-half method was used to compute the reliability of the test and the reliability index of 0.85 was obtained.

3.4.5 Students Mathematics Achievement Test (SMAT)

The SMAT is a forty-five item multiple choice tests with four options A–D constructed by the researcher to measure students' cognitive achievement in Mathematics. The SMAT has two sections; the first section contained the demographic variables of the students such as name, school, local government area, age, sex, and class. The second section consists of forty-five multiple choice items on the selected topics in SS II Mathematics curriculum. The content covered the following areas: Circles, Volumes, and Angles of elevation and depression. The SMAT consists of fifteen questions per topic which will reflect the aspect of the topic, concept, operation or skill that is giving the learners problem.

As categorized by Okpala, Onocha, and Oyedeji (1993) the test items focused on the first three levels of cognitive domain, knowledge, understanding and thinking. The table of specifications for the construction of SMAT is presented below:

Table 3.2: Table of Specifications for SMAT

S/N	Content Area	Knowledge	Understanding	Thinking	Total
1	Circles	5	6	4	15
2	Volumes	4	8	3	15
3	Angles of elevation and depression	7	5	3	15
	Total	16	19	10	45

SMAT was scored by allotting 1 mark per correct answer while a wrong answer was scored zero. The total mark obtainable was forty-five which was converted to percentage (100%).

To validate the instrument, the initial sixty items developed were presented to one doctoral student in Mathematics Unit of Teacher Education Department, University of Ibadan, one lecturer in Science and Mathematics unit, Teacher Education Department, as well as one lecturer in Educational Evaluation with the table of specification for experts review, criticism and advice. Their advices were used to modify the test items. After validation, fifteen items were dropped leaving forty-five items finally for SMAT. Then the restructured test items were presented to two SS II Mathematics teachers and their suggestions were incorporated into the final draft of the test items. The modified test was administered to thirty SS II students that were not involved in the real study in order to determine the reliability using Kuder-Richardson formula 20 (KR–20). The difficulty levels were computed and the result of the analysis was used to pick items that were neither too difficult nor too easy. These yielded difficulty indices of between 0.32 and 0.56 with a reliability index of 0.83.

3.4.6 Students Mathematics Attitude Questionnaire (SMAQ)

The SMAQ is made up of two Sections, Section A for the background information of the students like name of school, class, sex, and age; and Section B consists of 35 items adapted from Modified Fennema-Sherman (1976) attitude scale. The scale contains information on personal confidence about the subject matter, usefulness of the subject's content, and subject perceived as a male domain. Students method of response to the items was the closed response modes of 4 points scale of strongly agree, agree, disagree, and strongly disagree. The scoring for positive items was based on 4, 3, 2 and 1 for Strongly Agree (SA), Agree (A), Disagree (D) and Strongly Disagree (SD) respectively while these were reversed for negatively worded items.

To validate SMAQ, the instrument was given to one expert in Teacher Education Science Unit University of Ibadan and one expert in Social Sciences for review. Their advices were incorporated into the items. The modified test items were administered to thirty SS II students that were not involved in the main study to determine the reliability and internal consistency of the scores using Cronbach alpha formula. The standardized alpha value of 0.79 was obtained.

3.5 Research Procedure

The researcher visited the selected schools with letters of introduction to obtain permission from the principals and SS II Mathematics teachers for the use of the schools and students. This was necessary in order to seek the teachers' cooperation and active involvement because the topics taught during the treatment period were not in order with the school scheme of work. The required facilities were provided by the researcher.

(1) **Preliminary Activities**

(i) **Training of Teachers**

The first two weeks were used for the training of the research assistants and the Mathematics teachers in the study. The researcher was the resource person. Six teachers were trained to ensure that participating teachers adhere strictly to the instructional and experimental procedures. Two research assistants were also trained for distribution and collation of the materials used. Briefing sessions were organized for students participating in the study. Two participating teachers for first experimental group were trained with the instructional guide on Concrete – Representational – Abstract Instructional Strategy (CRAIS), and two teachers for second experimental group with the use of instructional guide on Explicit Instructional Strategy (EIS) and the materials to be used were given to the teacher. Two teachers for the control group were asked to use the steps on the instructional procedure on Modified Conventional Teaching Strategy (MCTS).

ii. Screening

The third and fourth weeks were used for screening of students for Mathematics learning difficulties. Test on Students' Mathematics Learning Difficulties (TOSMALD) were administered to one intact class selected randomly to screen students for Mathematics learning difficulties. Students who obtain 0-33% were classified as having high Mathematics learning difficulty, those ranging between 34-66% as moderate and 67-100% as low.

2. **Pretest**

The fifth week was used by the researcher, assistants and the trained teachers to administer the pretest on the participating students. The instruments were administered in the following order:

(i) Students' Mathematics Achievement Test (SMAT) and

(ii) Students' Mathematics Attitude Questionnaire (SMAQ)

3. **Treatment Procedure**

The instructional packages prepared by the researcher were used by the trained teachers to teach the students for six weeks. The instruments contain the step by step method of teaching as shown in appendices.

(i) **Experimental Group**

Students in this group were taught using the following steps.

(A) **Steps involved in Concrete-Representational-Abstract Instructional Strategy (CRAIS)**

- Introduction and presentation of materials (Concrete)
- Presentation of the concept to be taught:
    - Definition
    - Explanation of the topic by relating it with the concrete objects presented.
    - Students' activities: Using nets to make shapes of objects.
    - Drawing of relevant materials based on the concept (Representation).
    - Asking questions on drawing or picture of the material.
    - Asking question on topic without material and drawing (Abstract).
- Summary
- Evaluation
- Assignment

(ii) Experimental group 2

(B) **Instructional Guide on Explicit Instructional Strategy (EIS)**

Students in this group were taught using the following steps.

- Review of previous knowledge
- Presentation of the new topic
- Breaking down the task into small steps

- Providing a pictorial presentation
- Division of class into small groups
- Presenting questions to the groups; Group activities; listening to feedback and immediate correction
- Summary in whole class
- Evaluation
- Assignment

(iii) **Control Group**

Instructional guide on Modified Conventional Teaching Strategy (MCTS)

Steps involved in Modified Conventional Teaching Strategy (MCTS)

The steps to be followed are:

- Introduction
- Presentation
  - Definition
  - Drawing of the concept
  - Examples
- Summary
- Evaluation
- Assignment

4. Posttest

Students Mathematics Achievement Test (SMAT) and Students Mathematics Attitude Questionnaire (SMAQ) were administered at the end of instruction to all groups for one week. The study lasted for twelve weeks.

3.6 Method of Data Analysis

Data collected were analyzed using the Analysis of Covariance (ANCOVA) on the Statistical Package for Social Sciences version 16.0. The statistic was used to test the seven hypotheses stated. Also, the Multiple Classification Analysis (MCA) aspect of ANCOVA was used to determine the magnitude of the mean scores of the different groups. The Scheffe Post hoc test was used where significant main effects were obtained, while graphs were used to interpret significant interaction effects.

CHAPTER 4

RESULTS

This chapter presents results obtained based on the seven null hypotheses formulated and tested in the study. Thereafter, the findings as summarized were presented.

4.1 Presentation of Results

$H0_{1a}$: There is no significant main effect of treatment on students' achievement in Mathematics

To test hypothesis 1a, Tables 4.1, 4.2 and 4.3 are presented in succession.

Table 4.1: Summary of ANCOVA of Posttest Achievement Scores by Treatment, Mathematics Learning Difficulty and Gender

Source of Variance		Hierarchical Method				
		Sum of Squares	Df	Mean Square	F	Sig.
Covariates	Pretest	1036.1	1	1036.1	47.9	.00
Main Effects	(Combined)	9761.0	5	1952.2	90.3	.00
	Treatment	3736.7	2	1868.4	86.4	.00*
	Maths Diff Level	6015.0	2	3007.5	139.1	.00*
	Gender	9.2	1	9.2	.4	.51
2-Way Interactions	(Combined)	1137.5	8	142.2	6.6	.00
	Treatment x Maths Diff Level	801.5	4	200.4	9.3	.00*
	Treatment x Gender	130.5	2	65.2	3.0	.05
	Maths Diff Level x Gender	85.0	2	42.5	2.0	.14
3-Way Interactions	Treatment x Maths Diff Level X Gender	65.6	4	16.4	.8	.55
Model		12000.1	18	666.7	30.8	.00
Residual		5623.0	260	21.6		
Total		17623.1	278	63.4		

* Significant at p <.05

From Table 4.1, treatment has significant effect on students' achievement in Mathematics ($F_{(2, 260)} = 86.4$; $p < .05$). The implication of this is that there is significant difference in achievement in Mathematics of students exposed to Concrete-Representational-Abstract instructional strategy, explicit instructional strategy and those in the modified conventional instructional groups. Hypothesis 1a is therefore, rejected.

Table 4.2 displays the descriptive information on the relative performance of the various groups in posttest achievement.

Table 4.2: Multiple Classification Analysis (MCA) of Achievement Scores by Treatment, Mathematics Learning Difficulty and Gender

Grand Mean = 20.9

Variable + Category		N	Predicated Mean		Deviation		Eta	Beta
			Unadjusted	Adjusted for Factors and Covariates	Unadjusted	Adjusted for Factors and Covariates		
Treatment	Concrete-Rep-Abstract	82	26.6	25.1	5.7	4.2		
	Explicit	88	19.0	18.4	-2.0	-2.5	.5	.4
	Control	109	18.2	19.8	-2.7	-1.1		
Mathematics difficulty level	High	56	13.7	14.3	-7.2	-6.6		
	Moderate	89	17.0	17.8	-3.9	-3.1	.7	.6
	Low	134	26.5	25.8	5.6	4.8		
Gender	Male	122	21.1	21.1	.2	.2		
	Female	157	20.8	20.8	-.1	-.2	.0	.0
	R =	.8						
	R Squared =	.6						

From Table 4.2, students in the Concrete-Representational-Abstract instruction had higher adjusted posttest achievement score ($\bar{x}$ = 25.1; Dev.= 4.2) than their counterparts in the conventional instruction ($\bar{x}$ = 19.8; Dev. = -1.1) and those in explicit instruction ($\bar{x}$ = 18.4; Dev.= -2.5). The respective adjusted posttest mean scores in achievement are represented in Figure 4.1.

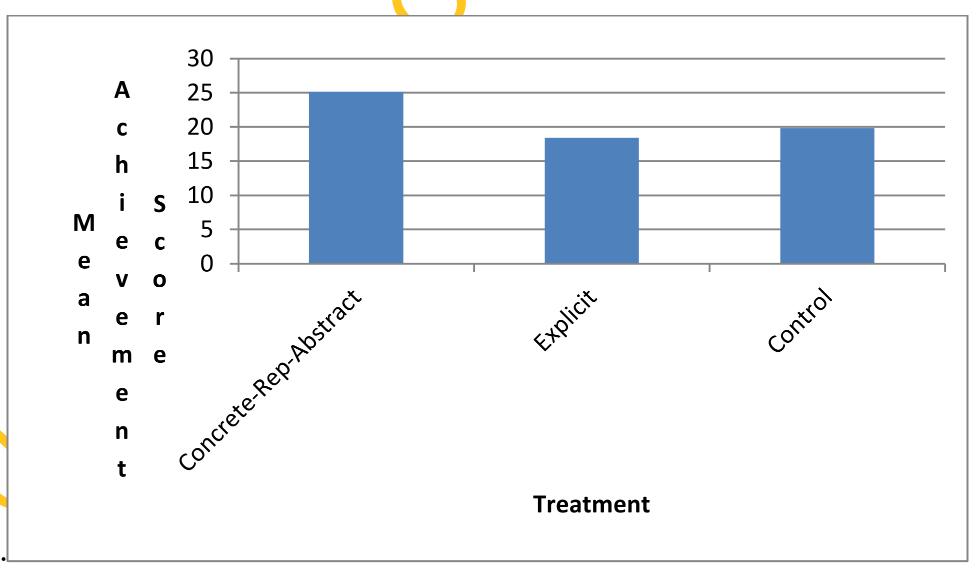


Figure 4.1: Bar Chart Showing Posttest Mean Scores of the Treatment and Control Groups.

Findings implies that the Concrete-Representational-Abstract instructional strategy proved most effective followed by the modified conventional instruction while the explicit instructional strategy was least effective on students' achievement in Mathematics. The respective adjusted posttest mean scores in achievement are represented in Figure 4.1.

Further, Table 4.3 traced the source of the significant effect of treatment on achievement.

Table 4.3: Scheffe Post hoc Test of Achievement by Treatment

			Treatment		
Treatment	N	$\bar{x}$	1. CRAIS	2. Explicit	3. Control
1. CRAIS	82	25.1		*	*
2. Explicit	88	18.4	*		
3. Control	109	19.8	*		

* Pairs significantly different at $p < .05$

Table 4.3 shows that the Concrete-Representational- Abstract instructional group was significantly different ($\bar{x}$ = 25.1) from both the explicit ($\bar{x}$ = 18.4) and control ($\bar{x}$ = 19.8) modes. Hence, the significant effect of treatment on achievement was due to the significant differences obtained between Concrete-Representational-Abstract instructional strategy and explicit instructional strategy as well as Concrete-Representational-Abstract instructional strategy and control.

$H0_{1b}$: There is no significant main effect of treatment on students' attitude to Mathematics

Table 4.4: Summary of ANCOVA of Posttest Attitude Scores by Treatment, Mathematics learning difficulty and Gender

Source of Variance		Hierarchical Method				
		Sum of Squares	Df	Mean Square	F	Sig.
Covariates	Pretest attitude	2637.0	1	2637.0	26.2	.00
Main Effects	(Combined)	6457.1	5	1291.4	12.8	.00
	Treatment	2339.7	2	1169.9	11.6	.00*
	Maths Diff Level	4072.9	2	2036.4	20.2	.00*
	Gender	44.5	1	44.5	.4	.51
2-Way Interactions	(Combined)	568.9	8	71.1	.7	.69
	Treatment x Maths Diff Level	439.3	4	109.8	1.1	.36
	Treatment xGender	134.7	2	67.4	.67	.51
	Maths Diff Level X Gender	32.4	2	16.2	.2	.85
3-Way Interactions	Treatment x Maths Diff Level X Gender	1022.6	4	255.7	2.5	.04*
Model		10685.6	18	593.6	5.9	.00
Residual		26204.9	260	100.8		
Total		36890.6	278	132.7		

* Significant at p< .05

Table 4.4 shows that there is significant effect of treatment on students' attitude to Mathematics ($F_{(2, 260)} = 11.6; <.05$). This implies that there is significant difference in the posttest attitude scores of students exposed to the Concrete-Representational-Abstract instructional strategy, explicit instructional strategy and those in the control group. Hypothesis 1b is therefore, rejected.

Table 4.5: MCA of Posttest Attitude Scores by Treatment, Mathematics Learning Difficulty and Gender

Grand Mean = 96.1

Variable + Category		N	Predicated Mean		Deviation		Eta	Beta
			Unadjusted	Adjusted for Factors and Covariates	Unadjusted	Adjusted for Factors and Covariates		
Treatment	Concrete-Rep-Abstract	82	99.9	99.0	3.8	3.0		
	Explicit	88	96.3	96.6	.2	.5	.2	.2
	Control	109	93.1	93.5	-3.0	-2.6		
Maths Diff Level	High	56	88.2	89.3	-7.9	-6.8		
	Moderates	89	95.0	95.2	-1.1	-.9	.4	.3
	Low	134	100.1	99.5	4.0	3.4		
Gender	Male	122	95.8	95.6	-.3	-.5		
	Female	157	96.3	96.4	.3	.4	.0	.0
	R =			.5				
	R Squared =			.3				

From Table 4.5, the Concrete-Representational-Abstract instructional strategy had higher adjusted posttest attitude score ($\bar{x}$ = 99.0; Dev. = 3.0) than the explicit instructional group ($\bar{x}$ = 96.6; Dev. = .5) and the control ($\bar{x}$ = 93.5; Dev. = -2.6).

These respective posttest scores are represented in Figure 4.2

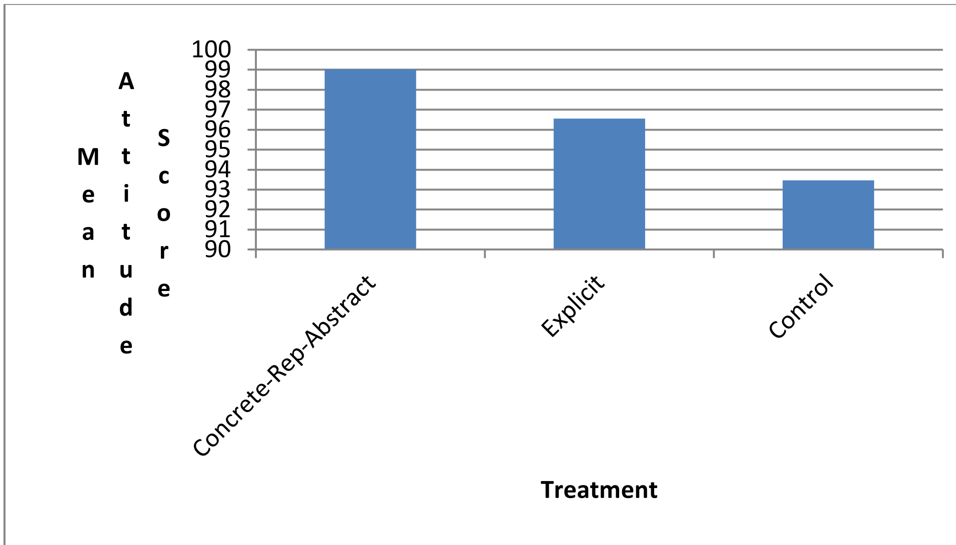


Figure 4.2: Bar Chart of Posttest Attitude Scores in Treatment and Control Group.

Figure 4.2 shows that the Concrete-Representational-Abstract instructional strategy was more effective than the explicit instructional strategy and the control groups.

Table 4.6 presents the summary of the post hoc tests carried out.

Table 4.6: Scheffe Post hoc Test of Attitude by Treatment

			Treatment		
Treatment	N	$\bar{x}$	1. CRAIS	2. Explicit	3. Control
1. Concrete-Repre-Abstract	82	99.0			*
2. Explicit	88	96.6			
3. Control	109	93.5	*		

* Pairs significantly different at p < .05

Table 4.6 shows that the significant effect of treatment on students attitude to Mathematics is due to the significant pairwise difference between the attitude scores of CRA instructional strategy ($\bar{x}$ = 99.0) and the control ($\bar{x}$ = 93.5) groups.

H0$_{2a}$: There is no significant main effect of Mathematics learning difficulty on students' achievement in Mathematics

Table 4.1 shows that Mathematics learning difficulty has significant effect on students' achievement in Mathematics ($F_{(2, 260)}$ = 139.1; p<.05). This means that there is significant difference in the posttest achievement scores of students with low, moderate and high levels of Mathematics learning difficulty. Hence, hypothesis 2a is rejected.

From Table 4.2, students with low Mathematics learning difficulty had higher adjusted posttest mean achievement score ($\bar{x}$ = 25.8; Dev. = 4.8) than their moderate ($\bar{x}$ = 17.8; Dev. = -3.1) and high Mathematics difficulty ($\bar{x}$ = 14.3; Dev. = -6.6) counterpart.

Figure 4.3 represents the relative performance of students across the three levels of Mathematics learning difficulty.

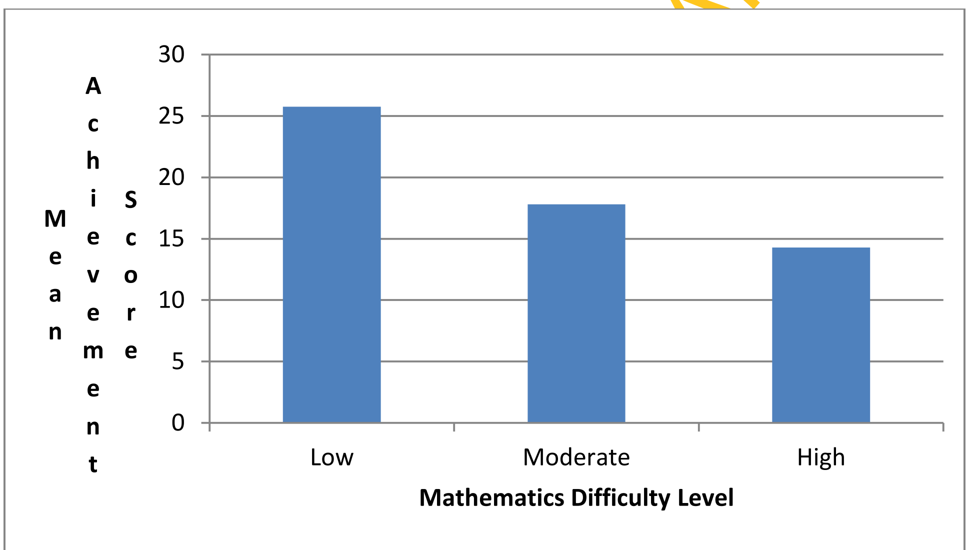


Figure 4.3: Bar Chart of Posttest Achievement Scores of Students' in the 3 Levels of Mathematics Learning Difficulty.

To trace the actual source(s) of significance, Table 4.7 presents the pairwise test carried out.

Table 4.7: Scheffe Post hoc Tests of Achievement by Mathematics Learning Difficulty

			Mathematics Learning Difficulty Level		
Mathematics Difficulty Level	N	$\bar{x}$	1. Low	2. Moderate	3. High
1. Low	134	25.8		*	*
2. Moderate	89	17.8	*		*
3. High	56	14.3	*	*	

* Pairs significantly different at p < .05

Table 4.7 shows that the significant effect of Mathematics learning difficulty on students' achievement in Mathematics was due to the significant pairwise differences between the low ($\bar{x}$ = 25.8) and moderate Mathematics learning difficulty ($\bar{x}$ = 17.8) groups; the low and the high Mathematics learning difficulty ($\bar{x}$ = 14.3) groups; and the moderate and high Mathematics learning difficulty groups.

$H0_{2b}$: There is no significant main effect of Mathematics learning difficulty on students Attitude to Mathematics

From Table 4.4, there is significant effect of Mathematics learning difficulty on students' attitude to Mathematics ($F_{(2, 260)}$ = 20.2; p <.05). Based on this, hypothesis 2b is rejected.

Table 4.5 further shows that students with low Mathematics learning difficulty had higher attitude to Mathematics ($\bar{x}$ = 99.5; Dev. = 3.4) than the moderate Mathematics learning difficulty group ($\bar{x}$ = 95.2; Dev. = -.9) and the high Mathematics learning difficulty group ($\bar{x}$ = 89.3; Dev. = -6.8).

Figure 4.4 shows the posttest attitude mean scores of students across the Mathematics learning difficulty levels.

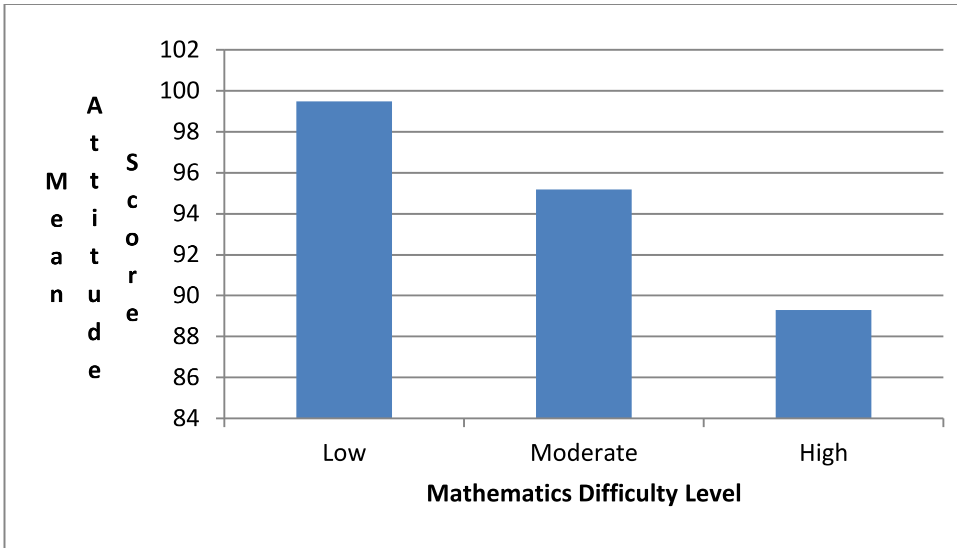


Figure 4.4: Bar Chart of Posttest Attitude Mean Scores of Low, Moderate and High Mathematics Learning Difficulty.

To trace the actual source of the significant effect of Mathematics learning difficulty on achievement, Table 4.8 is presented.

Table 4.8: Scheffe Post hoc Tests of Attitude by Mathematics Learning Difficulty

			Mathematics Learning Difficulty Level		
Mathematics Difficulty Level	N	$\bar{x}$	1. Low	2. Moderate	3. High
1. Low	134	99.5		*	*
2. Moderate	89	95.2	*		*
3. High	56	89.3	*	*	

* Pairs significantly different at p < .05

From Table 4.8, each of the three level of Mathematics learning difficulty differs significantly from one another in adjusted posttest attitude mean score. Specifically, the low difficulty group ($\bar{x}$ = 99.5) differs significantly from the moderate difficulty group ($\bar{x}$ = 95.2); the low difficulty group differs significantly from the high difficulty group ($\bar{x}$ = 89.3); and the moderate differs significantly from the high difficulty group. To this end, all the 3 possible pairs contributed to the significant effect of Mathematics learning difficulty on students' attitude to Mathematics.

H0$_{3a}$: There is no significant main effect of gender on students' achievement in Mathematics

Table 4.1 shows that gender has no significant effect on students' achievement in Mathematics ($F_{(1, 260)} = .4$; $p>.05$). Hence, hypothesis 3a is not rejected. From Table 4.2, however, the male students obtained slightly higher adjusted achievement mean score ($\bar{x}$ = 21.3; Dev. = .2) than their female counterparts ($\bar{x}$ = 20.8; Dev. = -.2).

H0$_{3b}$: There is no significant main effect of gender on students' attitude to Mathematics**.** Table 4.4 show that gender has no significant effect on students' attitude to Mathematics ($F_{(1, 260)} = .4$; $p>.05$). Hypothesis 3b is therefore not rejected. Further, Table 4.5 shows that the female students obtained slightly higher adjusted attitude mean score ($\bar{x}$ = 96.4; Dev. = .4) than their male counterparts ($\bar{x}$ = 95.6; Dev. = -.5).

H0$_{4a}$: There is no significant interaction effect of treatment and Mathematics learning difficulty on students' achievement in Mathematics.

Table 4.1 shows that there is significant interaction effect of treatment and Mathematics learning difficulty on students' achievement in Mathematics ($F_{(4, 260)} = 9.3$; $p<.05$). Hence, hypothesis 4a is rejected.

This significant interaction is represented in Figure 4.5.

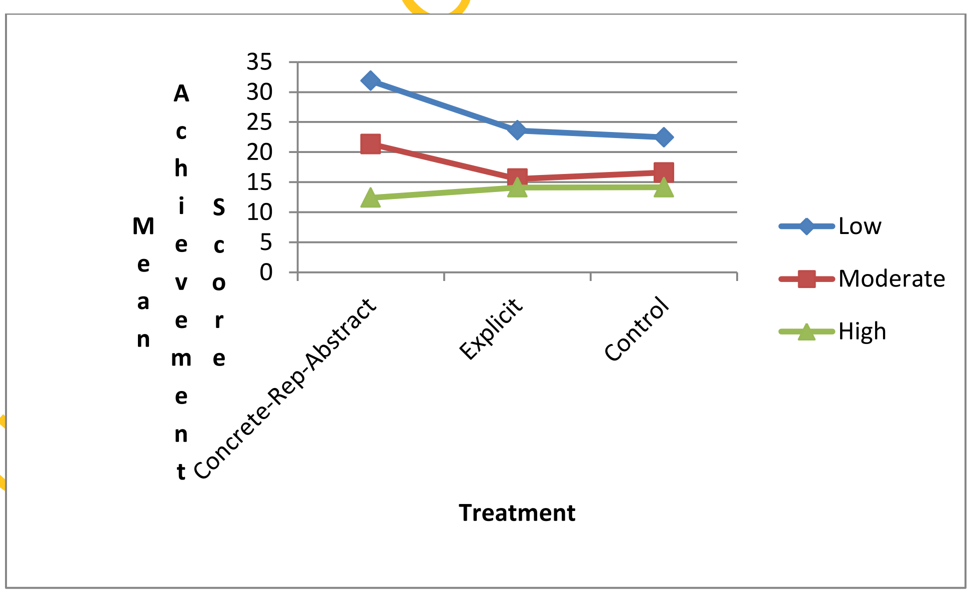


Figure 4.5: Graph of Interaction of Treatment and Mathematics Learning Difficulty on Students' achievement in Mathematics.

From Figure 4.5, among students in the CRA instructional strategy group, those with low Mathematics learning difficulty had highest achievement mean score ($\bar{x}$ = 31.9) followed by those with moderate ($\bar{x}$ = 21.3) and the high Mathematics learning difficulty ($\bar{x}$ = 12.4) respectively. This trend is similar to those of explicit and conventional groups where students with low Mathematics learning difficulty levels performed better in achievement than their moderate and high Mathematics learning difficulty counterparts. This interaction is therefore an ordinal one.

$H0_{4b}$: There is no significant interaction effect of treatment and Mathematics learning difficulty on students' attitude to Mathematics.

Table 4.4 shows that there is no significant interaction effect of treatment and Mathematics learning difficulty on students' attitude to Mathematics ($F_{(4, 260)}$ = 1.1; p>.05). Hence, hypothesis 4b is not rejected.

$H0_5$a: There is no significant interaction effect of treatment and gender on students' achievement in Mathematics.

From Table 4.1, the interaction effect of treatment and gender on students' achievement in Mathematics is not significant ($F_{(2, 260)}$ = 3.0; p>.05). Hypothesis 5a is therefore, not rejected.

$H0_{5b}$: There is no significant interaction effect of treatment and gender on students' attitude to Mathematics.

Table 4.4 shows that there is no significant interaction effect of treatment and gender on students' attitude to Mathematics ($F_{(2, 260)}$ = .7; p>.05). Hence, hypothesis 5b is not rejected.

$H0_{6a}$: There is no significant interaction effect of Mathematics learning difficulty and gender on students' achievement in Mathematics

Table 4.1 shows that the interaction effect of Mathematics learning difficulty and gender on students' achievement in Mathematics is not significant ($F_{(2, 260)}$) = 2.0; p >.05). Hence, hypothesis 6a is not rejected.

$H0_{6b}$: There is no significant interaction effect of Mathematics learning difficulty and gender on students' attitude to Mathematics.

From Table 4.4, there is no significant interaction effect of Mathematics learning difficulty and gender on students' attitude to Mathematics ($F_{(2, 260)}$ = .2; p>.05). Hypothesis 6b is therefore, not rejected.

H0$_{7a}$: There is no significant interaction effect of treatment on Mathematics learning difficulty and gender on students' achievement in Mathematics

From Table 4.1, the 3-Way interaction effect of treatment, Mathematics learning difficulty and gender on students' achievement in Mathematics is not significant ($F_{(4, 260)}$ = .8; p >.05). Hypothesis 7a is therefore, not rejected.

H0$_{7b}$: There is no significant interaction effect of treatment on Mathematics learning difficulty and gender on students' attitude to Mathematics.

Table 4.4 shows that there is significant 3-way interaction effect of treatment, Mathematics learning difficulty and gender on students' attitude to Mathematics ($F_{(4, 260)}$ = 2.5; p<.05). On this basis, hypothesis 7b is rejected.

4.2 Summary of Findings

The findings of the study revealed the following:

1. There was significant main effect of treatment on students' achievement in Mathematics. The mean scores show that Concrete-Representational-Abstract Instructional Strategy group was more effective than the Conventional Instructional strategy and the explicit Instructional Strategy. Also, there was significant main effect of treatment on students' attitude to Mathematics. Here students in CRAIS group had the highest mean score followed by explicit instructional strategy group and control group.
2. There was significant main effect of Mathematics learning difficulty on students' achievement in Mathematics. Students with low level of Mathematics difficulty had the higher mean score than the moderate group and the high Mathematics difficulty level respectively. Similarly, there was significant main effect of Mathematics learning difficulty on students' attitude to Mathematics. Students with low Mathematics difficulty had higher mean attitude score followed by the moderate group and those with high difficulty.

3. There were no significant main effects of gender on students' achievement and attitude to Mathematics. In this study the male students obtained slightly higher adjusted achievement mean score while female students obtained higher attitude mean score.

4. There was significant interaction effect of treatment and Mathematics learning difficulty on students' achievement in Mathematics. For each of the treatment, CRA instructional group, explicit instructional group and modified conventional group, students with low Mathematics learning difficulty had highest achievement mean score followed by those with moderate learning difficulty while the students with high difficulty had the least mean score. On the other hand, there was no significant interaction effect of treatment and Mathematics learning difficulty on students' attitude to Mathematics.

5. There were no significant interaction effects of treatment and gender on students' achievement and attitude to Mathematics.

6. There were no significant interaction effects of Mathematics learning difficulty and gender on students' achievement and attitude to Mathematics.

7. The 3-way interaction effect of treatment, Mathematics learning difficulty and gender was not significant on achievement but has significant effect on students' attitude towards Mathematics.

CHAPTER 5

DISCUSSION, CONCLUSION AND RECOMMENDATIONS

This chapter presents the discussion of the findings obtained in the study, the conclusion and recommendations.

5.1.1 Effect of Treatment on Students' Achievement and Attitude towards Mathematics

Findings of the study revealed a significant main effect of treatment on students' achievement in Mathematics. This result showed that Concrete-Representational-Abstract instructional strategy was more effective at improving students' achievement in Mathematics followed by the conventional strategy while the explicit instructional strategy was the least effective. The effectiveness of Concrete-Representational-Abstract Instructional strategy over both the Conventional and explicit instructional strategies could be due to the fact that Concrete-Representational-Abstract Instructional strategy is a learner-centered instructional strategy which provides learners with the opportunity to participate actively in the learning situation. During treatments learners were fully involved and especially, they physically explored and touched the concrete objects and real life items, using the materials to cut out and model the concepts taught and drawing the objects before moving to the abstract level using symbols.

The CRA instructional strategy provided a graduated and conceptually supported framework for students to create a meaningful connection between concrete, representational and abstract levels of understanding. Students move sequentially through the stages using models, verbalization, drawings, and numerical representations to indicate each of the steps. This high level of involvement of learners enabled them to solve real mathematical problems thereby gaining required knowledge which helped them to make meaning from information presented. Representations serve as tools for mathematical communication, thought and calculation, allowing personal mathematical ideas to be externalized, shared and preserved. These help in the clarification of ideas in ways that support reasoning and build understanding. This is line with the opinion of Devlin (2000) that hands-on experiences allow students to understand how numerical symbols and abstract equations operate at a concrete level, making the information more meaningful to students.

This finding also agree with the submission of Harrison and Harrison (1986) that the use of concrete materials develops more precise and more comprehensive mental representations, often provides more motivation, encourage on-task behaviour and leads to better understanding of Mathematics for better application to life situations. This finding corroborates the earlier findings of Witzel, Mercer and Miller (2003) that students taught using the Concrete-Representational-Abstract sequence of instruction performed fewer procedural errors in Mathematics.

Further, the conventional instruction was found to be more effective than explicit instructional strategy. This shows that the strategy remains a powerful means to communicate information so as to achieve instructional goals. Thus, the conventional teaching strategy cannot be totally ignored in the teaching and learning situation as it has its own benefits. This result negates the findings of Manalo, Bunnell and Stillman (2000) that students exposed to training with certain novel strategies achieved mathematical skills better than the conventional trained group. In agreement with NCTM (2000) however, there is the need to adopt some of the recent reform-based instructional strategies along with the conventional instructional strategy that has continually been condemned and rated very poorly in terms of teaching effectiveness not only in Mathematics but in other school subjects generally.

The explicit instructional strategy found to be the least effective strategy in this study might have performed low due to its characteristics as a more or less teacher-centered instructional strategy effective for teaching basic skills (Kroesbergen & Van, 2003). Also, the National Mathematics Advisory Panel Report (2008) emphasized that explicit instruction was primarily effective for computation of basic Mathematics operations, but not as effective for higher order problem solving. This result disagrees with the results from many studies (Jitendra, Edward, Sacks, and Jacobson, 2004; Mackenzie, Martella, Moore & Mantella, 2004; Sawalha, 2004; Kinder, Kubina & Marchand-Martella, 2005) which confirmed the effectiveness of explicit instructional strategy compared with conventional teaching strategy on the improved Mathematics achievement of students with learning difficulties and increasing their mastery of basic skills. The result, however, corroborates the findings of Wesley and Gersten (2001) that a strong effect of explicit instruction was recorded on spelling and reading skills whereas a moderate effect was reported on academic subjects such as Mathematics.

Also, there was significant main effect of treatment on students' attitude to Mathematics. Students in CRAIS group had the highest mean score, followed by the explicit and then the control group. CRAIS allows every student to participate and contributes his own quota, in terms of fun and play. The environment where cutting and modeling are taking place allows students to participate fully, freely, and relate directly with their peers encouraged them to develop positive attitude and increase their interest to what they are learning and also increase their attitude towards Mathematics. The mean score of students in explicit instructional group is more than that of students in conventional group. In explicit group, repetition and questions after every step which lead to mastery at every step might trill and gear up their interest towards the subject and this can make them think that Mathematics is all about fun. This repetition can be in form of song or recitation and this can increase students attitude towards Mathematics which receives support from many studies (Jitendra, Edward, Sacks, and Jacobson, 2004; Mackenzie, Martella, Moore & Mantella, 2004; Sawalha, 2004; Kinder, Kubina & Marchand-Martella, 2005) which confirmed the effect of a sequence, definite, and well-organized strategy on attitudes to Mathematics.

Conventional teaching strategy was the least effective on students' attitude to Mathematics. Several studies in the area of Mathematics have shown that instruction, especially at the secondary school level remains overwhelmingly teacher-centered, with greater emphasis being placed on lecturing especially conventional instruction rather than helping students to think critically across subject areas and applying their knowledge to real-life situation (Butty, 2001). This finding is in line with Akinsola and Olowojaiye's (2008) finding that the conventional teaching strategy was inadequate for improved students' attitude towards Mathematics. This suggests the need to shift from the conventional teaching strategy and embrace some other instructional strategies that have been found to have facilitative effect on students' attitude towards Mathematics.

5.1.2 Effect of Mathematics Learning Difficulty on Students' Achievement and Attitude towards Mathematics

It was found in this study that Mathematics learning difficulty had significant effect on students' achievement in Mathematics. Students with low Mathematics difficulty had higher mean achievement score than their moderate and high counterparts. These results concurred with the findings of Tapia (2004) that student with low difficulty scoring significantly higher than students with moderate or high Mathematics difficulty. The students with low Mathematics learning difficulty have self-confidence, good self-concept, high self efficacy and belief in having high scores in Mathematics. All these lead to high achievement in Mathematics. This result also corroborates the findings of Cawley, Parmar, Yan and Miller (1996) that students' with high learning difficulties have low retention rates. Students with moderate Mathematics learning difficulty also have higher mean achievement score than students with high difficulty. The result was in line with the findings of Tapia (2004) that as Mathematics difficulty increase, achievement scores decrease. Students with high level of Mathematics difficulty performed poorly in this study. They already have phobia, anxiety, poor attitude and no interest for this subject, all these will definitely lead to low achievement in Mathematics. This finding is consistent with the studies of Ma (1999) and Woodard (2004) which revealed a negative relationship between these two variables i.e. Mathematics learning difficulty and achievement in Mathematics.

On attitude, findings of this study agree with the submission of Burstein (1992) that there is a direct link between students' difficulty in Mathematics and attitude towards Mathematics. The data revealed that the three different levels of Mathematics learning difficulty produced impacted on students' attitude to Mathematics, students with low Mathematics difficulty had higher mean attitude score, followed by moderate difficulty students and high Mathematics difficulty students. Since the students with low Mathematics learning difficulty already have interest and eagerness in solving Mathematics problems always this will lead to positive attitude to the subject. The fear that students with high Mathematics learning difficulty have for this subject will reduce their interest to the subject and lead to negative attitude to the subject.

5.1.3 Effect of Gender on Students' Achievement and Attitude towards Mathematics

The findings on gender show that it has no significant influence on students' achievement in Mathematics. Though the male students obtained slightly higher mean achievement score than female but the difference was not significant. This study has shown that Mathematics is neither a male-dominated nor a female-dominated subject in line with the assumptions of Akinsola and Tijani (1999). This result negates the findings of Onabanjo (2000), Odogwu (2002) and Ojo (2003) that found significant main effects of gender on students' achievement in favour of boys. Findings of this study are in line with the findings of Popoola (2002), Adegoke (2003) and Akinola (2003). The result also concurred with the findings of Home (2003), Nguyen (2003), Abiam and Odok (2006), Vale (2009), Hydea and Mertzb (2009) who found no significant relationship between gender and achievement in Mathematics.

Also gender was found to have no significant effect on students' attitude to Mathematics although the female students obtained slightly mean attitude score than their male counterparts. The result of this study also corroborates the findings of Tymms (2001) which suggested that gender was weakly associated with attitudes.

5.1.4 2-way and 3-way Interaction Effects

Findings of this study also showed that there was significant interaction effect of treatment and Mathematics learning difficulty on students' achievement. This result shows that the treatment is sensitive to students' learning difficulty towards improving their achievement in Mathematics. In other words, the use of concrete-representational-abstract instructional strategies to teach students with learning difficulties becomes inevitable. This result agreed with the findings of Butler, Miller, Crehan, Babbitt and Pierce (2003) that CRAIS has proven to be effective method of teaching students with Mathematics learning difficulties. However, this interaction was not significant on students' attitude towards Mathematics.

The interaction effects of treatment and gender on students' achievement and attitude were not significant and the interaction effects of Mathematics learning difficulty and gender on students' achievement in Mathematics were also not significant. In the same vein, the findings indicated that there was no significant interaction effect of treatment, Mathematics learning difficulty and gender on students' achievement in Mathematics but, this has significant effect on students' attitude towards Mathematics.

5.2 Implications of the Findings

Based on the findings of this study, the effectiveness of Concrete-Representational-Abstract instructional strategy in acquisition of knowledge and high achievement in Mathematics has been established. This study has also shown the effectiveness of conventional teaching strategy and weakness of explicit instructional strategy. The use of Concrete-Representational-Abstract instructional strategy will help the students to see Mathematics as one of the best school subject which is very easy to learn, this will reduce the tension of abstraction imposed on students while learning Mathematics. The strategy could be used to reduce the level of taking Mathematics as difficult subject. This will also allow students to have good grade, reduce the level of mass failure and increase the number of students that will register for science oriented subject which will also increase scientific and technological development of the nation.

The use of CRAIS will reduce the effort of the teachers of Mathematics and make teaching easier but yielding good result. It will also reduce the blame receiving from the government, parent and even the students that the teachers of Mathematics are not competent.

5.3 Recommendations

The following recommendations are made based on the findings of this study.

In order to improve students' achievement in Mathematics, strategies which involves active participation of students' in learning such as the Concrete-Representational-Abstract Instructional Strategy should be adopted for teaching Mathematics to move teachers away from teaching Mathematics in abstract. The teachers of Mathematics should always screen their students to know their level of Mathematics difficulty using the basic Mathematical skills like addition, subtraction, multiplication and division so that they will be able to know the exact teaching strategies that can be adopted in teaching that set of students.

Training and retraining programmes such as seminars and workshops should be organized by government and professional associations like Science Teachers Association of Nigeria (STAN) and Mathematical Association of Nigeria (MAN) for the teachers of Mathematics to learn more on Concrete-Representational-Abstract and explicit instructional strategies to improve and enhance students' achievement in Mathematics. Teaching materials are very important in teaching and learning of Mathematics, government should purchase these materials and distribute round the schools to make learning real and increase students' level of assimilation.

5.4 Limitations of the Study

The limitation to the generalization of the results of this study includes 3% of the participants in the selected schools that did not take the posttest and this rendered their pretest scores useless. The moderator variables investigated in this study are Mathematics learning difficulty and gender, it is possible that other moderator variables such as mental ability, verbal ability and others could influence the findings of the study.

5.5 Contributions of the Study to Knowledge

This study has contributed to knowledge in the following ways:

- Concrete-Representational-Abstract Instructional Strategy is effective in enhancing achievement in Mathematics of students' with Mathematics learning difficulty. The strategy can be used to improve students' attitude to Mathematics positively. Students' achievement in Mathematics is determined by their Mathematics learning difficulties. Also Attitude of students to Mathematics is a function of Mathematics learning difficulties. This result has provided a basis for curriculum innovation in training and re-training of instructors through in-service programmes to expose them to these teaching strategies.
- The Concrete-Representational-Abstract Instructional Strategy encourages cutting, modelling and drawing of objects that represent the entire content to be taught, it also encourages group work. This will help all categories of learners with high, moderate or low levels of Mathematics difficulties to participate actively and discover fact of the concepts to be learnt and this will put

Mathematics phobia into minimal rate, turning learning of Mathematics into fun, increasing students' interest rate, change their attitude to this subject and increasing achievement in Mathematics. Additional information has been provided towards the fact that Mathematics is not a male dominated subject, both male and female have the same opportunity to learn and understand Mathematics in order to pursue any Mathematics related careers.

5.6 Suggestion for Further Studies

The following suggestions are made based on the findings of the study. The study should be replicated at the primary and tertiary levels of education. The study should be carried out in other science subjects such as physics, chemistry, biology and agricultural science.

- The study could be replicated with Mathematics students in other geo-political zones of the country to make the findings more generalizable.
- Other instructional strategies which could be used to help in the improvement of achievement and attitude of students with Mathematics learning difficulty apart from Concrete-Representational-Abstract and explicit instructional strategies should be investigated.
- Studies could be carried out on other factors such as mental ability, and reading ability which could be influenced students' achievement and attitude in Mathematics.

5.7 Conclusion

Based on the findings, it could be concluded that the Concrete-Representational-Abstract instructional strategy, when employed in the teaching and learning of Mathematics concepts, has great potential for improving both achievement in Mathematics and attitude of every student towards Mathematics. Active participation of students in cutting, modeling and drawing before symbolic representation of the concepts would not only lead to the achievement of the desired objectives of Mathematics learning but would develop greater confidence in the students especially the students with Mathematics learning difficulty to have the same chance in learning Mathematics as their counterparts without the problem and this will eradicate the problem of mass failure in Mathematics and increase enrolment in science-oriented subject and also improve the technological development of the country. In this case, Mathematics teachers should make conscious efforts towards learning about the Concrete-Representational-Abstract instructional strategy and training adopting it in their teaching. Also, the government, educational agencies and teachers' professional associations should organize periodic workshops for Mathematics teachers and encourage continuous in-service teacher education programmes through which innovations in Mathematics education pedagogy will be provided.

REFERENCES

Abiam, P. O. and Odok, J. K. 2006. Factors in students' achievement in different branches of secondary school Mathematics. *Journal of Education and Technology. 1 (1), 161-168.*

Abimbade, A. A. 1997. Problem-Solving and Electronic Calculator in Mathematics Instructional. *Journal of the Science Teachers' Association of Nigeria, 3(12), 9-14.*

Access Center, 2004. Concrete-representational-abstract instructional approach. *Retrieved March 21, 2008, from Access Center Web site*: http://www.k8accesscenter.org/training_resources/CRA_Instructional_ Approach.asp

Access Center: Improving Out comes for all students K-8. *Concrete- representational-abstract instructional approach.* Retrieved January 17, 2008 from http://www.k8 accesscenter.org/training_resources/CRA_Instructional_Approach.asp

Adams G, and Carmine, D. 2003. Direct instruction in H. L. Swanson, K. R. Harris and S. Graham (Eds.). *New York: Guilford Press.*

Adegoke, A. B. 2003. Teacher influence as a determinant of dependent– prone students' learning outcomes in senior secondary school geometry in Ibadan, Nigeria Unpublished Ph.D Thesis, University of Ibadan.

Adekoya, S. O. 2008. Secondary school Mathematics and education reforms in Nigeria. *Journal of Mathematical Association of Nigeria* 40-43.

Adeosun, O. V. 2008. Gender differences in the achievement and retention of Nigerian Students exposed to concepts in social studies through multi-media packages. *Asian Journal of Information Technology, Medwell 7 (5), 240-244.*

Adesoji, F. A. 1995. Students' ability levels and their competence in a self-directed problem-solving task. *Journal of Curriculum Studies and Development.* 1 (1): 55-61.

______1997. Average students and effectiveness of problem-solving instructional strategies. *Ife Journal of Educational Studies,* 4(1), 16-19.

______2008. Students' ability levels and effectiveness problem-solving instructional strategy. *Kanla-Raj Journal of Social Science, 17(1): 5-8.*

Adesoji, F. A. and Fisuyi, M. O. 2001. Analysis of problem solving difficulties of student in volumetric analysis according to gender Ibadan. *Ibadan Journals of Educational Study. 1(1), 106-117*

Adetunji, F. O. 2000. A study of some Nigerian primary school teachers' attitude to Mathematics. *African Journal of Educational Research, 6(1), 2 93-99.*

Afolabi, S. S. 2010. Teaching method and textual material variables as correlates of students learning outcomes. Unpublished Ph.D Thesis, University of Ibadan.

Aiken, J. R. 2002a. *Psychological Testing and Assessment* (10th ed.) Boston, MA: Allyn and Bacon.

______ 2002b. Attitude and related psychosocial construct: *theories and assessment, 18.*

Ajiboye, J. O. 1996. A self learning programme, the modified lecture method and students' cognitive and effective outcomes in some population education concepts. Ph.D. Thesis. University of Ibadan.

Akinola, E. M. 2003. The Impact of Self-Assessment on Achievement. *Journal of Educational Research*, 87, 60 – 75.

Akinoso, S. O. 2011. Correlates of some factors affecting students' achievement in secondary school Mathematics in Osun State, Nigeria. *International Journal of Education, Science, Mathematics and Environmental Studies 3(1), 83-95.*

Akinsola, O. S. 2001. The extent of dominant factors of girls and women's low participation in Mathematics education: *Mathematics Educators' views. Proceedings of the 42nd Annual Conference of the Science Teachers Association of Nigerian Mathematics panel 295-298.*

Akinsola, M. K. 1994. Comparative effects of learning and enhanced mastery learning strategies on students' achievement and self-concept Mathematics. Ph.D. Thesis U.I. Ibadan.

______ 2000. Teachers' perception of communication and language skill in Mathematics teaching and learning situations. *Africa Journal of Educational Research.* 6, 1 and 2 100-114.

Akinsola, M. K. and Tijani, R. D. 1999. The Relationship between Mathematics self-concept and achievement in Mathematics. *Nigerian Journal of Applied Psychology, 5, 1 and 2 58-64.*

Akinsola, M. K. and Ogunleye, B.O. 2003. Improving Mathematics curriculum at the implementation stage. In O. A. Bamisaiye, I. A., Nwazuoke and Okediran (Eds.). Education this Millennium: Innovation in theory and practice. Lagos: Macmillan Nigeria Publisher Limited, 211-218.

Akinsola, M. K. and Olowojaiye, F. B. 2008. Teacher instructional methods and student attitudes towards Mathematics. *International Electronic Journal of Mathematics Education,* 3(1), 60-73.

Allsopp, D. H and Minskoff, E. 2003. The learning toolbox: An on-line instructional resources for students with learning disabilities and ADHD, Their teachers, and parents. *Journal of international special Needs Education,* 6, 43-45.

Alternmatt, E. R., Pomerantz, E. M., Ruble, D. N., Frey, K. S. and Greulich, F. K. 2002. Predicting changes in children's self –perceptions of academic competence: *A naturalistic examination of evaluative discourse among classmates.* Developmental Psychology, 38, 903-917.

American Association for the Advancement of Science, 1989. *Project 2061: Science for all Americans.* Washington, DC: Author.

______ 1990. *Science for all Americans.* New York: Oxford University Press.

American Association of University Women 1992. *Shortcoming Girls, shortcoming America: A Call to Action.* ERIC Document Reproduction Service No. ED 340658.

Andrews, P. and Hatch, G. 2000. A comparison of Hungarian and English teachers' conceptions of Mathematics and its teaching. *Educational Studies in Mathematics* 43(1), 31-34.

Arnot, M., David, M. and Weiner, G. 1999. Closing the gender gap. Cambridge: Polity Press.

Ashcraft, M. H. Krause, J. A. and Hopko D. R. 2007. Is Math anxiety a Mathematics learning disability? In D. B. Berch and M. M. M. Mazzocco why is Mathematics so hard for some Children*? The nature and origins of Children's Mathematical learning difficulties and disabilities.* Balltuonore Brookes Publishers.

Audiblox, 2000 Math learning disabilities: Overcoming Math Problem. *Retrieved March* 19, 2011 From http://.audibox 2000. com /math_ problems.

Auzmendi, E. 1992. *Las actitudes hacia la matematica/estadistica en las ensenanzas medias y universitarias.* Editorial *Paidos: Caracteristicas y medicion. Ediciones Mensajero-Bilbao.*

Bakers, S. Gersten, R. and Lee, D. 2002. A synthesis of empirical research on teaching Mathematics to low-achieving students. *Elementary School Journal,* 103, 51-73.

Baroody, A. and Hume, J. 1991. Meaningful Mathematics instruction: The case of fractions. *Remedial and Special Education.* 12, 54-68.

Bassey, S. W., Joshua, M. T. and Asim, A. E. 2007. Gender differences and Mathematics achievement of rural senior secondary students in Cross River State, Nigeria. November, 11, 2009 from http://web.gnowledge.org/episteme3/pro_pdfs/09-bassavjoshua-asim.pdf.

Berch, D. B. and Mazzocco, M. M. M. 2007. Why is Math so hard for some children? The nature and origins of mathematical learning difficulties and disabilities. Baltimore, MD: Paul H. Brookes publishing co.

Berkas, N. and Pattison, C. 2007. Manipulative: More than a special education intervention. NCTM News Bulletin. *Retrieved March 20, 2008, from the National Council of Teachers of Mathematics Website:*http://www.K8accesscenter.org/training_%20resources/%20CRA_Instructional.

Bolaji, C. 2005. A study of factors influencing students attitudes towards Mathematics in the Junior Secondary schools; Mathematics teaching in Nigeria.

Bordo, S. 2001. Selection from the flight to objective in Leadernman, M. and Barrtsg. I Eds. *The Gender and Science reader, London. Routledge.*

Booker, G. 2000. The Mathematics game: Using *instructional games to teach Mathematics.* NZ: NZCER.

_____ 2004. Difficulties in Mathematics: Errors, origin and implications. In B.A. Knight and W. Scott (Eds.), *learning difficulties. Multiple perspectives. Frenchs Forests, NSW: Pearson Education.*

Booker, G., Bond, D., Sparrow, L. and Swan, P. 2004. Teaching primary Mathematics. 3. Australia: Pearson Education.

Bottge, B. A. 2001. Reconceptualising Mathematics problem solving for low-achieving students Edu. *Remedial and special cation*. 22. 102-112.

Bowen, J. 1972, 1981. *A history of western education three volumes* London: Methuen.

Brew, L. 2011. Mathematical activities and classroom based fctors that support senior high school students' mathematical performance. *British Journal of Arts and Social Sciences 2(1), 11-20.*

Brigham, F. J., Wilson, R., Jones, E., and Moisio, M. 1996. Best practices: Teaching decimals, fractions, and percents to students with learning disabilities. LD Forum, 21 (3), 10-15.

Bruner, J. 1960. The process of education. Cambridge, MA: Harvard University press. Retrieved on 3/3/ 2012 from http://www.instructionaldesign.org/theories/constructivist.html

_______1966. Toward a theory of instruction. Cambridge, HA: Harvard University press. http://www.instructionaldesign.org/theories/constructivist.html

_______1973. Going beyond the information given. New York: Notion. http://www.instructionaldesign.org/theories/constructivist.html

_______1996. *The Culture of Education*, Cambridge, Mass. Harvard UniversityPress. 224.

_______ 2004. *Toward a theory of instruction (Belknap Press*). Cambridge, MA: Belknap Press of the Harvard University Press.

Burstein, L. 1992. *The analysis of multilevel data in educational research and evaluation*. Review of Research in education; 8, 158 -223. *Federal Republic of Nigeria 2004: National Policy on Education, NERC.*

Butler, F. M., Miller, S. P., Crehan, K., Babbitt, B. and Pierce, T. 2003. Fraction instruction for students with Mathematics disabilities: Comparing two teaching sequences. *Learning Disabilities Research & Practice*, 18(2), 99-111.

Butty, J. L. M. 2001. Teacher instruction, student attitudes, and Mathematics performance among 10th and 12th grade black and Hispanic students. *Howard University Provided by Pro Quest Information and Learning company.*

Campos, C. 2006. *Actitud hacia las matematicas: Diferencias de genero entre estudiantes de sexton de primaria y tercer grado de secundaria.* Unpublished masters thesis. Cinvestav-IPN, Mexico: Department of Mathematics Education.

Cangiano, A. 2008. On the importance of Mathematics an outline. Retrieve on 22/03/2011 from http://math-blog.com/2008/03/31 on_the_importance-of-Mathematics.

Carnellor, Y. 2004. *Encouraging mathematical success for children with learning difficulties.* Southbank, VIC: Social Science Press.

Casey, M. B. and Tucker, E. C. 1994. Problem-centered classrooms: Creating lifelong learners. Phi Delta Kappan, 139-143.

Cass, M., Cates, D., Jackson, C. W. and Smith, M. 2002. Facilitating adolescents with disabilities understanding of area and perimeter concepts via manipulative instruction. In No child left behind: The vital role of rural schools. Annual National Conference Proceedings of the American Council on Rural Special Education.

Cawley, J. E., Miller, J. H., and School, B. A. 1987. A brief inquiry of arithmetic work-problem-solving among learning disabled secondary students. *Learning Disabilities Focus.* 2, 87-93.

Cawley, J. F. and Miller, J. H. 1989. Cross-sectional comparisons of the mathematical performance of children with learning disabilities: Are we on the right track toward comprehensive programming? *Journal of Learning Disabilities*, 23, 250-254, 259.

Cawley, J. F., Parmar, R. S., Yan, W. E. and Miller, J. H. 1996. Arithematic computation abilities of students with learning disabilities: Implications for instruction. *Learning Disabilities Research and Practice*. 11, 230-237.

Cawley, J. F., Miller, J. H. 1989. Cross-sectional comparisons of the mathematical performance of children with learning disabilities: Are we on the right track toward comprehensive programming? *Journal of Learning Disabilities.* 23, 250-254, 259.

Costello, J. 1991. *Teaching and Learning Mathematics.* 11-16. London: Routledge.

Cusimano, A. 2001. Learning disabilities: *There is a cure*. Achieve Publication. DSM-IVT-TR 2009 DSM-IV-TR- Current Version. Retrieved September 7, 2009 from http://en.wikipedia.org/wiki/diagnotic_and_statistical_manual_of_mental_disorders

Dale, E. 1969. Audiovisual methods in teaching, third edition. New York: The Dryden Press; Holt, Rinehart and Winston. Retrieved June 7, 2012 fromhttp://rapidbi.com/created/coneofexperience-dale.html

David, M. B. 1997. The History of Mathematics: An Introduction, McGraw-hill, 374.

David, H. A 1999. Using modelling, manipulative, and mnemonics with eight-grade Mathematics students. *Teaching Exceptional Chidlren* 32(2), 74-81 N/D.

David, A. S. 2007. *How brain learns Mathematics*. Corwin press, 186-189.

Denton, K. and West, J. 2002. *Children's reading and Mathematics achievement in kindergarten and first grade.* (NCES 2002-125). Washington, DC: National Center for Education Statistics.

Devlin, K. 2000. Finding your inner mathematician. *The chronicle of higher education*, 46, B5.

_____2009. Archive for the Mathematics: *The science of patterns' category*. Mathematics for real reasons? Retrieved on April 2 ,2011 from http://www.teachildmath.com/ wordpress/?category_name=mathematics

Dey, J. G. and Hill, C. 2007. *Beyond the pay gap.* Washington, DC: American Association of University Women Educational Foundation.

Dickens, M. N. and Cornell, D. G. 1993. Parental influences on the Mathematics self-concept of high ability adolescent girls. *Journal for the Education of the Gifted*, 176, 53-73.

Dossey, J. 1992. The nature of Mathematics: *It's role and its influence*. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* pp. 39-48. New York: Macmillan.

Dragoo, K. Arefeh, S. Stephen, D. and Luke. 2010. Effective Mathematics instruction. National Dissemination Center for Children with Disabilities. Evidence for education, 3, Issue 1, 2008.

Duncan, W. 1989. Engendering school learning: science attitudes and achievement among girls and boys in Botswana. Studies in comparative and International Education Journal, 6, 12-21.

Edward, L. A. 1994. Education that work, The child is always right, part five. Direct instruction this paper is based upon a talk that Dr Anderson gave on august 2, 1994, sponsored by the Chautauqua Institution, Chautauqua, New York.

Edwards-Groves, C. J. 1998. *The Reconceptualization of classroom events as structured lessons: Documenting changing the teaching of literacy in the primary school.* Unpublished doctoral thesis, Griffith University.

______ 2002. *Building an inclusive classroom through explicit pedagogy: A focus on the language of teaching.* Literacy Lexicon Sydney: Prentice Hall, Austral Pty Ltd.

Edwards, S. 2004. "Thirty Years of Current Account Imbalances, Current Account Reversals, and Sudden Stops" *IMF Staff Papers*, (Forthcoming).

Elkins, J. 2005. Numeracy. In A. Ashman and J. Elkins. (Eds.) *Educating children with diverse abilities.* French forest, NSW: Pearson Education.

Engelmann, S., Kelly, B. and Carnine, D. 1996. Connecting Mathematics Concepts: Level, F (Teacher's Presentation Book, Student, Material, and Teacher's Guide). Chicago: Science Research Associates.

Englert, C. S., Culatta, B.E. and Horn, D. G. 1987. Influence of irrelevant information in addition word problems on problem solving. *Learning Disability quarterly*, 10, 29-36.

Esan, A. O. 1999. Effect of cooperative and individualistic problem solving strategies on students' learning outcomes in secondary school Mathematics. Unpublished Ph. D Thesis, University of Ibadan.

Ethington, C. A. 1990. 'Gender Differences in Mathematics; An international perspective'. Journal for Research in Mathematics Education. 21(1), 74-80.

______ 1992. Gender difference in a psychological model of Mathematics achievement. *Journal for Research in Mathematics Education,* 23(2). 166-181.

Etukudo, U. E 2002. The effect computer assisted instruction on gender and performance of junior secondary school students in Mathematics. ABACUS. *The Journal of the Mathematics Association of Nigeria (27) 1-8.*

Eze J. E 2007. Respositing Mathematics education for job creation: Implications for tertiary education. ABACUS. *Journal of Mathematical Association of Nigeria*. 32 (1), 145-153.

Ezeani, and Ebere, A. 1999. Towards improved enrolment of females in the basic sciences and Mathematics. *Journal of Educational Research and Development Institute of Education, University of Lagos.1 (1), 66.*

Farooq, M. S. and Shah, S. Z. U. 2008. Students attitude towards Mathematics. Pakistan Economic and Social review. 46(1): 75-83.

Federal Republic of Nigeria, 2004. National policy on education, Lagos: NERDC.

Fennema, E. 2000. Gender and Mathematics. What is known and what I wish was known? Unpublished manuscript. Madison, Wisconsin: Wisconsin Centre for Educational Research.

Fennema, E. and Leder, G. C. 1990. Mathematics and gender. New York: Teachers College Press.

Fennema, E. Sherman, J. A. 1976. Fennema-Sherman Mathematics attitudes scales: *Instruments designed to measure attitudes toward the learning of Mathematics by males and females. Catalog of Selected Documents in Psychology,* 6(1), 31.

______ 1977. Sex-related differences in Mathematics achievement, spatial visualization, and affective factors. *American Educational Research Journal,* 14 (1), 51-71.

Freebody, P., Ludwig,C. and Gunn, S. 1995. The literacy practices in and out of Schools in low socio-economic urban communities, commonwealth department of employment, education and training, curriculum corporation, commonwealth of Australia.

Fryer, R. G. and Levitt, S. D., 2009. An Empirical Analysis of the Gender Gap in Mathematics. NBER Working Paper.

Fuchs, L. S. and Fuchs, D. 2002. Enhancing first-grade children's mathematical development with peer assisted learning strategies. School psychology Review. 2002; (31): 569-584.

______ 2003. Enhancing the mathematical problem solving of students with Mathematics disabilities. In H. J. Swanson, K. R. Harris and S.E. Graham (Eds.) *Handbook\ on Learning Disabilities* , 306-322, O. New York: Guilford.

______ 2006. Strategies to chance young children's mathematical development. In: Tremblay R. E., Barr R. G., Peters R. De V., eds. *Encyclopedia on Early Childhood Development (online).* Montreal, Quebeec: Center of Excellence for Early Childhood Development; 1-17.

Fuchs, L. S., Fuchs, D. Prentice, K. Burch, M., Hamlet, C. L., Owen, R., Hosp, M., Jancek, D. 2004. Explicitly teaching for transfer: Effects on third-grade students' mathematical problem-solving among third-grade students with schema-based instruction. *Journal of Educational Psychology* ; 96(4), 635-647.

Fullarton, S. 1993. *Confidence in Mathematics: The Effects of Gender.* Geelong: Deakin University Press.

Garnett, K. 1992. Developing fluency with basic number facts: *Intervention for students with learning disabilities. Learning Disabilities Research & Practice,* 7, 210-216.

______1998a. Mathematics learning disabilities. Retrieved November 10, 2006, from the LD Online Web sit: http://www.idonline.org/article/5896.

______1998b. Mathematics learning disabilities. Retrieved 17/04/2011, from the LD online Web site: http://www.Idonline:org/article/5881.

Garnett, K., Frank, B. and Fleisehener, J. X. 1983. A strategies generalization approach to basic fact learning addition and subtraction lessons, manual #3; multiplication lessons, manual #5. Research institute for the study of learning disabilities. New York, NY: Teacher's College, Columbia University.

Gauss, C. F. 2010. Mathematics, from citizendium, the citizens' compendium: An online. Retrieved on 19/03/2011 from http://en.citizendium.org/wiki/mathematics.

Gavin, M. K. 1997. A gender study of students with high Mathematics ability: Personological, educational and parental influences on the intent to pursue quantitative fields of study in college. Retrieved February 26, 2007 from http://www.gifted.uconn.nrcgt/newletter/spring097/sprng974.html.

Geary, D. C. 1993. Mathematical disabilities: Cognitive, neuropsychological, and genetic components. Psych Bull. 114:345–362.

_____ 2001, How Disabilities can affect Mathematics achievement NICHY, The National Dissemination Center for Children with Disabilities.

______ 2001. Mathematical disabilities: What we know and don't know. Retrieved November 10, 2006, from the LD Online Web site: http://www.idonline.org/article/5881

______2004. Mathematics and learning disabilities. *Journal of Learning Disabilities*, 37(1), 4-15.

Gersten, R., Baker, S. K., Pugach, M., Scanlon, D., and Chard, D. 2001. Contemporary research on special education teaching. In V. Richardson (Ed.), *Handbook for research on teaching* 4, 695-722. Washington, DC: American Educational Research Association.

Gersten, R., David, C., Madhavi, J. and Scott, B. 2006. Experimental and Quasi-experimental research on instructional approaches for teaching Mathematics to students with learning disabilities: A research synthesis. Signal Hill, Calif: Center on Instruction RG Research Group in NCTM 2012.

Gersten, R. Ferrini-Mundy, J. Benbow, C. Clements, D. Loveless, T., Willaims, V. Arispe, I. and Barifield, M. 2008. Report of the task group on instructional practices National Mathematics Advisory Panel. Retrieved March 20, 2008, from the U.S Department of Education Website: http:.//www.ed.gov/ about%20bdscomm.

Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R. and Witzel, B. 2009. *Assisting students struggling with mathematics: response to Intervention (RTI) for elementary and middle schools (NCEE 20094060)*. Washington, DC: U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance.

Gomez-Chacon, I. M. 2000. *Matematica emociona. Santiago:* Narcea.

Graham, S., Harris, K. R. and Reide, R. 1992. Developing self-regulated learners. *Focus on Exceptional Children*, 24(6), 1-16.

Graham, K. and Fennell, F. 2001. Principles and standards for school Mathematics and teacher education: *Preparing and empowering teachers*. School Science and Mathematics, 10(6), 39-327.

Grootenboer, P. J. 2003. The affective views of primary school children. In Pateman, N. A., Doughhtery, B. J and Zillioz, J. (Eds.), *Navigating between theory and practice* (*Proceedings of the 27th Conference of the International Group for the Psychology of Mathematics Education*, 3, 1-8. Honolulu, HI: University of Hawaii.

Grouws, D. A. and Cebulla, K. J. 2000. *Improving student achievement in Mathematics*. Genf, Switzerland: International Academy of Education/International Bureau of Education

Hall, N. 1991. Teaching Mathematics using the procedural analogy theory. 335-341.

Hall, T. 2002. Explicit instruction. Wakefied, MA: National Center on Accessing the General Curriculum. Retrieved November 12, 2012, from http://aim.cast.org/learn/historyarchive/backgroundpapers/explicit_instruction.

Hannell, G. 2005. Dyscalculia. Action plans for successful learning in Mathematics. London: David Fulton.

Hannula, M. S. 2002. Attitude towards Mathematics: Emotions, expectations and values. *Educational Studies in Mathematics* 49, 25-46.

Harley, 1995. http://tip.psychology.\org/bruner.html

Harris, C. A., Miller, S. P. and Mercer, C. D. 1995. Teaching initial multiplication skills to students with disabilities in general education classrooms. *Learning Disabilities Research and Practice*, 10, 180-195.

Harrison, M. and Harrison, B. 1986. Developing numeration concepts and skills. *Arithmetic Teacher*, 33, 18-21
http://www.cast.org/publications/ncac/ncaca_explicit.html.
http://www.child_encylopedia.com/documents/FuchsANGxp.pdf.
http://www.ed.gov/inits/Math/glenn/index.html.

Heidmann and Humphrey, 2002. Students Mathematics attitude in the Course Hero Portal. Retrieved on May 01, 2011, from http://www.course hero. com/prefix/5f/documents/5999.

Heng-Yuku and Howard, P. 2000. Personalization of Mathematics word problems in Taiwan. Educational Technology Research and Development, 48(3), 49-59.

Holton, D. 1993. What Mathematicians do and why it is important in the classroom set: Research information for teachers, 1(10), 1-6.

Home, M. 2003. Gender differences in the early years in addition and subtraction. In N. A. Pateman, B. J. Dougherty, and J. T. Zilliox eds. Proceedings of the 2003 joint meeting of PME and PMENA. 79-86 Hawaii: University of Hawaii.

Hoyles, C., Wolf, A., Kent, P. and Molyneux-Hodgson, S. 2002. School Mathematics, Science and Technology. Sheffield University.

Hydea, J. S. and Mertzb, J. E. 2009. Gender, culture and Mathematics performance. Retrieved on December, 1, 2009 from http://tetvideo.madison.com/uw/gender.pdf

Hyde, J. S., Lindberg, S. M., Linn, M. C., Ellis, A. B. and Williams, C. C. 2008. Gender similarities characterize Mathematics performance. *Science,* 321 (5888), 494-495.

Ifamuyiwa, A. S. 2006. Effects of self and cooperative instructional strategies on Senior Secondary school students' learning outcomes in Mathematics. Unpublished Ph.D Thesis, University of Ibadan.

Individual with Disabilities Education Act. 2011. in NICHY National Dissemination Center for Children with Disabilities. NICHCY's *The Power of Strategy Instruction.* Retrieved on June 16, 2011 from http://www.ncld.org/images/stories/downloads/parent_center/idea2004parentguide.pdf

Instructional Strategies Online, 2011. What is Explicit Teaching? *Understanding Explicit Instruction.* A Chapter excerpt from Teaching Written Response to Text by Boyles. N. Retrieved on June 15, 2011 from http://www.nichcy.org/research/ee/learning-strategies/sp

Jain, P. 2010. Mathematics queries. The role of Mathematics in other disciplines. Accessed on http://drpjain_math problems. Blogspect. Com/2010/03 role-of-mathematics.

Jegede, O. 2001. Hong Kong. In O. Jegede and G. Shive . Open and distance education in the Asia Pacific region, 44-79. Hong Kong: Open University of Hong Kong Press.

Jitendra, A., Edwards, L., Sacks, G. and Jacobson, L. 2004. What research says about vocabulary instruction for students with learning disabilities. *Exceptional Children, 70,* 299–311.

Jordan, L., Miller, M. and Mercer, C. D. 1998. The effects of concrete to semi-concrete to abstract instruction in the acquisition and retention of fraction concepts and skills. Learning disabilities: A Multidisciplinary Journal, 9,115-122.

Jung, L. A. and Guskey, T. R. 2007. Standards-based grading and reporting: A model for special education. *Teaching Exceptional Children, 40*, 2, 48–53.

Kaino, L. M. 2001. Students' attitudes and interactions in learning Mathematics in Botswana junior Secondary Schools. Retrieved July, 17, 2009 from http://www.hiceducation.org/EduProceedings/Luckson%20M.%20Kaino.pdf.

Karp, K. and Niemi, R. 2000. The Mathematics club for girls and other problem solvers. *Teaching Mathematics in the middle School*, 5, 426-432.

Kelly, B. and Carnine, D. 1996. Connecting Mathematics Concepts: Level F *(Teacher's presentation book, student material and teachers guide*). Chicago: Science Research Association.

Kelvin M. 2008. Beyond I. Q. A Model of Academic Competence and Motivation (MACM), Institute for applied psychometrics, 11c.

Kheong. F. H. 2009. Mathematics in Focus: The underpinning concept. The Singapore approach.

Kilpatrick, J. 2001. Productive disposition-habitual inclination to see the Mathematics as sensible useful, and worthwhile, coupled with a belief in diligence and one's own efficacy pg. 11.

Kilpatrick, J., Swafford, J. and Findell, B. (Eds.). 2001. Adding it up: *Helping children learn Mathematics.* Retrieved March 20, 2008, from the National Academies Press Web Site: http://www/nap.edu/catalog. php?record_id=9822

Kinder, D., Kubina, R. and Marchand-martella, N. 2005. Special education and direct instruction: An effective combination. *Journal of Direct Instruction,* 5(1), 1-36.

Kloosterman, P. 1990. Attributions, performance following failure, and motivation in Mathematics. In E Fennema and G. C. Leder (Eds.) *Mathematics and gender*, 96-127. New York: Teachers College Press.

Kolawole, E. B. 2002. Sex difference in academic achievement in science subjects in a Nigerian tertiary institution Res. Curriculum studies 2(1) 168-173.

Kolb, D. A. 1984 Experience learning: Experience as the source of learning and Development. Prentice-Hall, Inc. Englewood Cliff, N. J. Retrieved on 28/11/2010 from http://www.learning-theories .com/experiential-learning-kol

Kolb, D. and Chapman, A. 2003-2010. Original concept relating to Kolb's learning styles model review, code and diagrams artwork.

Kolb, A. Y. and Kolb, D. A. 2006. Learning styles and learning spaces: A review of interdisciplinary application of experimental learning in higher education. http://www.learningfromexperience.com

Koehler,M. S. 1990. "Classrooms, teachers and gender differences in Mathematics." In E. Fennema and G. Leder (Ed.), *Mathematics and Gender*. New York: Teacher College Press.

Koller, O., Baumert, J. and Schnabel, K. 2001. Does interest matter? The relationship between academic interest and achievement in Mathematics. *Journal for Research in Mathematics Education,* 32(5), 448-470.

Kroesbergen, E. H. and Van, J. E. H. 2003. Mathematics interventions for children with special educational needs. Remedial and special education, 24(2), 97-114.

Lachance, J. and Mazzocco, M. M. M. 2006. A longitudinal analysis of sex differences in Mathematics and spatial skills in primary school age children learning and individual differences, 16, 195-216.

Leeson, N. 1995. Performance of sixth-grades in the Australian primary school Mathematics competition: Gender and others factors. *Mathematics Educational Research Journal,* 7(1) 37-49.

Lubienski, S. T. and Crane, C. C. 2009. *Girls' and Boys' early home experiences and School Mathematics achievement: An exploration using ECLS-K.* Paper presented at the annual meeting of the American Educational Research Association, San Diego.

Ma, X., and Kishor, N. 1997. Assessing the relationship between attitude toward Mathematics and achievement in Mathematics: A meta-analysis. *Journal for Research in Mathematics Education* 28(1), 26-47.

Ma, X. 1999. A meta-analysis of the relationship between anxiety toward Mathematics and achievement in Mathematics. *Journal for Research in Mathematics Education,* 30(5), 520-540.

Maccini, P. and Gagnon, J. C. 2000. Best practices for teaching Mathematics to secondary students with special needs. *Focus on Exceptional Children,* 32, 1-21.

Mackenzie, A. M. R., Martella, E., Moore, M. and Mantella, R. C. 2004. Teaching basic Mathematics skills to pre-school using connecting Mathematics concepts level K. *Journal of Direct Instruction,* 4 85-94.

Magliaro, S. G., Lockee, B. B. and Burton, J. K. 2005. Direct instruction revisited: A key model for instructional technology. *Educational Technology Research and Development, 53,* 4, 41-55.

Mahadevan, L. 2009. Mathematics the language of science. The Hindu online edition of India's National Newspaper Monday, Jan. 11, 2010.

Mahesh, C. S. 1999. Levels of learning in Mathematics. Learning problems in Mathematics: Diagnostic and remedial perspectives.

Mahlomaholo, S. and Sematle, M. 2005. Gender differences and black students' attitudes towards mathematics in selected high schools in South Africa. Retrieved October, 31, 2009 from http://www.icme-organisers.dk/tsg26/2SechabaMZ.doc

Malone, J. and Miller, D. 1993. Communicating Mathematical terms in writing: Some influential variables. In M. Stephens, A., Waywood, D., Clark, and J. Izard (Eds.), *Communicating Mathematics: Perspectives from classroom practice and current research* 177-190. Australian Council for Educational Research.

Manalo, E. Bunnell, J. K. and Stillman, J. A. 2000. The use of process mnemonics in teaching students with Mathematics learning disabilities. Learning disability quarterly, 23(2), 137-156. http://dx.odi.org/10.2307/1511142.

Manassero, Ma, A. Vazquez, A. and Acevedo, J. A. 2001. *Avaluacio dels temes de ciencia, Tecnologia i societat.* Mallorca (Espana): Consejeria de Educacion y Cultura del Gobierno de las Islas Baleares.

Marcus, S. 2009. Without Maths we're lost in a dark labyrinth. The Sunday times Newspaper Ltd 2010, registered in England No. 894646 registered office: 3 Thomas more square, London, E 18XY.

Marope, P. T. M. 1992. Determinants of academic achievement in Botswana Junior Secondary Schools (diss.) Chicago: University of Chicago.

Mason, J. and Johnson-Wilder, S. 2004. *Designing and using mathematical tasks*, Open University, Milton Keynes.

Mason, J. 2001. Modelling: where is the centre of gravity of-for-when modelling? In J. Matos, W. Blum, S. Houston and S. Carreira (Eds.). *Modelling and Mathematics Education: ICTMA 9 application in science and technology,* Horwood Publishing, Chichester, 39-61.

Martino, P. and Zan, R. 2001. Attitude toward Mathematics: Some theoretical issues. *Proceedings of 25th Annual Conference for the Psychology of Mathematics Education. 3, 351-358. Utrecht*, Northlands: Utrecht University Press.

Mastropieri, M. A., Scruggs, T. E. and Shiah, S. 1991. Mathematics instruction for learning disabled students: A review of research. *Learning Disabilities Research and Practice,* 6, 89-98.

Mathematical Sciences Education Board and National Research council (MSEBNRC). 1989. *Everybody Counts: A Report to the Nation on the Future of Mathematics Education.* Washington: National Academy Press.

Mathematics Worksheet Center, 2011. 10 Everyday reasons why algebra is important in your life. Retrieved March 19, 2011 from http://www.mathworksheetscenter.com

Mazzocco, M. M. M. 2007. Defining and differentiating Mathematical learning and difficulties. In D. B. Berch and Mazzocco M. M. M (Eds.), Why is math so hard for some children? 29-47. Baltimore, MD: Brookes Publishing.

Mazzocco M. M. M. and Thompson R. E. 2005. kindergarten predictors of Mathematics learning disabilities Research and practice, 20,142-155.

Mboto, F. A. 2001. Relative effectiveness of Gagne and Ausubel learning sequences on Mathematics students acquisition of laboratory skills. Unpublished Ph.D Thesis, Faculty of Education, University of Uyo, Nigeria.

Mc Gill, I. and Beaty, L. 1995. Action learning, second edition: a guide for professional, management and educational development London: Kogan page.

McLeod, D. B. 1992. Research on affect in Mathematics education: A reconceptualization. In D. A. rows (Ed.), *Handbook of research on Mathematics teaching and learning*, 575-596. New York: Macmillan.

Mercado, M. 2007. *Actitud hacia las matematicas rendimiento* Unpublished masters thesis. Cinvestav-PIN, Mexico: Department of Mathematics Education.

Mercer C. D. and Mercer, A. R 1998. Teaching students with learning problems 5th ed. Columb OH: prentice-hall.

Mercer, C. D., Jordan, L. and Miller, S. P. 1996. Constructivistic Mathematics instruction for diverse learners. Learning Disabilities Research and Practice, 11, 147-156.

Mercer, C. D. and Miller, M. D. 2003. Teaching algebra to students with learning difficulties: An investigation of an explicit instruction model. *Learning Disabilities Research and Practice*, 18, 121-131.

Middleton, J. A. and Spanias, P. A. 1999. Motivation for achievement in Mathematics: Findings, generalizations and criticism of the research. *Journal for Research in Mathematics Education* 30, 65-88.

Miles, D. D. and Forcht, J. P. 1995. Mathematics strategies for secondary students with learning disabilities or Mathematics deficiencies: A cognitive approach. Intervention in School and Clinic 91-96.

Miles, D. D., Jordon, L. and Miller, S. P. 1996. Mathematics strategies for secondary students with learning disabilities or Mathematics deficiencies: A cognitive approach. *Intervention in School and Clinics*, 31, 91-96.

Miller, S. P. and Mercer, C. D. 1997. Educational aspects of Mathematics disabilities. *Journal of Learning disabilities*. 30(1), 47-56.

Mills D. 2011. Mathematics Learning Difficulties Retrieved March 19, 2011, from http://www.mathlearningdifficulties.com

Minato, S. 1983. Some Mathematical attitudinal data on eighth grade students in Japan measured by a semantic differential. *Educational Studies in Mathematics 14(1), 19-38.*

Minato, S. and Yanasse, C.S. 1984. On the Relationship between Student attitude towards school Mathematics and there level of intelligence. *Educational studies in mathematics*, 15, 313-320

Ministry of Education 2008. *A guide to teaching and learning of A-level Mathematics 2010.* Singapore: Ministry of Education, Curriculum Planning and Development Division.

Montague, M. 2004. Mathematic problem solving for middle school students with disabilities. Retrieved March 21, 2008, from the Access Center Website: http://www.k8accesscenter.org/trainingressources/MathProblemSolving.asp.

_____ 2007. Self-regulation and Mathematics instruction. Learning Disabilities Research and Practice. 22(1), 75-83.

Morin, V. A. and Miller, S. P. 1998. Teaching multiplication to middle school students with mental retardation. Education and treatment of children, 21, 22-36.

Moyer, P. S. 2001. Are we having fun yet? How teachers use manipulatives to teach Mathematics. *Educational Studies in Mathematics*, 47, 175-197.

Mullis, I. V. S., Martin, M. O., Gonzalez, E. J., K. Conner, K. M., Chrostowski, S. J., Gregory, K. D., Garden R. A. and Smith, T. A. 2001, *Mathematics Benchmarking Report*. Retrieved January 12, 2013, from http://timss.bc.edu/timss1999b/pdf/TB99_Math_all.pdf .

Mutemeri, J. and Mugweni, R. 2005. The extent to which Mathematics instructional practices in early childhood education in Zimbabwe relates to or makes use of children's experiences. African Journal of Research in Mathematics, Science and Technology Education. 9(1), 49-54.

Nakonia, H. 2005: Reclining real "Basic Skills" in mathematics education. Retrieved February 22, 2011 from http.//home.blarg.net/- building/trans/hayes

National Adult Literacy and Learning Disabilities Center, 1995. Adults with learning disabilities: Definitions and issues.

National Centre for the Study of Adult learning and Literacy, NCSALL, 2000. Accommodating Mathematics student with learning disabilities. Retrieved on 19/09/2008 from file : local host/c:/Documents % 19/09/2008 from file: //location host/c/: Documents accommodating % 20 Math % 20 students % 20w

National Commission on Mathematics and Science Teaching for the 21st Century. 2000. *Before it's too late: A report to the nation from the National Commission on Mathematics and science Teaching for the 21st Century*. Retrieved March 20, 2008, from the US Department of Education Website

National Council of Teachers of Mathematics. 1988. The ideas of Algebra, K-12, Yearbook, Reston, VA: NCTM ISBN: 0-87353-250-3.

________1989. Curriculum and Evaluation Standards for School Mathematics. Reston, VA: NCTM.

_______ 2000. *Principle and Standards of School Mathematics.* Reston, V.A: NCTM ISBN: 0-87353-480-8.

_______ 2006. Overview of principles and standards for School Mathematics. Retrieved February 24, 2011 from www.nctm.org

National Mathematics Advisory Panel. 2008. Foundations for success: The final report of the National Mathematics Advisory Panel. Retrieved March 20, 2008 from the Rand Web site: http://www.ed.gov/about/bdscomm/list/mathpanel/report/final-reportpdf

National Mathematical Centre, 2011. Why AIMS?

National Policy on Education. 2004. 4th Edition. Federal republic of Nigeria.

National Dissemination Centre for Children with Disabilities. 2009. How Disabilities can affect Mathematic achievement. Retrieved March 20, 2011 from http://www.nichy.org/Research/ Evidence for Education/Pages/matt.

Nguyen, T. K. C., 2003. *Family and student influences on withdrawal from lower secondary school in rural Vietnam.* Unpublished Ph.D Thesis. Melbourne: University of Melbourne.

Northwest Regional Advisory Committee, 2005. A report to the U.S. Department of Education on educational challenges and technical assistance needs for the Northwest Region. Washington, DC: U.S. Department of Education.

Obioma, G. 2006. Nigeria reviews education goals. Leaders hope to reinvigorate policy on education.

Odogwu, H. N. 2002a. The Effect of computer assisted instruction of students' performance in Mathematics. *Proceedings of September 2002, Annual National conference of Mathematical Association of Nigeria* M. A. N, 1-9

______ 2002b. Female Students' Perception and Attitude to Mathematics: A Bane to their S. M. T Education. ABACUS. *The Journal of the Mathematics Association of Nigeria, 27(1), 19-29.*

Oginni, O. I. 2007. Effectiveness of laboratory method of teaching on Students' performance in senior secondary school Mathematics in Ondo State. Unpublished M.Ed. Thesis Faculty of Education, University of Ado-Ekiti, Nigeria.

Ogunkola, B. J. 2007. An Assessment of Teaching Ability of Elementary Science Teachers in Universal Basic Education (UBE) Programme in Ogun State, Nigeria. *Journal of Applied Education and Vocational Research*, 2 (1), 40 – 46.

Ogunniyi, O. 2000. A causal model and student factors as determinants of achievement in secondary school integrated Science. *Unpublished Ph. D Thesis, University of Ibadan.*

Ogunleye, B. O. and Ogunsanwo, T. 2001. Women's scientific research productivity in Nigeria: Problems and prospects. *42nd Annual Conference of Science Teachers Association of Nigeria.139-141.*

Ojo, O. J. 2003, Relative effectiveness of self-regulatory and cooperative learning strategies on learning outcomes in senior Secondary School Mathematics. Unpublished Ph.D. Thesis, University of Ibadan.

Okpala, N. P. and Onocha, C. O. 1995. Students' factors as correlates of achievement in physics. Physics Education: A.J. Br. Inst. Phys., 6:361-365.

Okpala, P. N, Onocha, C. O. and Oyedeji, O.A, 1993. Measurement and evaluation in education. Ibadan: sterling – Horden published Limited.

Olusi F. I. and Anolu, E. 2010. Mathematics as a foundation for children education in science and technology. *Pakistan Journal of Education and Technology. 1(1)65-168.*

Oluwaniyi, O. O. Ibiyemi, S. A. and Usman, L. A. 2007. Effect of detoxification on the nutrient content of thevetia peruviana seed cake. *Research Journal of Applied sciences 2(2): 188-191, Medwell Journals.*

Onabanjo I. O. 2000. Peer tutoring- assisted instruction, parent supportiveness and students locus of control as determinants of learning outcomes in Senior Secondary school Mathematics. Unpublished Ph.D Thesis, University of Ibadan.

Onaolapo, O. 2001. Elementary Mathematics for Schools and colleges (1), 1-75

Opara V. O. 2004 Child Abuse: Dimension, causes, effects and suggested solutions. Journal of the Nigerian society for Educational psychologi*es* 3(1), 237-246

Opolot – Okurot, C. 2005. Student attitudes towards Mathematics in Uganda secondary schools. *African Journal of research in Mathematics science and technology education* 9(2), 167-174.

Otung, I. E. 2001. Reassessing the Mathematics content of engineering education, 130-138.

Owolabi, O. T. 2002. Analysis of Students' performance in JSS integrated science examination. Res curriculum stud, 2(1) 31-36

Oyedeji, O. A. 2000. Effective teaching of Mathematics in Erinosho, S. Y. Adesanya, A and Ogunyemi, A. Eds. Teaching effectiveness in Nigeria Schools. 147-164. Ibadan Sam Bookman publishers.

Pandit, R. P. 2000. "A model for the identification of learning disabilities in Mathematics*." Shikshak Journal*. 3, Kathmandu: Mahendra Ratna Campus, 11-17.

Paula, L. 2012. Mathematics Remediation and Learning Strategies. Retrieved on 7/3/2012 from http://www.pbs.org/wgbh/misunderstoodminds/mathstrats.html.

Penner, A. M., and Paret, M. 2008. Gender differences in Mathematics achievement: Exploring the early grades and the extremes. *Social Science Research*, 37(1), 239-253.

Peterson, M., Hittie, M. and Tamor, L. 2002. Authentic multi-level teaching. Teaching children with diverse academic abilities together well. Retrieved February 24, 2011 from http://www.coe.wayne.edu/wholeschooling/ws/wspress/Authentic%206-25-02.pdf

Pincott, R. 2004. Are we responsible for our children's Mathematics difficulties? In B.A. Knight and W. Scott (Eds.), Learning difficulties. Multiple perspectives. Frenchs Forest, NSW: Pearson Education Australia.

Popoola, A. A. 2002. Gender difference in Nigeria research in curriculum studies, 1 (2), 37- 44.

Presmeg, N. 2002. Beliefs about the nature of Mathematics in the bridging of everyday and school mathematical practices. In Leder, G., Pehkonen, E. and Torner G. (eds.), Beliefs: A hidden variable in Mathematics education? 293-312. Dordrecht: Kluwer.

Randhawa, B. S., Beamer, and Lundberg, I. 1993. Role of the Mathematics self-efficacy in the structural model of Mathematics achievement. *Journal of Educational Psychology* 85, 41-48.

Rathbun, A. H., West, J. and Germino-Hausken, E. 2004. *From kindergarten through third grade: Children's beginning school experiences* (NCES 2004-2007). Washington, DC: National Center for Education Statistics.

Republic, 2001. Plato: Education and the value of justice. Retrieved on 20/03/2011 from http://www.philosophypages.com/hy/2h.htn

Robson, J. 1996. Some outcomes of learning through teleconferencing. *Journal of Instructional Science and Technology,* 1(3). Retrieved October 26, 2006 from http://www.usq.edu.au/electpub/e-jist/docs/old/vo11no3/article1.htm.

Roblyer, M. D, Edwards, J. and Havriluk, M. A 1997. Integrating educational technology into teaching. Upper Saddle River, N. J: prentice- Hall.

Rosenshine, B. V. 1986. Synthesis of research on explicit teaching. Educational Leadership, 43, 7, 60-69. Ross, J. A., Hogaboam-Gray, A., McDougall, D., and Bruce, C. 2001-02. The contribution of technology to the implementation of mathematics education reform: Case studies of grade 1-3 teachers. *Journal of Educational Computing Research, 26(* 1), 123-140.

Ross, J. A., Hogaboam-Gray, A., and Hannay, L. 2001. Effects of teacher efficacy on computer skills and computer cognitions of K-3 students. *Elementary School Journal, 102*(2), 141-156.

_____ 2001. Collateral benefits of an interactive literacy program. *Journal of Research on Computing in Education, 33(* 3), 219-234.

Ross, J. A., McDougall, D. and Hogaboam-Gray, A. 2002. Research on reform in mathematics education, 1993-2000. The Alberta Journal of Educational Research, 49(2), 122-138.

Ruffell, M., Mason, J., and Allen, B. 1998. Studying attitude to Mathematics. *Educational Studies in Mathematics* 35, 1-18.

Saha, S. 2007. A study of gender attitude to Mathematics, cognitive style and achievement in Mathematics. Experiment in Education. 35, 6.

Salawu A. 2001. The Yoruba and their language Newspaper: origin, nature, problems and prospects. Kanla- Raj 2004. Stud. Tribes Tribals 2 (2), 97-104.

Sawalha, A. 2004. The role of direct instruction strategy on education of some Mathematical skills to students with learning difficulties. Learning difficulties Review. (5), 41-47.

Schiefelee, U. and Csikszentmihalyi, M. 1995. Motivation and ability as factors in Mathematics experience and achievement. *Journal f*or research in *Mathematics education*, 26(2), 163-181.

Science daily, 2010. Few gender differences in Mathematic abilities, Worldwide study finds. An outline Retrieved March 19, 2011. http:www.sciencedaily.com/2010/01/100105112303.

Sheldon, H. H. 2013. Planning for Transition to College for Students with Learning Disabilities. National Center for Learning Dissabilities. The power to hope, to learn and to succeed.

Sherman, H. J., Richardson, L. I. and Yard, G. J. 2005. Teaching children who struggle in Mathematics. A systematic approach to analysis and correction. Upper Saddle River, NJ:Pearson Education. Site.//c:/Documents and settings/user/Desktop/Learning disability.

Skinner, E. A. 1996. A Guide to Constructs of control. *Journal of Personality and Social Psychology, 71, 549-570.*

Slavin, R. E. 2009. Educational Psychology : Theory and Practice (9th ed.). Upper Saddle River, NJ: Pearson Education, Inc.

Smith, A. 2004. Making Mathematics count: The report of Professor Adrian Smith's inquiry into post-14 Mathematics Education. London, The stationery Office.

_______2009. How disabilities can affect Mathematic achievement. *Retrieved on* 10/1/2011 from http://int.forum.com/ archieve/index. Php/t-4715 htt

Spear-Swerling, L. 2005. *Components of Effective Mathematics Instruction*. Retrieved September 26, 2012, from the LD Online Web site: http://www.ldonline.org/article/5588

Stanic, G. M. A. and Hart, L. G. 1995. Attitudes, persistence and Mathematics achievement: qualifying race and sex differences, in W. G. Secada, E. Fennema and L.B. Adajian (eds.) *New Directions for Equity in Mathematics Education.* Cambridge: Cambridge University Press

Steedly, K., Dragoo, K., Arefeh, S. and Luke, S. D. 2008. Effective Mathematics instruction. Evidence for Education. III, Issue I.

Stroessiger, R. 2002. An introduction to critical numeracy. *Australia Mathematics Teacher*, 58 (4), 17-20.

Suydam, M. N. and Higgins, J. L. 1977. Activity-based learning in elementary school Mathematics: Recommendations from research. Columbus, OH: ERIC Center for Science, Mathematics, and Environmental Education.

Swanson, H. L. 1999. Instructional components that predict treatment outcomes for students with learning disabilities: Support for a combined strategy and direct instruction model. Learning Disabilities Research and Practice, 14(3), 129-140.

____ 2001. Searching for the best model for instructing students with learning disabilities. Focus on Exceptional Children, 34(2), 1-15.

Swanson, H. L. and Rhine, B. 1985. Strategy transformations in learning disabled children's Mathematics performance. Clues to the development of expertise. *Journal of Learning Disabilities*, 18, 596-603.

Tapia, M. and Marsh, G. E. 2002. *Confirmatory factor analysis of the Attitudes toward Mathematics Inventory*. Paper presented at the Annual Meeting of the Mid-South Educational Research Association, Chattanooga, TN. (ERIC Documents Reproduction Service No. ED 471 3021).

Tapia, M. 2004. The relationship of Mathematics anxiety and gender. *Academic Exchange Quarterly*, 8, 130-134.

_____ 2004. An instrument to measure Mathematics attitudes. Berry college, GA.

Tymms, P. B. 2001. Students' attitudes toward Mathematics. Academic exchange quarterly, summer, 2004 by Kathryn Sanchez, Laurie Zimmerman, Renmin Ye.

Udousoro, U. J. 2000. The relative effects of computer and text-assisted programmed instruction on students' learning outcome in Mathematics. *Unpublished doctoral Thesis, University of Ibadan.*

Ugboduma, S. O. 2008. Introducing Mathematics laboratory into secondary schools Mathematics curriculum to promote students' enthusian for Natural Development. *Journal of Mathematical Association of Nigeria 187-196.*

UNESCO, 2003. Gender and education for all: the leap for equality. Global monitoring report 2003/2004. http.//www.unesco/oc.unesco.org/education/eta-report/2003-pdf/chapter3.pdf

Ursine, S. and Sanchez, G. 2008. Gender, technology and attitude towards Mathematics: A comparative longitudinal study with Mexican students. ZDM Mathematics Education 40: 559-577.

Vale, C. 2009. Trends and factors concerning gender and Mathematics in Australasia. Retrieved on November 21, 2009 from http://tsg.icmell.org/document/get/169

WAEC 2004, 2005, 2007, 2011. Chief Examiners' Reports. Yaba, Lagos: Amao Press Limited.

Walden, R., and Walkerdine, V. 1985. *Girls and Mathematics: From primary to secondary schooling, Bedford Way Papers 24*. London: Heinemann.

Webb, R. M., Lubinski, D. and Benbow, C. P. 2002.Mathematically facile adolescents with math-science aspirations: New perspectives on their educational and vocational development. *Journal of Educational Psychology, 94, 785 - 794.*

Wesley, C. and Gersten, R. 2001. Follow-up of follow through: The later effects of the direct instruction model on children in fifth and sixth grades. *Journal of Direct Instruction.* 3(2), Winter.

Westwood, P. 2000. *Numeracy and learning difficulties. Approaches to teaching and assessment.* Camberwell, VIC: ACER.

Wilson J. P. 2010. Family engagement in education. Offices of research and education accountability (OREA). *Legislative Brief.*

Willis, S. 1995. Gender Justice and the Mathematics Curriculum: Four perspectives. In Parker, L., Rennie, L. and Fraser, B. (eds.), *Gender, Science and Mathematics shortcoming the shadow*, 41-51. Dordecht: Kluwer.

Winter, J. 2001. Personal, spiritual, moral, social and cultural issues in teaching Mathematics. In P. Gates (Ed.), *issues in Mathematics teaching* . 197-213. London: *Routledge Falmer.*

Wisniewski, Z. G. and Smith, D. 2002. How effective is Touch Math for improving students with special needs academic achievement on math addition Mad Minute timed tests? Eugene, OR: ERIC Clearinghouse on Educational Management. (ERIC Document No. 469445).

Witzel, B. S. and David, A. (in press). Dynamic modeling of manipulative instruction. *Mathematics teaching in the middle school.*

Witzel, B. S., Smith, S. W. and Brownell, B. T. 2001. How can I help students with learning disabilities in algebra? *Intervention in School and Clinic*, 37, 101-104.

Witzel, B. S., Mercer, C. D. and Miller, M. D. 2003. Teaching algebra to students with learning difficulties: An investigation of an explicit instruction model. *Learning Disabilities Research and Practice, 18(2), 121-131.*

Wolfe, P. 2001. *Brain matters: Translating research into classroom practice*. Alexandria, VA: Association for Supervision and Curriculum Development.

Woodard, T. 2004. The effects of Mathematics anxiety on post-secondary developmental students as related to achievement, gender and age, *inquiry*, 9(1), 1-3. ERIC Document Reproduction Server, No EJ876845

Yara, P. O. 2009. Relationship between teachers' attitude and students' academic achievement in Mathematics in some selected senior secondary schools in South western Nigeria. *European Journal of Social Science. 11. 364-369.*

Zan, R. and Martino, P. D. 2007. Attitudes toward Mathematics: Overcoming positive/negative dichotomy. The Montana Mathematics enthusiast monograph 3. 157-168. *The Montana Council of Teacher of Mathematics.*

Zhu, Z. 2007. Gender differences in mathematical problem solving patterns: A review of literature. International Education Journal, 8, (2), 187-203.

APPENDIX 1

INSTRUCTIONAL GUIDE FOR CONCRETE-REPRESENTATIONAL-ABSTRACT INSTRUCTIONAL STRATEGY (CRAIS)

(i) **General Information**

Subject: Mathematics

Topics: (i) Circles (ii) Volumes (iii) Angles of Elevation and Depression

Class: SS II

Sex: Male [ ] Female [ ]

(ii) **About the Guide**

This guide was designed to teach the students in senior secondary school two the above topics. There are eighteen periods in the guide; the lessons are designed for 6 weeks of 3 periods each week.

(iii) **To the Teacher**

The procedure was used by the teacher to teach the students. The teacher followed the guide accordingly and make sure that every student gets in touch with the materials provided.

(iv) **General Objectives**

At the end of the lessons in this guide, students' were able to:

(i) define circle.
(ii) name object with circular face.
(iii) find the area and sector of a circle.
(iv) work sums on chords and arcs of circles.
(v) state some circle theorems.
(vi) apply circle theorems when necessary.
(vii) work sums on cyclic quadrilaterals .
(viii) define prism with examples.
(ix) find the volume of cube and cuboid.
(x) identify triangular prism and find its volume.
(xi) identify cylinder and find its volume.

(xii) find the volume of pyramid.

(xiii) differentiate between angles of elevation and depression.

(xiv) calculate angles of elevation and depression.

(vi) **To the Students**

This learning guide was specially designed to teach you some topics within your school syllabus. Your total cooperation is very essential for your own benefit. The lesson will cover six weeks, if anything is not clear to you during the period of teaching, endeavour to ask questions.

Contents for each of the 6 Weeks

Week 1:

Topic: Circle Geometry

Period 1: Area and arc of a circle

2. Sectors and Chords of circles

3. Circle theorems

Week 2:

Topic: Circle Geometry

Period 1: Cyclic quadrilaterals

2. Tangent to a circle

3. Practice exercises on Circle geometry

Week 3:

Topic: Volume of Solids

Period 1: Volume of Solids (Prism)

2. Volume of Cube and Cuboids

3. Volume of Triangular Prism

Week 4:

Topic: Volume of Solids

Period 1: Volume of Cylinder

2. Volume and frustum of cone

3. Volume of Pyramid

Week 5:

Topic: Angles of Elevation and Depression

Period 1**:** Angles of Elevation

2. Angles of elevation

3. Practice exercises on angles of elevation

Week 6:

Topic: Angles of Elevation and Depression

Period 1**:** Angles of Depression

2. Angles of Depression

3. Practice exercise on angles of depression

WEEK 1

EXPERIMENTAL GROUP 1

Instructional Guide for Concrete-Representational-Abstract Instructional Strategy (CRAIS)

Topic: Circle Geometry

Period 1: Area and arc of a Circle

Introduction and Presentation of Materials (Concrete): Different materials with circular faces were displayed in class. These include: Cups, Water bottle cover and concrete material with circular face showing different parts of circle like, radius, diameter, arc, sector, chord, tangent and circumference.

Examination of the materials: Pass the materials round the class for students to touch and examine.

Presentation of the concept to be taught:

Step 1: Definition

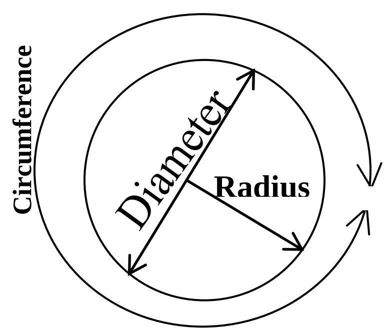
The concept circle and different parts of the circle were defined with reference to the materials. Thus: A circle is a closed curve in a plane. It is the locus of all points equidistant from a central point.

The radius of a circle is the distance from the centre of the circle to the outside edge.

Step 2: Explanation

Diameter**:** The diameter starts at one side of the circle, goes through the centre and ends on the other side. It is the longest distance across a circle, the diameter is twice the radius.

Diameter = 2 x radius



Circumference: The circumference is the distance around the edge of the circle. It is exactly π times the diameter.

Circumference = π x diameter

$= \pi \times 2 \times \text{radius}$

$= 2\pi r$

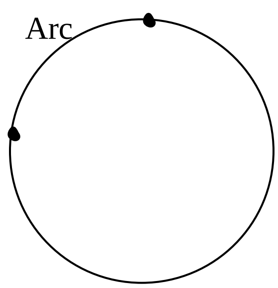
$$\pi = \frac{Circumference}{Diameter}$$

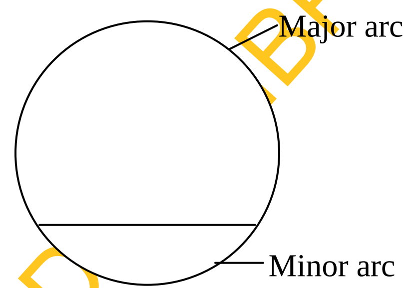
Area**:** The area of a circle is π times the radius squared.

$A = \pi \times r^2$

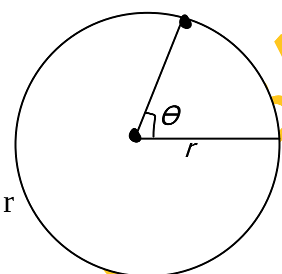
$= \pi r^2, \ \pi = 3.141592.$

An Arc of a Circle**:** An arc of a circle is a segment of the circumference of the circle.



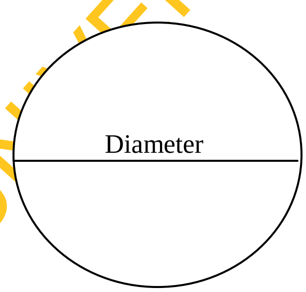


$$Arc \text{ length} = \frac{\theta}{360} X\, 2\pi \text{ r}$$

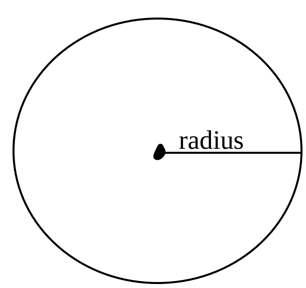


Step 3: Students' activities

The instructor will guide the students on how to cut circle, locate radius, diameter and arc of a circle group by group.



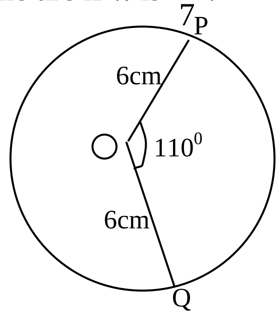
A Circle



Step 4: Example

The following question is given as an example with the use of concrete object.

1. An arc subtends an angle of 110^0 at the centre of a circle of radius 6cm. Find the length of the arc if π is $\frac{22}{7}$.



$$Arc \text{ length} = \frac{\theta}{360} X \ 2\pi \ r$$

$$\therefore Arc \ PQ = \frac{110^0}{360} X \ 2 \text{ X} \frac{22}{7} X \frac{6}{1}$$

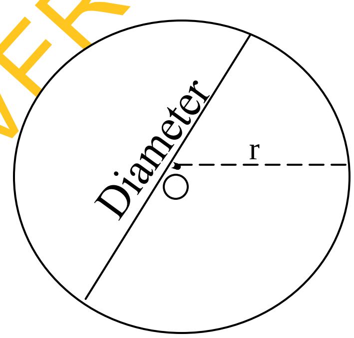
$$= \frac{11X\,22}{3X\,7}$$

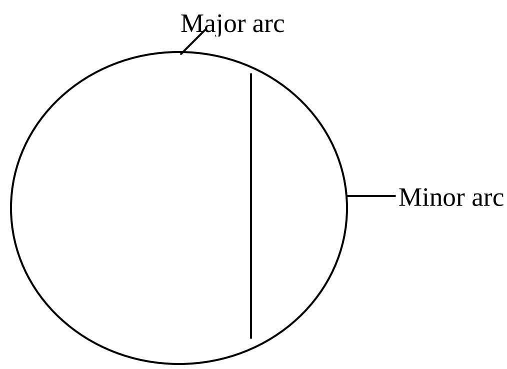
$$= \frac{242}{21}$$

$$= 11.5cm$$

Step 5: Drawing of relevant materials (representation)

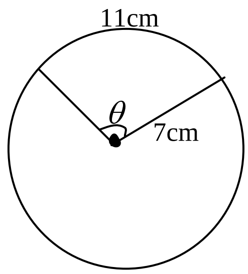
The students were allowed to draw the structure they have made in their note books and relate the concrete material to the diagram.





Step 6: Another example is given with the use of diagram.

(2) Find the value of Θ from the following diagram.



$$Arc \text{ length} = \frac{\theta}{360} \text{ X } 2\pi \text{ r}$$

$$11 = \frac{\theta}{360} \text{ X } 2\pi \text{ X } 7$$

$$\frac{11}{1} = \frac{\theta \text{ X } 2 \text{ X} \pi \text{ } 7}{360^0}$$

$$\theta = \frac{11 \text{ X } 360}{2 \text{ X } \pi \text{ x } 7}$$

$$= \frac{11 \text{ X } 360 \text{ X } 7}{2 \text{ X } 22 \text{ x } 7}$$

$$\therefore \theta = 90^0$$

Step 7: Question without material and drawing (Abstract)

(3) Calculate the area and arc length of a circle of radius 7cm which subtends 108^0 at center, if $\pi = \frac{22}{7}$

Solution:

Area of a circle = π r^2

$$= \frac{22}{7} \text{ X } \frac{7}{1} \text{ X } \frac{7}{1}$$

$$= 154cm^2$$

$$Arc\ \text{length} = \frac{\theta}{360} X\, 2\,\pi\, \text{r}$$

$$= \frac{108^0}{360^0} X\, 2 X \frac{22}{7} X \frac{7}{1}$$

$$= \frac{108 X 11}{90^0} = \frac{1{,}188}{90^0}$$

$$= 13.2cm$$

Summary: A circle is a closed curve in a plane. Circumference of a circle can be calculated by using the following formula:

Circumference of a circle = 2π r

Area of a circle = π r^2

Arc length = $\frac{\theta}{360^0} X\, 2\pi$ r

Evaluation: Allow them to solve the following question in class.

Question: What is the length of an arc which subtends an angle of 60^0 at the centre of a circle of radius $\frac{1}{2}m$?

Assignment

1. In terms of π, what is the length of an arc which subtends an angle of 30^0 at the centre of a circle of radius $3^1/_2$cm?
2. Find area of a circle with diameter 7cm.

Period 2

Topic: Sectors and Chords of Circles

Introduction: Materials with circular shape showing sectors and chords made of cardboard and plywood were displayed in class.

Examination of the Materials: The materials were passed round the class. Different parts of the material were identified with the help of the instructor.

Presentation:

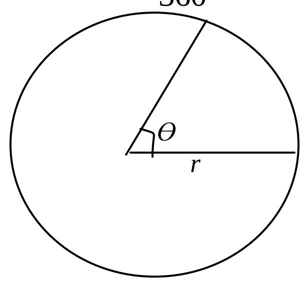
Step 1: Definition

A sector of a circle is a portion enclosed by two radii and an arc of the circle. A chord of a circle is a line segment that connects one point on the edge of the circle with another point on the circle. The diameter is the longest chord.

Step 2: Explanation

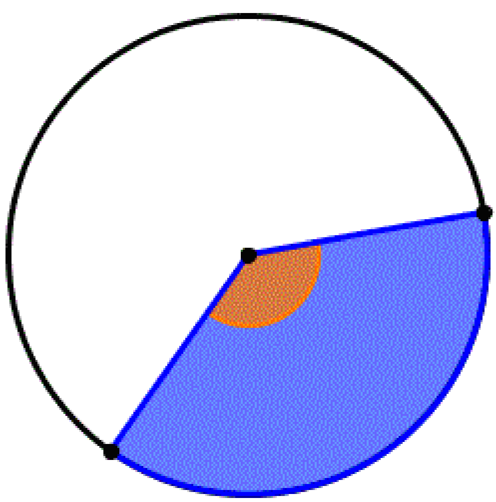
The chord divides the circle into two segments of different sizes, a major and minor segment. Also, a chord which is not a diameter divides the circumference into two arcs of different sizes, a major arc and a minor arc.

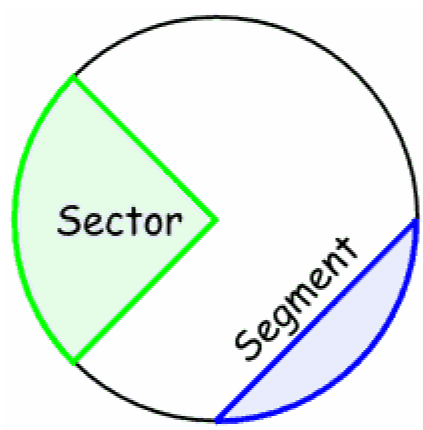
Area of a sector = $\frac{\theta}{360^0} X \pi r^2$

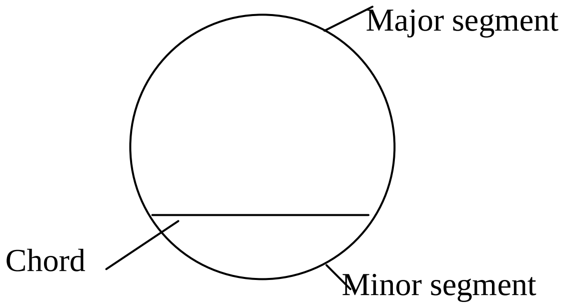


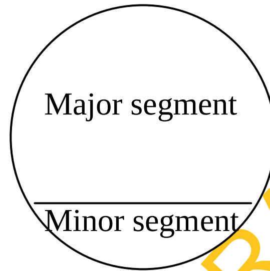
Step 3: Students' activities

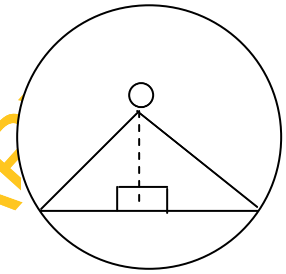
With the guide of instructor, the students will use cardboard either in group or individually to cut sector, indicate major arc and minor arc, also major segment and minor segment.











Step 4: Example (with concrete material)

1. Calculate the area of a sector of a circle which subtends an angle of 45^0 at the centre of the circle radius 14cm if $\pi = \frac{22}{7}$

Solution:

$$Area \text{ of a sector} = \frac{\theta}{360} X \pi \, r^2$$

$$\theta = 45^0, \pi = \frac{22}{7}, r = 14$$

$$\text{Area} = \frac{45^0}{360^0} X \frac{22}{7} X \frac{14}{1} X 14$$

$$\frac{45 X 22 X 7}{90} = 77$$

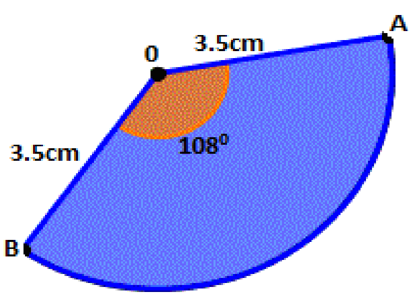
$$Area = 77cm$$

Step 5: Drawing of the materials (representation)

The students are allowed to draw the materials made with cardboard in their note and compare the structure made with the diagram.

Step 6: another examples are given with diagrams.

(2) Calculate the perimeter of the following diagram.



Solution:

Perimeter is the distance round the shape.

Perimeter of sector AOB = AB + AO + BO

$$Arc\ \text{AB} = \frac{\theta}{360} X\, 2\pi\ \text{r}$$

$$= \frac{108}{360^0} X\, 2 X \frac{22}{7} X\, 3.5$$

$$= \frac{108}{360^0} X\, 2 X \frac{22}{7} X \frac{7}{2}$$

$$= \frac{33}{5}$$

$$= 6.6cm$$

Arc AB

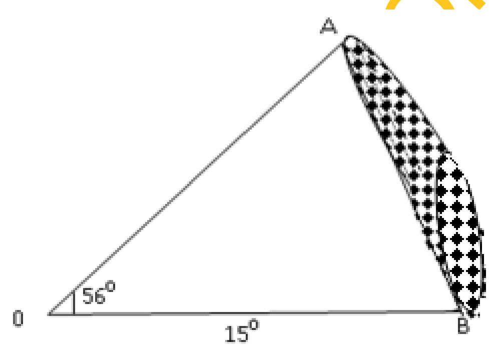
$\overline{\text{AO}} = 3.5cm$

$\text{BO} = 3.5\text{cm},$

$\therefore$ Perimeter $= (6.6 + 3.5 + 3.5)cm$

$= 13.6\text{cm}$

3. Calculate the area of the shaded segment of the following diagram; if $\pi = \frac{22}{7}$.



Area of the shaded part= Area of sector AOB – Area of Δ AOB

$$Area \text{ of sector AOB} = \frac{\theta}{360^0} X \pi\ \text{r}^2$$

$$= \frac{56}{360^0} X \frac{22}{7} X \frac{15}{1} X \frac{15}{1}$$

$110cm^2$

$$Area \text{ of } \Delta \text{ AOB} = \frac{1}{2}\overline{AO}X\,\overline{BO}\ Sin\ 56^0$$

$$= \frac{1}{2}X15X15X0.8290$$

$$= \frac{186.525}{2}$$

$$= 93.2625cm^2$$

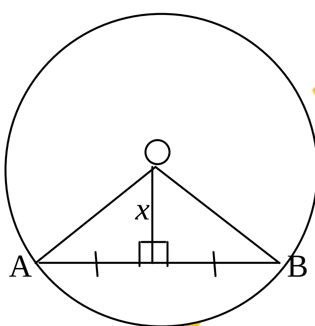
$\therefore$ *Area* of the shaded part

$$= (110 - 93.26)\text{cm}^2$$

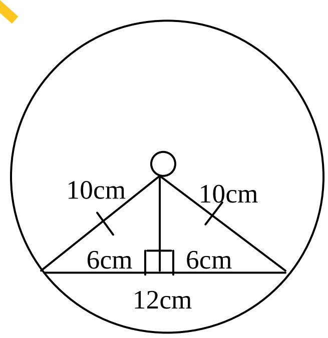
$$= 16.74\text{cm}^2$$

Chord of a circle

A straight line drawn from the centre of a circle to bisect a chord which is not diameter is at right angles to the chord.



4. The radius of a circle is 10cm. The length of a chord is 12cm. Calculate the distance of the chord from the centre of the circle.



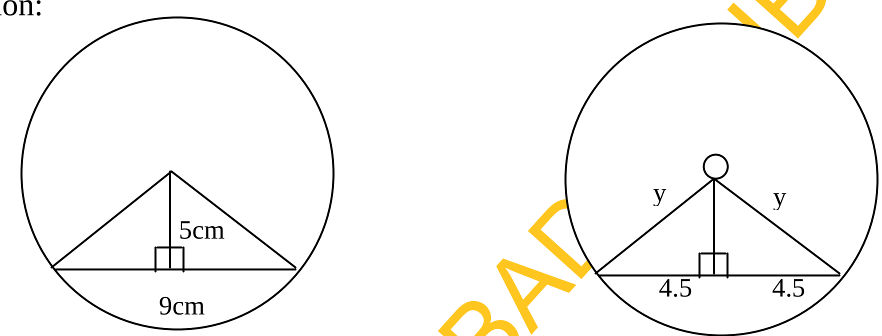
Solution:

$$x^2 = 10^2 - 6^2 \quad \text{(Pythagoras)}$$
$$= 100\text{-}36$$
$$= 64$$
$$x = \sqrt{64}$$
$$= 8\text{cm}$$

Step 7: Questions without material and drawing (abstract).

(5) A chord of a circle is 9cm long. If its distance from the centre is 5cm, calculate the radius of the circle.

Solution:



$$y^2 = 5^2 + 4.5^2 \quad \text{(Pythagoras)}$$
$$= 25 + 20.25$$
$$= 45.25$$
$$y = \sqrt{45.25}$$
$$= 6.7cm$$
$$\therefore radius = 6.7cm$$

(6) Calculate the area of a sector which subtends an angle of 90^0 at the centre of the circle with radius 7cm $\left(\pi = \frac{22}{7}\right)$

Solution:

$$Area \text{ of a sector} = \frac{\theta}{360^0} X \pi \text{ r}^2$$
$$= \frac{90}{360} X \frac{22}{7} X \frac{7}{1} X \frac{7}{1}$$
$$= \frac{154}{4}$$
$$= 38.5cm^2$$

Summary: A chord is an internal segment of a circle that has both of its endpoints on the circumference of the circle.

The diameter of a circle is the largest chord possible. A sector is enclosed by two radii and an arc in the circle.

$$Area \text{ of sector} = \frac{\theta}{360} X \pi \, r^2$$

Evaluation: Ask them the following questions.

1. What is a sector of a circle?
2. How do we find the area of a sector?
3. Define chord of a circle?
4. What do we call the largest possible chord?

Assignment

1. The radius of a circle is 10cm. The length of a chord is 16cm. Calculate the distance of the chord from the centre of the circle.
2. A sector with radius 14cm subtends an angle of 45^0 at the centre of a circle. What is the area of the sector?

Period 3

Topic: Circle Theorems

Introduction: Materials with circular shape showing angle that an arc of a circle subtends at the centre, angles in the same segment, and angle in a semicircle were displayed in class.

Examination of the materials: The materials were passed round the class for students to identify.

Presentation

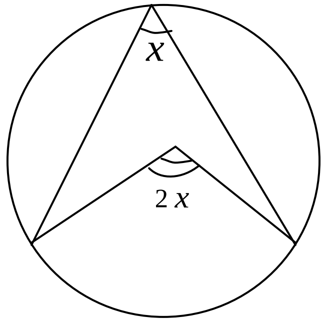
Step 1: Theorem

(1) The angle that an arc of a circle subtends at the centre is twice that which it subtends at any point on the remaining part of the circumference.

Step 2: Explain with concrete object, show students (i) the angle an arc subtends at centre and the angle at the circumference.

Step 3: Students' activities

The students were allowed to cut the structure showing angle at centre and angles at the circumference.

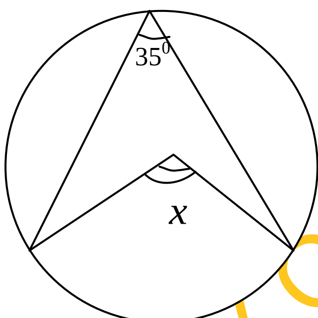


Step 4: Example

From the structure the students cut allow them to measure the angle at the circumference, then measure the angle at centre and compare the result.

If the angle at the circumference is 35^0 find the angle at centre?

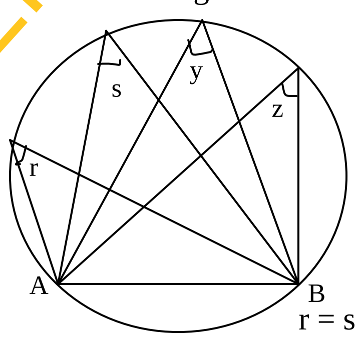
Solution:



With the theorem**:**

$x = 2 \text{ x } 35^0 = 70^0$

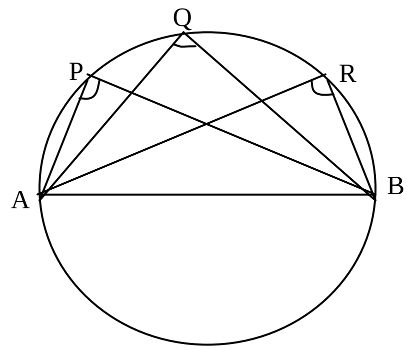
(ii) Angles in the same segment of a circle are equal.



r = s = y = z

If $r = 42^0 \Rightarrow s = 42^0$, $y = 42^0$ and $z = 42^0$

(iii) The angle in a semicircle is a right angle

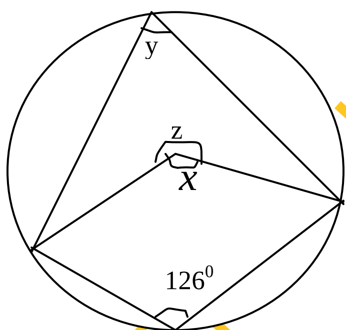


$\Rightarrow P = Q = R = 90^0$

Step 5: The students were allowed to draw the structure made and compare the structure with the diagrams.

Step 6: Examples with diagram

1. Find the angles x, y, and z



Solution:

z = 126^0 x 2 (Angle at centre twice that at circumference)

$\Rightarrow$ z = 252^0

x + z = 360^0 (Sum of angles at a point)

$x + 252^0 = 360^0$

$x = 360^0 - 252^0$

$= 108^0$

2y = x (Angle at centre twice that at the circumference)

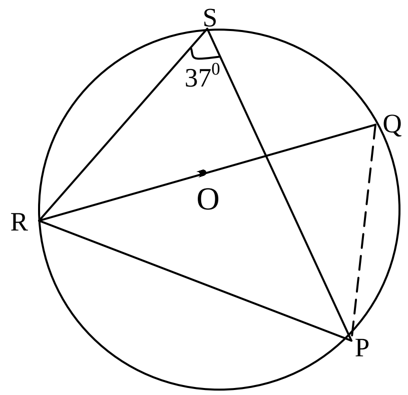
$2y = 108^0$

$$y = \frac{108}{2} = 54^0$$

$\therefore x = 108^0, y = 54^0$ z=252

2. 0 is the centre of the following circle and QOR is the diameter.

PSR = 37^0. Find $P\hat{R}Q$



Join P and Q

RSP = RQP = 37^0 (Angles in the same segment)

RPQ = 90^0 Angle subtended by diameter)

$\therefore$ PRQ= $180^0 - (90^0 + 37^0)$ (Sum of angles of a triangle)

$= 180^0 - 127^0 = 53^0$

Step 7: Another example is given without diagram

(3) If A, B, C and D are angles in the same segment of a circle and A = 36^0, find the values of B, C, and D.

Solution:

Since A, B, C and D are in the same segment of a circle,

$\therefore$ A = B = C = D = 36^0

$\Rightarrow$ A = 36^0, B = 36^0, C = 36^0 and D = 36^0

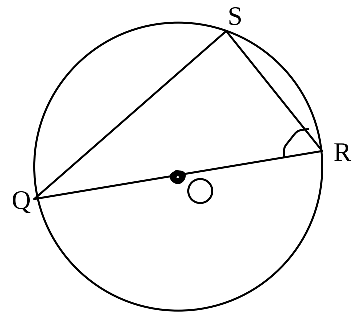
Summary: The angle that an arc of a circle subtends at the centre is twice that which it subtends at any point on the remaining part of the circumference. Angles in the same segment of a circle are equal. The angle in a semicircle is a right angle.

Evaluation: Ask them the following questions:

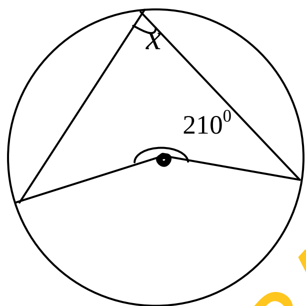
1. Angles in the same segment of circle are ______ (equal/not equal)
2. The angle in a semicircle is a ______ (obtuse angle/right angle)
3. The angle that an arc of a circle subtends at the centre is ____ that which it subtends at any point on the remaining part of the circumference (twice/three times)

Assignment: Solve the following questions.

(1) In the diagram below, 0 is the centre of the circle. If $Q\hat{R}S = 62^0$, find the value of SQR. (a) 14^0 (b) 28^0 (c) 31^0 (d) 90^0



(2) If 0 is the centre of the following circle find x



WEEK 2

Topic: Circle Geometry

Period 1: Cyclic Quadrilaterals

Introduction: Materials with circular shape showing cyclic quadrilateral were displayed in class.

Examination of the materials: The materials were passed round the class. Different parts of the material were identified with the help of the instructor.

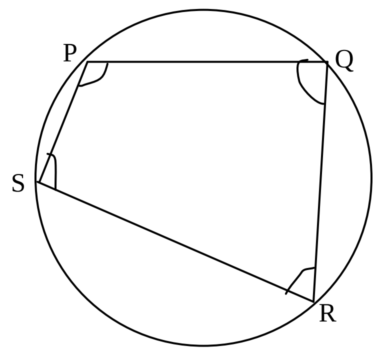
Presentation

Step 1: Theorems

1. Angles in opposite segments are supplementary.

Step 2: Explanation

Explain with concrete object with cyclic quadrilateral.



$P + R = 180^0$

$S + Q = 180^0$

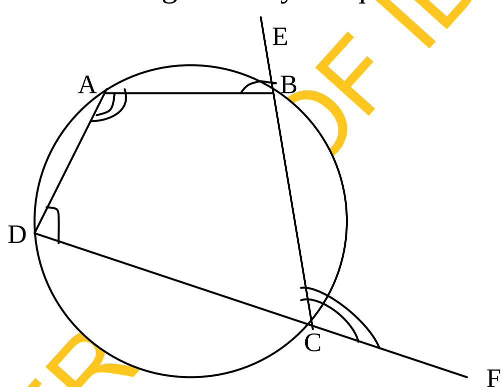
Step 3: Students' activities

The students were allowed to cut the structure showing opposite angles of a cyclic quadrilateral.

Step 4: Example

From the structure cut by the students, allow them to measure the opposite angles, sum it up and say the result.

(2) An exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.



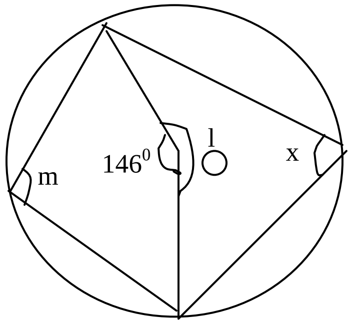
$\Rightarrow B\hat{A}D = BCF$

ADC = ABE

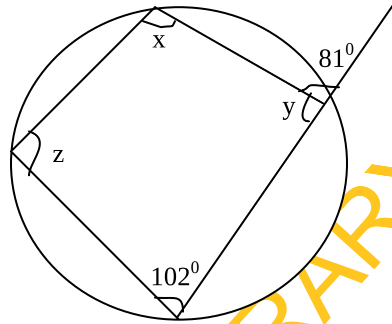
Step 5: Allow the students to draw the structure made and compare it with the diagram.

Step 6: Examples with diagram. Write down the sizes of the lettered angles in the following diagrams where O is given it is the centre of the circle.

(1)



(2)



Solution:

(1) $x = \frac{1}{2} X \, 146$ (Angle at center)

$= 73^0$

$l = 360^0 - 146^0$ (Angles at a point)

$= 214^0$

$m + x = 180^0$ (Opposite∠s of a cyclic quadrilateral)

$m = 180^0 - x$

$= 180^0 - 73^0$

$= 107^0$

(2) $y + 81^0$ (Angles on a straight line)

$y = 180^0 - 81^0$

$= 99^0$

$x + 102^0 = 180^0$ (Opposite angles of a cyclic quadrilateral)

$x = 180^0 - 102^0 = 78^0$

$z = 81^0$ (Exterior angle of a cyclic quad. equal interior opposite ∠)

$\therefore \quad x = 78^0, \quad y = 99^0, z = 81^0$

Step 7: Give another example without diagram.

(3) From a cyclic quadrilateral ABCD, A is opposite C and B opposite D and line DC is produced to E to form exterior angle BCE = 35^0. Find angles A, and B if D = 100^0.

Solution:

DCB = $180^0 - 35^0$ (Angles on a straight-line)

$\therefore$ DCB = 145^0

$A + C = 180^0$ (Opposite angles of a cyclic quadrilateral)

$A + 145 = 180^0$

$\therefore\ A = 180^0 - 145$

$= 35^0$

$B + 100^0 = 180^0$ (Opposite $\angle s$ of cyclic quadrilateral)

$B = 180^0 - 100^0$

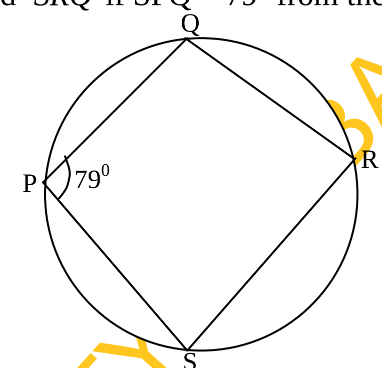
$= 80^0$

Summary: Opposite angles of a cyclic quadrilateral are added up to 180^0. Also, the exterior angle of a cyclic quadrilateral equal interior opposite angle.

Evaluation: Ask students the following questions.

1. Opposite angles of a cyclic quadrilateral are supplementary? True/False
2. What can you say about the exterior angle of a cyclic quadrilateral?

Assignment: Find $S\hat{R}Q$ if SPQ = 79^0 from the following diagram.



(a) 79^0 (b) $39^1/_2{}^0$ (c) 101^0 (d) 89^0

Period 2

Topic: Tangent to a Circle

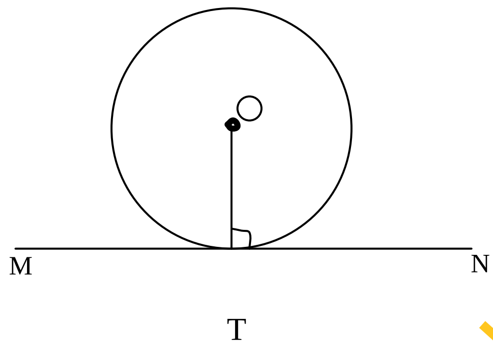
Introduction: Display the materials that contain tangent to a circle.

Examination of the materials: Students to look and touch the material.

Presentation:

Sept 1: Theorems

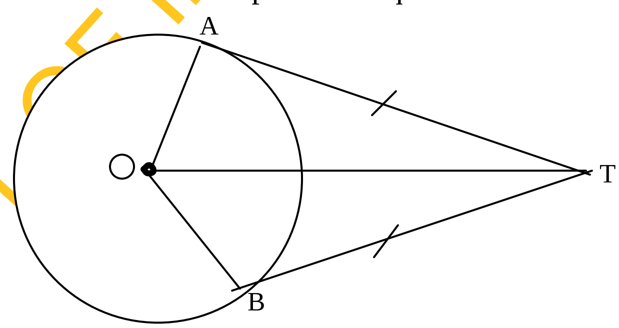
1. A tangent to a cycle is perpendicular to the radius drawn to its point of contact.
2. The perpendicular to a tangent at its point of contact passes through the centre of the circle.



Step 2: Explanation

Explain with concrete object showing tangent to a circle. Also explain that the tangent does not cut the circle, it touches the circle. The line $\overline{MN}$ is said to be a tangent to the circle.

3. The tangents to a circle from an external point are equal.



$\overline{TA} = \overline{TB}$

Step 3: Students' activities

The students were allowed to cut the structure showing tangent to a circle as well as tangents from an external point. The cutting was done group by group.

Step 4: Examples

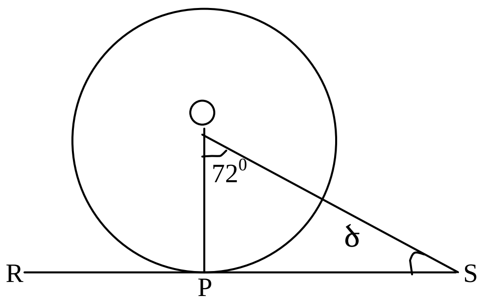
From the structure cut, allow them to measure (i) the angles equivalent to MTO and NTO from theorem 2 above. (ii) Measure lines equivalent to $\overline{AT}$ and $\overline{\mathrm{BT}}$ from theorem 3 above.

Step 5: Allow them to draw the structure made.

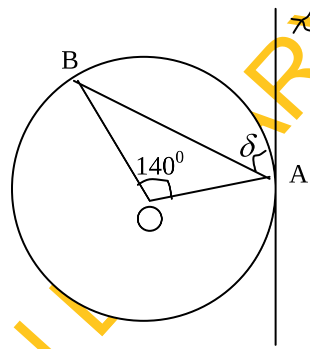
Step 6: Examples with diagram.

Question: Find the size of the angle from the following diagrams.

(1)



(2)



Solution:

(1) $O\hat{P}S = 90^0$ (Line from centre forms a $\perp$ line with the tangent)

$\therefore 90^0 + 72^0 + \delta = 180^0$ (Sum of angles of a triangle)

$162^0 + \delta = 180^0$

$\delta = 180^0 - 162^0$

$= 18^0$

$\therefore \delta = 18^0$

(2) $\overline{OA} = \overline{OB}$ (radii)

$O\hat{B}A = O\hat{A}B$ (Base angles of isosceles)

$\therefore O\hat{B}A + O\hat{A}B + 140^0 = 180^0$ (Sum of angles of a triangle)

$O\hat{B}A + O\hat{A}B = 180^0 - 140^0 = 40^0$

But $O\hat{B}A = O\hat{A}B$

$\Rightarrow 2O\hat{A}B = 40^0$

$OAB = \dfrac{40^0}{2}$

$= 20^0$

$OAB = 20^0$

$\Rightarrow OBA = 20^0$

OAB + BAX = 90^0 (Line from centre)

OAB + $\delta = 90^0$

$20^0 + \delta = 90^0$

$\delta = 90^0 - 20^0$

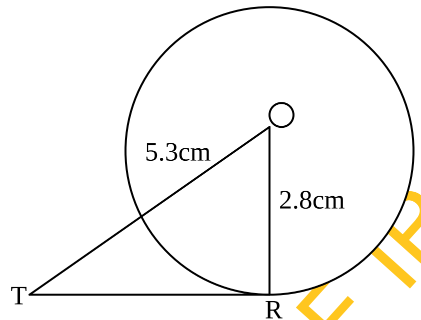
$= 70^0$

Step 7: Another example is given without diagram.

Question: The tangent from a point T touches a circle at R. If the radius of the circle is 2.8cm and T is 5.3cm from the centre, calculate $\overline{TR}$.

Solution:

The teacher translates the question to diagram before solution.



$(TR)^2 = (5.3)^2 — (2.8)^2$ (Pythagoras)

$(TR)^2 = 28.09 — 7.84$

$= 20.25$

$TR = \sqrt{20.25}$

$= 4.5cm$

Summary: The tangent does not cut the circle, it touches the circle. A tangent to a circle is perpendicular to the radius drawn to its point of contact. Also, the perpendicular to a tangent at its point of contact passes through the centre of the circle.

Evaluation: Ask the following questions:

1. A tangent to a circle ___the circle? Cut/touches.
2. The perpendicular to a tangent at its point of contact passes through ___ of the circle.

Assignment: Two circles have the same centre and their radii are 15cm and 17cm. A tangent to the inner circle at P cuts the outer circle at Q. Calculate $\overline{PQ}$.

Period 3

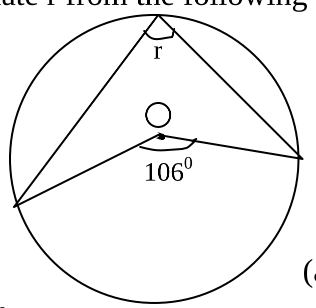
Practice Exercises on Circle Geometry

Step 1: The following questions are given to students as class exercise.

Pick the correct answer from the following options.

1. The radius of a circle is 10cm. The length of a chord of the circle is 16cm. Calculate the distance of the chord from the centre of the circle. (a) 5cm (b) 6cm (c) 7cm (d) 8cm

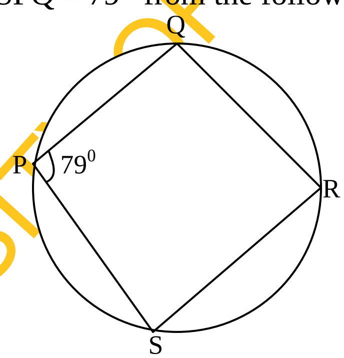
2. Calculate r from the following diagram



(a) 53^0 (b) 63^0 (c) 35^0 (d) 36^0

(3) A 216^0 sector of a circle of radius 5cm is bent to form a cone. Find the radius of the cone. (a) 3cm (b) 5cm (c) 6cm (d) 7cm

(4) Find $S\hat{R}Q$ if $S\hat{P}Q = 79^0$ from the following diagram.

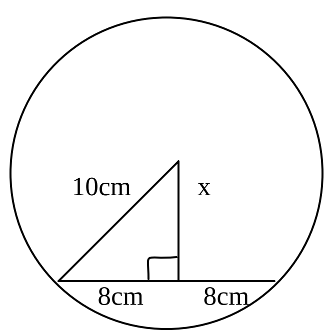


(a) 79^0 (b) $39^1/_2{}^0$ (c) 101^0 (d) 89^0

Step 2: Supervise their work and mark.

Step 3: Solution:

(1)



$x^2 = 10^2 - 8^2$ (Pythagoras)
$x^2 = 100 - 64$
$x^2 = 36$
$x = \sqrt{36}$
$= 6cm$

(2) $2r = 106^0$ (Angles at centre twice that at the circumference)

$r = \frac{106}{2}$

$=53^0$

(3) Area of a sector $= \frac{\theta}{360} X\pi\, r^2,\ r = 5cm$

Curve surface of cone $= \pi\, rl,\ l = 5cm$

$\Rightarrow$

$\pi rl = \frac{\theta}{360} X\pi\, r^2$

$\pi rl = \frac{216 X\pi\, r^2}{360}$

$360^0 X\pi\, rl = 216\ \pi\ X\ 5\ X\ 5$

$r = \frac{216\pi\ X\ 5\ X\ 5}{\pi\ X\ 5\ X\ 360^0}$

$= \frac{216 X 5}{360}$

$= \frac{1080}{360^0}$

$= 3cm$

(4) SRQ + SPQ = 180^0 (Opposite angles of a cyclic quadrilateral)

SPQ = 79^0

SRQ + 79^0 = 180^0

$\therefore SRQ = 180^0 - 79^0$

$= 101^0$

Step 4: Allow them to ask questions.

WEEK 3

Topic: Volume of Solids

Period 1: Prism– Volume of Cube

Introduction: Different materials like sugar box, sugar cubes, matches box, magi cubes, textbooks, chalk box, cardboard and scissors were displayed in class.

Examination of the materials: Pass the materials round the class for students to touch, name, and compare. Different parts of the concrete materials were identified with the help of the instructors, the faces, edges and vertices.

Presentation

Step 1: Definition

Point: A point is one of the basic terms in geometry. It is an entity that has only one characteristic, that is, it has position. A point has no length or width, it just specifies an exact location.

B•

A•

C•

Point A, B & C

Straight line: A straight line is the shortest distance between two points. A straight line has one dimension which is length.

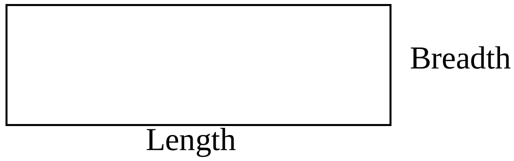
Prism is a solid which has a uniform cross-section. It is a solid that has two congruent parallel bases that are polygons. Examples of prism are cubes, cuboids, cylinder, and pyramid. All prisms are 3 dimensional shapes.

Step 2: Explanation

Here is a line, it has got one dimension which is length (ID).

Length

The following is a rectangle, it has got two dimensions length and breadth, so it is 2D shape.

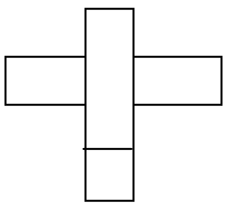


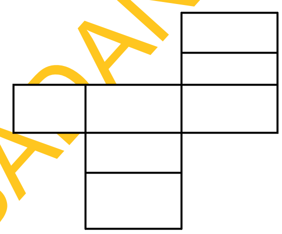
Examples of 2D shapes are squares, circles, triangles, parallelograms and all the polygons. 2D shapes have area but no depth.

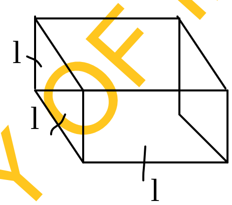
3D shapes have length, breadth and depth, so all 3D shapes have volume. Examples of 3D shapes are cube, cuboid, triangular, prism, cone, and pyramid.

Step 3: Students' activities

Guide them on how to make cube with cardboard. A cube is a 3D shape with square faces.







Step 4: Example

The volume of a prism is given by the product of the area of its base and its height.

Volume of prism = Area of base x height

Volume of cube = length x length x length

$= l \times l \times l$

$= l^3$

Height = length = breadth

Area of base = $l \times l = l^2$

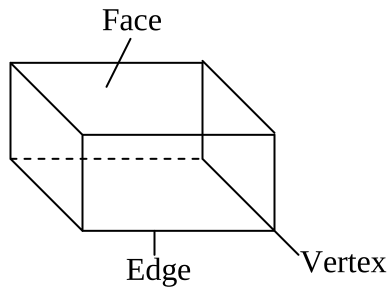
Volume of cube = Base area x height

$= l^2 \times l$

$= l^3$

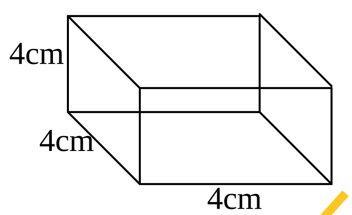
Step 5: Drawing of the material (representation)

Allow the students to draw the structure made in their note books; count the faces, edges and vertices. Identify the base and compare the drawing part with concrete material by labelling.



Step 6: Examples with diagram

1. Find the volume of a cube below



Solution:

Volume = (l x l x l) unit cube

= (4 x 4 x 4) unit cube

= $64cm^3$

Step 7: Another example is given without diagram

2. Calculate the volume of a cube whose edges are 8cm each

Solution:

l = 8cm, b = 8cm, h = 8cm

Volume of cube = (8 x 8 x 8) cm^3

= $512cm^3$

Summary: A prism is a solid with uniform cross section. A cube is a 3D shape. It has faces, edges and vertices.

Evaluation: Allow them to solve the following questions in class.

1. Find the volume of a cube with sides 70cm.
2. If the volume of a cube is 64cm. Find the sides of the cube.

Assignment: Give the following questions as assignment.

1. Find the volume of a cube with sides 25cm.
2. If the volume of a box in the shape of a cube is 2,197cm^3 find the length of the edges.

Period 2

Topic: Volume of Cuboid

Presentation of materials: Display matches box in class and other materials with cuboid shape.

Examination of the material: Distribute the matches' boxes to students.

Presentation:

Step 1: Definition

A cuboid is a 3D shape with rectangular faces.

Step 2: Matches box has a cuboid shape.

Step 3: Students' activities

Use cardboard to cut cuboid shape and allow them to do the same as assignment.

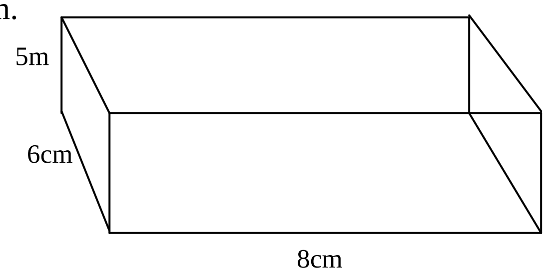
Volume of cuboid = (l x b x h) unit cube

$\Rightarrow$ Base area x height

Where Base area = l x b

Step 4: Examples

1. Find the volume of a cuboid with the following dimensions, length 8cm, breadth 6cm and height 5cm.



Solution:

Volume of cuboid = (l x b x h)

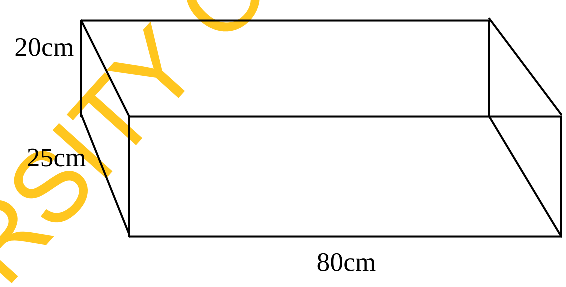
= (8 x 6 x 5) cm^3

= $240cm^3$

Step 5: Allow them to draw cuboid shape and relate vertices, edges and faces of the matches' box with the drawing.

Step 6: Examples with diagram.

2. A car petrol tank is 0.8m long, 25cm wide and 20cm deep. How many litres of petrol can it hold?



Solution:

Volume of tank = 0.8m x 25cm x 20cm

0.8m — cm

$\Rightarrow 1m = 100cm$

$0.8m = \frac{100}{1} X 0.8$

= 80cm

Volume 80cm x 25cm x 20cm

$= 40{,}000cm^3$

$1000cm^3 = 1$ litre

Capacity of tank = $\frac{40{,}000}{1{,}000} = 40\,litres$

Step 7: Questions without material and drawing

3. Find the volume of a tank with length 30cm breadth 20cm and height 15cm.

4. Calculate the volume of the rectangular solid with the dimension 4cm length by 3cm breadth and 2cm height?

Solution:

3. Volume = l x b x h

$= (30 \times 20 \times 15)\ cm^3$

$= 9000cm^3$

4. Volume = l x b x h

$= (4 \times 3 \times 2)\ cm^3$

$=24cm^3$

Summary: Cuboid is also a 3 dimensional figure. It has faces, edges and vertices. It is an example of prism.

Evaluation: Solve the following questions:

Find the volume of a box with length 15cm, breadth 6cm and height 4cm.

Assignment: The following questions are given as assignment.

1. Find the volume of a cuboid with height 12cm, breadth 6cm and length 8cm.
2. Find the volume of a cuboid with 3cm by 4cm by 6cm.

Period 3

Topic: Volume of Triangular Prism

Presentation of materials: Display material with the shape of triangular prism in class.

Examination of the material: Pass the material round the class for students to identify.

Presentation:

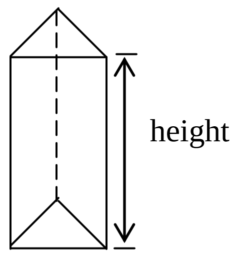
Step 1: Definition:

A triangular based prism is a prism with a triangle-shaped base.

Step 2: The volume of a prism is given by the product of the area of its base and its height.

Step 3: Activities

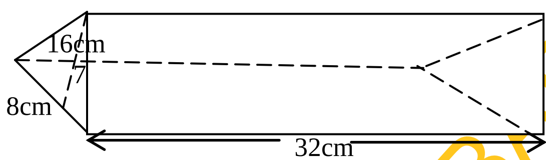
Use cardboard to cut a triangular based prism



Volume = Area of base x height

Step 4: Example with concrete material

(1) Find the volume of a triangular prism below



Solution:

Volume of prism (Area of triangular base x height)

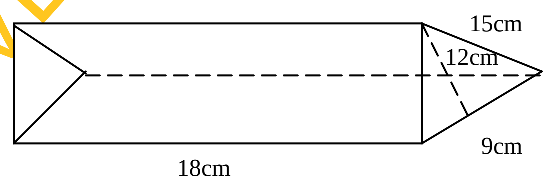
$$\frac{1}{2} X \frac{8}{1} X \frac{16}{1} X\, 32$$

$$= 2048 cm^3$$

Step 5: Allow the students to draw the shape, triangular prism in their note books.

Step 6: Example with diagram

(2)



Find the volume of the above shape.

Solution: $Vol. = \frac{1}{2} \times \frac{12}{1} \times \frac{9}{1} \times \frac{18}{1} = 972 cm^2$

(3) Find the volume of a triangular prism with a triangle of base 9.3cm, height 8.1 and the length 14.4cm

Solution:

Volume of prism = Base area X Height

$$= \frac{1}{2} X \frac{9.3X8.1X14.4}{1}$$

$$= 542.376cm^3$$

Summary: A prism is a solid of uniform cross section.

Volume of prism = Area of cross section x Height

A right triangular prism is a solid having a uniform triangular cross section.

Volume = Area of triangular base x Length or height of the prism

Evaluation: Ask the following questions?

1. What is a prism?
2. How do we find the volume of a prism?
3. Find the volume of the following triangular prism with a right-angled triangular 3cm by 5cm, length 6cm.

Assignment: A right prism of length 10cm has as its cross-section equilateral triangle of side 6cm. Calculate its volume.

WEEK 4

Topic: Volume of Solids

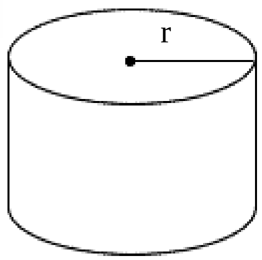
Period 1: Volume of Cylinder

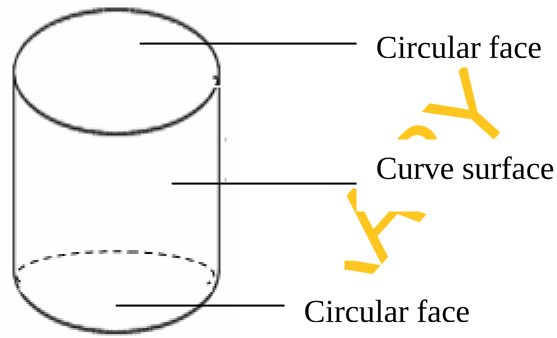
Introduction: Materials like tin of milk, tin of Milo, cone made of cardboard scissors were displayed in class.

Examination of the materials: Pass the materials round the class for students to touch and examine.

Presentation:

Step 1: A cylinder is a solid with two congruent circles joined by a curved surface.





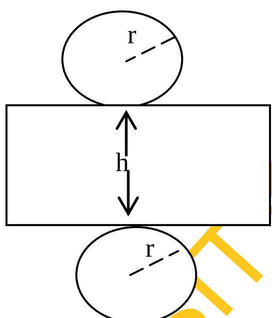
Step 2: The volume of the cylinder is the area of the base x height.

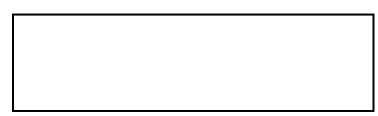
Volume of cylinder = $\pi r^2 h$

Step 3: Activities

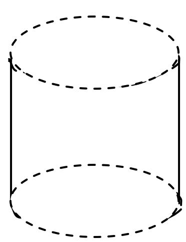
Use cardboard to cut a cylindrical shape. Also, the students to do the same against next class.

Thus:





Roll the shape above to make a cylindrical surface, then insert the two circular shapes.



Step 4: Example with the use of concrete material.

1. Find the volume of a cylinder of radius 43.4cm and length 550cm.

Solution

Volume of cylinder = $\pi r^2 h$

r = radius, h = height

r = 43.4cm, h = 550cm, $\pi = \frac{22}{7}$

$$Volume = \frac{22}{7} X (43.4)^2 X 550$$

$$= \frac{22}{7} X 43.3 X 43.4 X 550$$

= (22 x 6.2 x 43.4 x 550) cm^3

= (12100 x269.08) cm^3

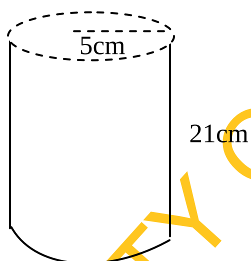
= 3255868cm^3

= 3.26 x 10^6 cm^3

Step 5: Allow them to draw the shape in their note.

Step 6: Example with diagram.

2. Find the volume of the cylinder below.



Solution

$$Volume \text{ of cylinder} = \pi r^2 h$$

$$= \frac{22}{7} X \frac{5}{1} X \frac{5}{1} X \frac{21}{1}$$

$$= 22 X 25 X 3$$

$$= 1{,}650 cm^3$$

Step 7: Give another example without diagram.

3. A cylindrical container, closed at both ends, has a radius of 7cm and height 5cm. Find the volume of the containers $\left(\pi = \frac{22}{7}\right)$.

Solution

$Volume \text{ of cylinder} = \pi r^2 h$

$\pi = \frac{22}{7};\ r = 7, h = 5$

$\therefore Vol = \frac{22}{7} X 7 X 7 X 5$

$= 154 X 5$

$= 770cm^3$

Summary: A cylinder is a solid with two congruent circles joined by a curved surface.

$Volume \text{ of cylinder} = \pi r^2 h$

Evaluation: (1) A cylinder has how many circular faces?

(2) What is the formula for finding the volume of a cylinder?

Assignment: Find, correct to 1 decimal place, the volume of a cylinder of height 8cm and base radius 3cm $(Take\, \pi = 3.142)$.

Period 2

Topic: Volume of frustum of cone

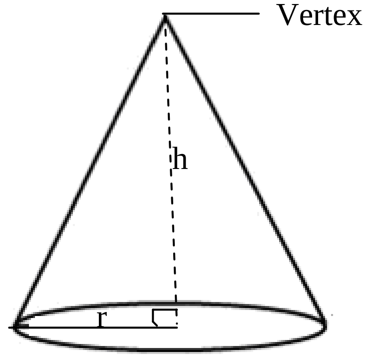
Introduction and Presentation of materials: Display materials with the shape of cone and frustum of come in class.

Examination of the materials: Allow students to examine the shapes and name the materials with the shape.

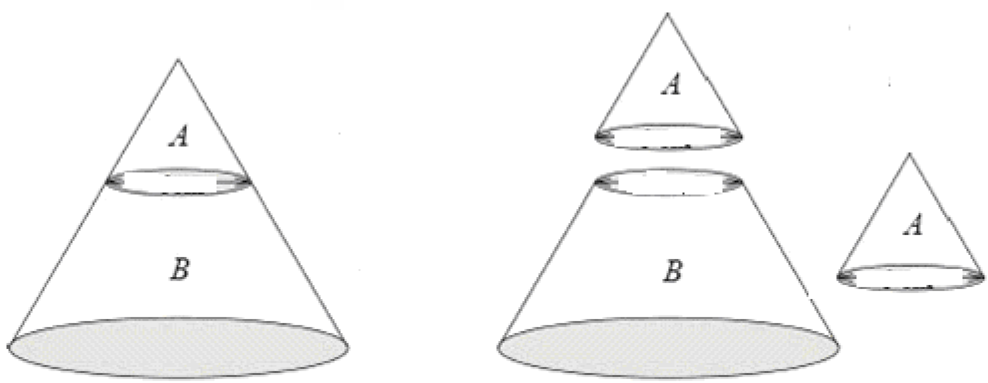
Presentation:

Step 1: A circular cone has a circular base, which is connected by a curved surface to its vertex. A cone is called a right circular cone, if the line from the vertex of the cone to the centre of its base is perpendicular to the base. A cone is also referred to as circular based pyramid.

Cone



Frustum of cone



Step 2: Volume of cone = $\frac{1}{3}\pi r^2 h$

Volume of frustum of cone =

Volume of big cone – Volume of small cone

$$\frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h$$

Or

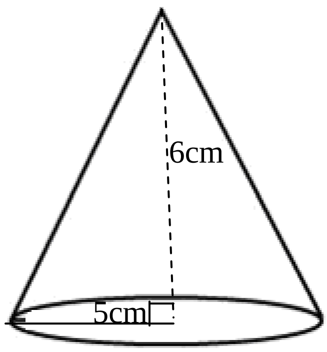
Volume of frustum of cone = $\frac{1}{3}\pi\, h\,\left(R^2 + r^2 + Rr\right)$

Step 3: Activities

From the concrete material with cone shape provided, cut off small cone, the remaining part is frustum of cone.

Step 4: Example with the use of concrete material.

Calculate the volume of cone with base radius 5cm and height 6cm



Solution:

$$\textit{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} X \frac{22}{7} X \frac{5}{1} X \frac{5}{1} X \frac{6}{1}$$

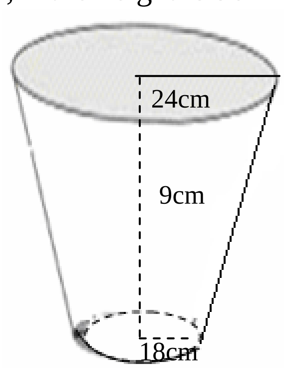
$$\frac{1{,}100}{7}$$

$$= 157.14cm^3$$

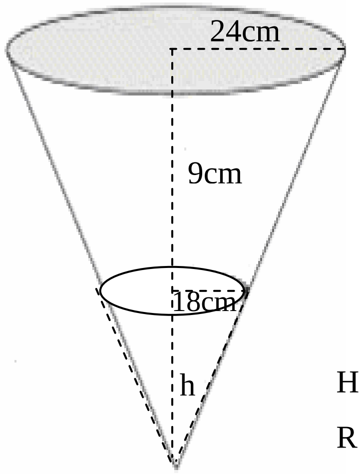
Step 5: Allow them to draw the shape in their notes.

Step 6: Example with diagram

2. A bucket inform of a frustum has upper and lower radii as 24cm and 18cm respectively, if the height is 9cm find the volume of the bucket in terms of π.



Solution



H = 9 + h

R = 24cm

r = 18cm

h = ?

$$\frac{H}{h} = \frac{R}{r}$$

$$\frac{9+h}{h} = \frac{24}{18}$$

4h = 3 (9 + h)

4h = 27 + 3h

4h – 3h = 27

$$\therefore H = 9 + 27 = 36cm$$

$\therefore$ *Volume* of bucket = Volume of big - Volume of small

$$= \frac{1}{3}\pi \mathrm{R}^2 H - \frac{1}{3}\pi \mathrm{r}^2 h$$

$$= \frac{1}{3}\pi \left(\mathrm{R}^2\mathrm{H} - \mathrm{r}^2 h\right)$$

$$= \frac{1}{3} X\pi \left(20^2 \mathrm{X}36 - 18^2 \mathrm{X}27\right)$$

$$= \frac{1}{3}\pi \left(20737 - 8748\right)$$

$$= \frac{1}{3}\pi \left(11988\right)$$

$$= 3996\pi cm^3$$

Step 7: Another example is given without diagram.

3. Calculate the volume of a cone with base diameter 10cm and height $2^1/_2$cm

Solution:

$$Volume \text{ of cone} = \frac{1}{3}\pi r^2 h \qquad r = \frac{10}{2} = 5cm$$

$$= \frac{1}{3} X \frac{22}{7} X 5 X 5 X 2\frac{1}{2}$$

$$= \frac{1}{3} X \frac{22}{7} X 5 X 5 X \frac{5}{2}$$

$$= \frac{1375}{21} = 65.47$$

$$= 65.47cm^3$$

Summary: Volume of cone $= \frac{1}{3}\pi r^2 h$ and Volume of frustum of cone =

Volume of big cone – Volume of small cone

Or

Volume of frustum of cone = $\frac{1}{3}\pi h \left(R^2 + r^2 + Rr\right)$

Evaluation: (1) What is the formula for finding the volume of cone?

(2) What is the formula for finding the volume of frustum of cone?

Assignment: Calculate the volume of cone with base radius 3.5cm and height 15cm

Period 3

Topic: Volume of Pyramid

Introduction and Presentation of materials: Display material with pyramid shape in class.

Examination of the material: Allow students to touch and study the shape.

Presentation:

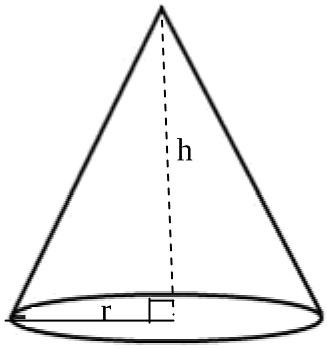
Step 1: Volume of a pyramid

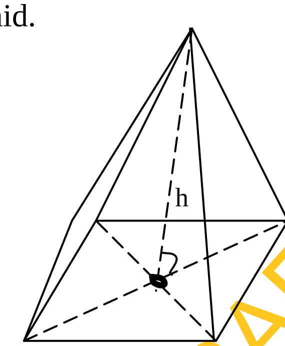
A pyramid is a solid with a polygon base which meets at common point called the apex or vertex.

Different types of pyramid are: cone (circle based-pyramid), square-based pyramid, rectangular-based pyramid and triangular based pyramid.

Step 2:

Circular-based pyramid





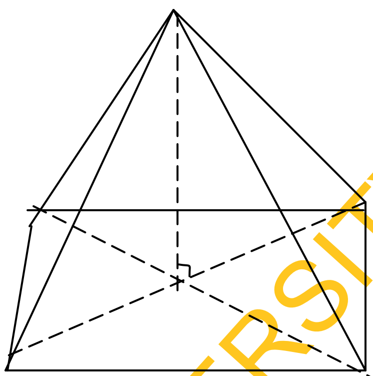
Square-based pyramid

$$Volume = \frac{1}{3}\pi r^2 h$$

$Volume = \frac{1}{3}Ah$ r = radius

$= {}^1/_3\, l^2$

h= height A = area, l = length, h=height



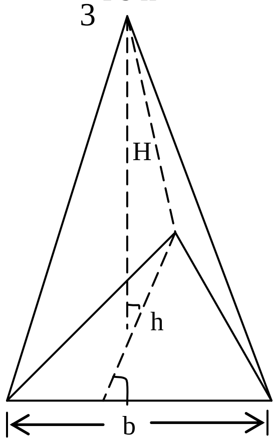
Rectangular-based pyramid

$Volume = \frac{1}{3}Ah$ where A = area

h = height

$= \frac{1}{3} l\, b\, h$ where l = length

b = breadth
h = height



$$Volume = \frac{1}{3} Ah$$

$$= \frac{1}{3}\left(\frac{1}{2} bh\right)H$$

$$= \frac{1}{6} bhH$$

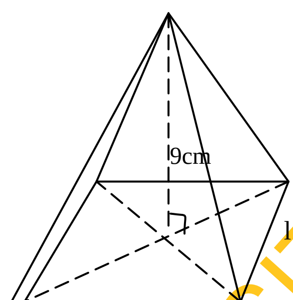
Triangular-based pyramid

Step 3: Activities

Show them how to cut the pyramid and ask each group to cut one against next class.

Step 4: Example with the use of materials.

1. A pyramid holding by the teacher has a square base of side 4cm and height of 9cm. Find its volume.



Volume of a square based pyramid

Solution:

$$Volume = \frac{1}{3} Ah$$

$$= \frac{1}{3} \; l^2 h$$

$$= \frac{1}{3} X\, 4 X\, 4 X\, 9$$

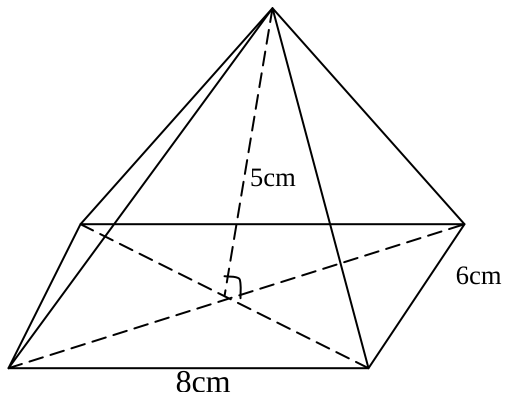
$$= 16 \text{ X } 3$$

$$= 48\text{cm}^3$$

Step 5: Allow them to draw the shape.

Step 6: Example with diagram. Volume of a rectangular-based pyramid

2. Find the volume of a rectangular-based pyramid whose base is 8cm by 6cm and height is 5cm.



$$Volume = \frac{1}{3}Ah$$

$$= \frac{1}{3} l \ b \ h$$

$$= \frac{1}{3} X \frac{8}{1} X 6 X 5$$

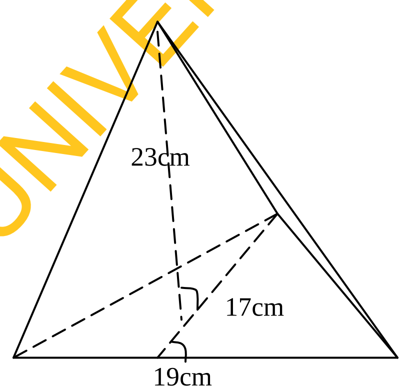
$$= 80cm^3$$

Example without diagram

Step 7: Volume of a triangular-based pyramid.

3. Find the volume of the triangular-based pyramid with base 19cm, height of the triangle 17cm and height of the pyramid is 23cm, correct the answer to 2 decimal places.

Solution:



$$Volume = \frac{1}{3} AH$$

$$= \frac{1}{3}\left(\frac{1}{2} bh\right) H$$

$$= \frac{1}{6} bhH$$

$$= \frac{1}{6} X 19 X 17 X 23$$

$$= 1238.166667 cm^3$$

$$= 1238.17 cm^3 \text{ to } 2 \text{ d.p.}$$

Summary: Volume of pyramid = $\frac{1}{3} Ah$

Different types of pyramid are: Circular-based pyramid (cone), square-based pyramid, rectangular-based pyramid, and triangular-based pyramid.

Evaluation: (1) What is a pyramid?

(2) What is the formula for finding the volume of a pyramid?

Assignment: (1) A pyramid has a square base of side 6cm and height of 15cm. Find its volume.

(2) Find the volume of a rectangular-based pyramid whose base is 10cm by 8cm and height 5cm.

Week 5

Topic: Angles of Elevation and Depression

Period 1: Angles of elevation

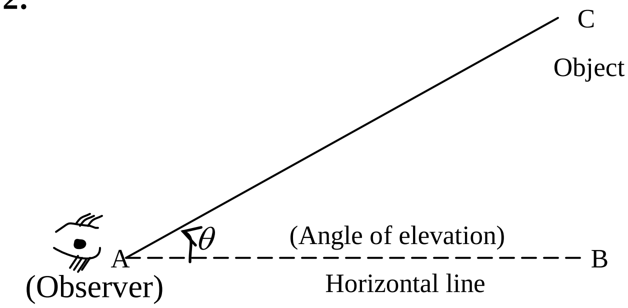
Introduction and presentation of materials (concrete): Display the materials showing angles of elevation and depression in class.

Examination of the materials: Allow students to move round and examine the materials.

Presentation:

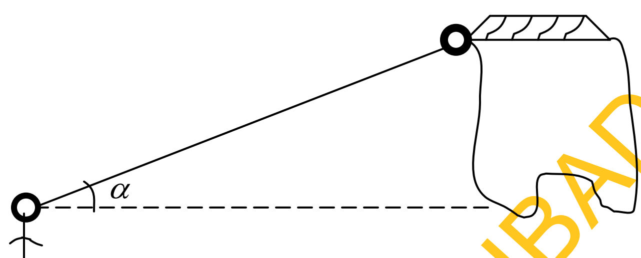
Step 1: Explain that the angle of elevation is the angle between a horizontal line from the observer and the line of sight to an object that is above the horizontal line.

Step 2:



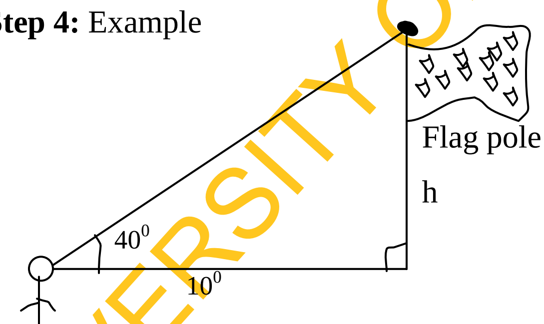
Step 3: Activities

Demonstrate how to observe the object on top of the roof and show them the angle of elevation by raising up the eyes and the initial eyes level.



α is the angle of elevation.

Step 4: Example



Find the height of the flagpole.

Solution

$$\tan 40^0 = \frac{h}{10}$$

$h = 10 \tan 40^0$

$= 10 \text{ X } 0.839$

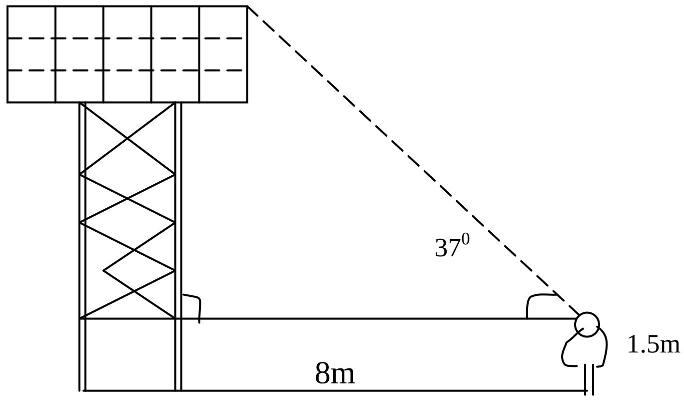
$= 8.39$

$= 8.4\text{m}$

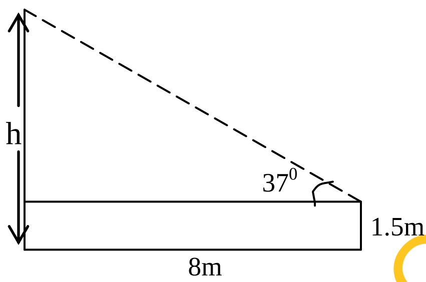
Step 5: Allow them to draw diagrams on angles of elevation.

Step 6: Example with diagram

2. Find the height of the water tower below to the nearest $^1/_2$m



Solution:



$$\tan 37^0 = \frac{h-1.5}{8}$$

$$\Rightarrow h - 1.5 = 8\tan 37^0$$

$$= 8 \text{ X } 0.7534$$

$$= 6.028$$

h -1.5 = 6.028

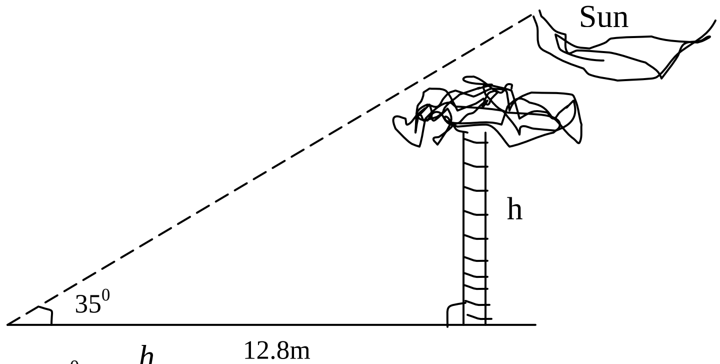
h = 6.028 + 1.5

= 7.5284

= 7.5m

Step 7: The angle of elevation of the sun is 35^0. A tree has a shadow 12.8m long. Find the height of the tree.

Solution:



$$\tan 35^0 = \frac{h}{12.8}$$

$$h = 12.8 \tan 35^0$$
$$= 12.8 \text{ X } 0.7002$$
$$= 8.96$$
$$= 9\text{m}$$

Summary: The angle of elevation of an object as seen by an observer is the angle between the horizontal and the line from the object to the observer's eye (the line of sight).

Evaluation: How do we find tangent of an angle?

Assignment: The angle of elevation of the top of a tower as seen by a man 5cm away from the foot of the tower is 37^0. The angle of elevation of the top of the tower as seen by a second man standing on the opposite side of the tower is 49^0 how far are the men?

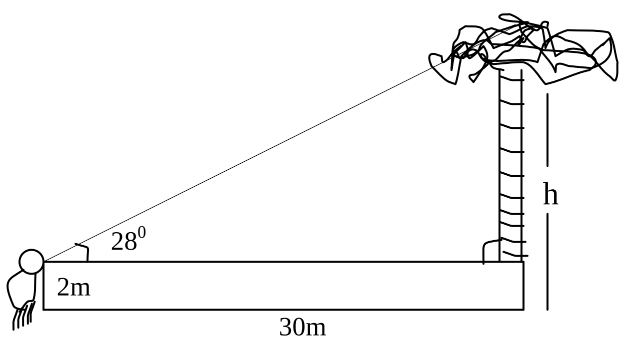
Period 2

More examples on angles of elevation

Examples

1. A man who is 2m tall stands on horizontal ground 30m from a tree. The angle of elevation of the top of the tree from his eyes is 28^0. Find the height of the tree.

Solution:



$$\tan 28^0 = \frac{h-2}{30}$$

$$h - 2 = 30 \tan 28^0$$

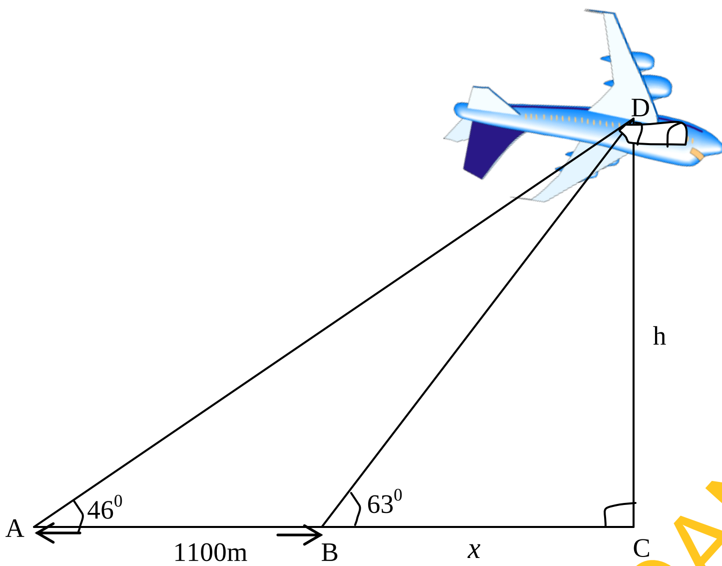
$$= 30 X 0.5317$$

$$h - 2 = 15.95$$

$$h = 15.95 + 2$$

$$= 17.95m$$

2. The following figure shows the angle of elevation of an aircraft from two points 1100m, apart. Find the height of the aircraft above the ground to the nearest 100m.



Let BC = x

$$\tan 46^0 = \frac{h}{1100+x}$$

$$h = (1100+x)\ \tan 46^0 \qquad \text{.......................}(i)$$

$$\tan\ 63^0 = \frac{h}{x}$$

$$h = x \tan 63^0 \qquad \text{..........................}(ii)$$

Since h = h

Equate equations (i) and (ii)

$\Rightarrow (1100 + \ x)\tan 46^0 = x\ \tan 63^0$

$1100 \tan 46^0 + x \tan 46^0 = \ x \tan 63^0$

$1100 \tan 46^0 = x \tan 63^0 — x \tan 46^0$

$1100 \tan 46^0 = \ x(\tan 63^0 — \tan 46^0)$

$$x = \frac{1100\ \tan 46^0}{\tan 63^0 \text{ - } \tan 46^0}$$

$$= \frac{1100 X 1.0355}{1.9626 - 1.0355}$$

$$= \frac{1139.08}{0.9271}$$

$\therefore x = 1228.64$

From equation (i)

h = (1100 + x) tan 46^0

= (1100 + 1228.64) 1.0355

=2328.64 x 1.0355

= 2411.306

To the nearest 100m

$\Rightarrow$ h = 2400m

Period 3

Practice exercise on angles of elevation

Allow them to practice the following questions in class.

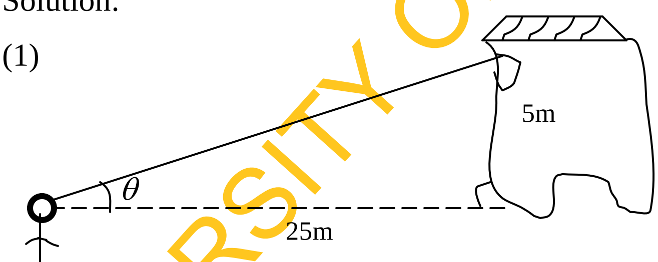
1. A man standing 25m away from a building observes the window of the house 5m above the ground. Calculate the angle of elevation of the window from the man.

2. The angle of elevation of the top of a tower from a point 42cm away from its base on level ground is 36. Find the height of the tower.

3. The angle of elevation of the sun is 27^0. A man is 180cm tall. How long is his shadow? Give your answer to the nearest 10cm.

Solution:

(1)



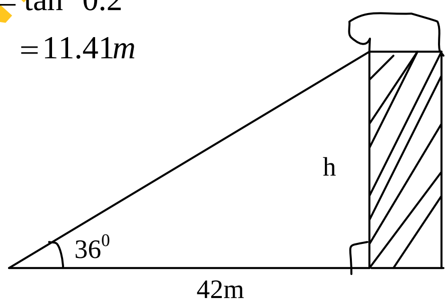
$$\tan\ \theta = \frac{5}{25}$$

$$\tan\ \theta = 0.2$$

$$\theta = \tan^{-1} 0.2$$

$$= 11.41m$$

(2)



$$\tan 36^0 = \frac{h}{42}$$

$$h = 42 \tan 36^0$$

$$h = 42 \text{ X } 0.7265$$

$$= 30.5\text{m}$$

(3) $\tan 27^0 = \frac{180}{x}$

$$x = \frac{180}{\tan 27^0} = \frac{180}{0.509}$$

$$x = 353.6$$

to the nearest 10cm

= 350cm

Week 6

Topic: Angles of Elevation and Depression

Period 1: Angles of Depression

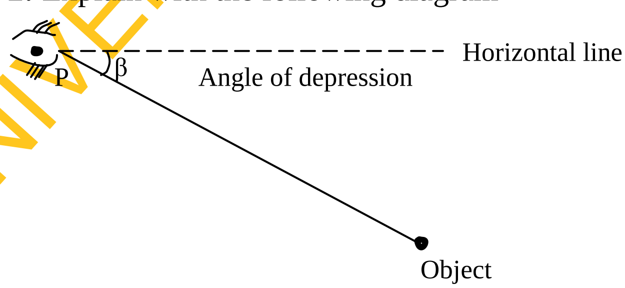
Introduction and Presentation of materials: Display the materials showing angles of depression.

Examination of the materials: Allow students to move round and examine the materials.

Presentation:

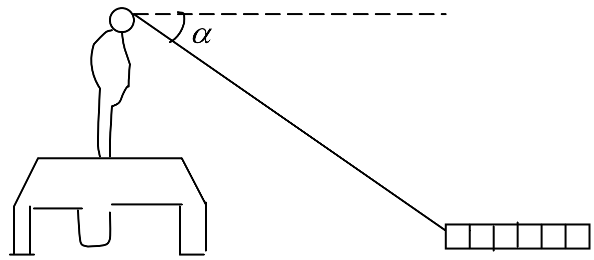
Step 1: The angle of depression is the angle between a horizontal line from the observer and the line of sight to an object that is below the horizontal line.

Step 2: Explain with the following diagram



Step 3: Activities

Demonstrate how to observe the object on the floor while climbing table.

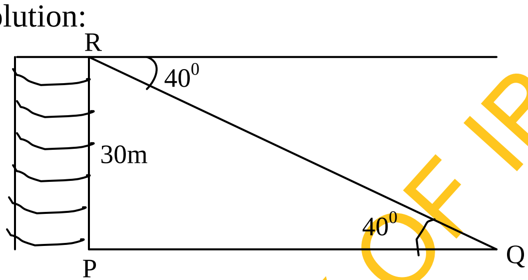


α is the angle of depression.

Step 4: Example

1. The angle of depression of a point Q from a vertical tower PR, 30m high is 40. If the foot P of the tower is on the same horizontal level as Q. Find correct to 2 decimal; places /PQ/.

Solution:



$$\tan 40^0 = \frac{30}{y}$$

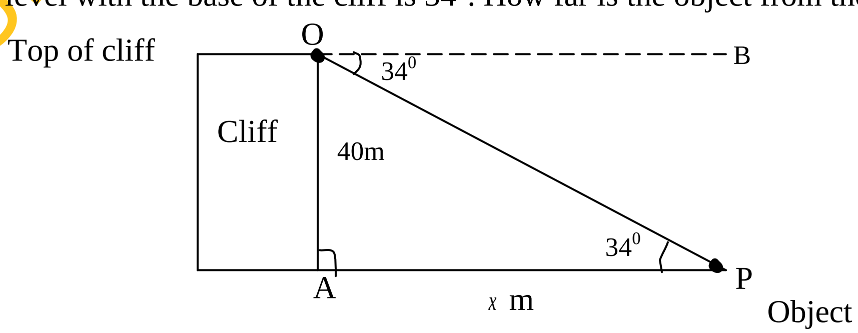
$$y = \frac{30}{\tan 40^0} = \frac{30}{0.8390}$$

$$= 35.75m$$ PQ = 35.75

Step 5: Allow them to draw diagrams on angles of depression.

Step 6: Example with diagram

2. From the top of a vertical cliff 40m high, the angle of depression of an object that is level with the base of the cliff is 34^0. How far is the object from the base of the cliff?



Solution:

$A\hat{P}O = B\hat{O}P$ (*Alternate* angles)

$A\hat{P}O = 34^0$

$\tan 34^0 = \frac{40}{x}$

Multiply both sides by x

$x \tan 34^0 = 40$

$0.6745x = 40$

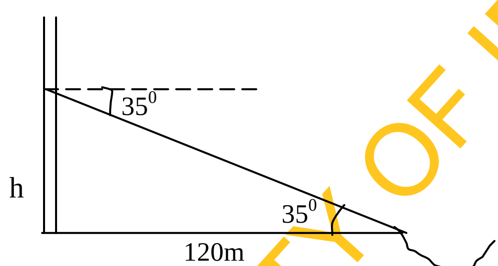
$x = \frac{40}{0.6745}$

$= 59.30m$

$\therefore$ The object is 59.30m from the base of the cliff.

3. The angle of depression of a boat from the mid-point of a vertical cliff is 35^0. If the boat is 120m from the foot of the cliff, calculate the height of the cliff.

Solution:



$\tan 35^0 = \frac{h}{120^0}$

$h = 120^0 \times \tan 35^0$

$= 120^0 \times 0.7002$

$= 84.02$

$\therefore 2h = 84.02 \times 2$

$= 168m$

Summary: If the object is below the level of the observer, then the angle between the horizontal and the observer's line of sight is called the angle of depression.

Evaluation: What is angle of depression?

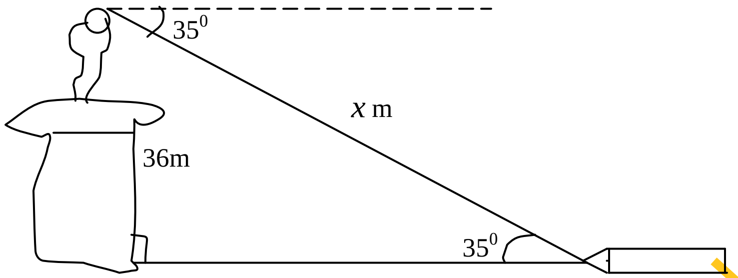
Assignment: From the top of a building 50m high, the angle of depression of a car is 55^0. Find the distance of the car from the foot of the building?

Period 2

More examples on angles of Depression.

1. A girl on top of a storey building 36m high dropped a pencil on the ground below. If the angle of depression of the pencil is 36^0, how far is the pencil away from the girl?

Solution



$$Sin \quad 35^0 = \frac{36}{x}$$

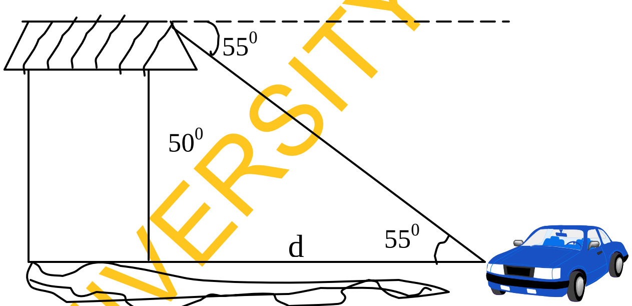
$$= \frac{36}{Sin\ 35^0} = \frac{36}{0.5735}$$

$$= 62.77m$$

2. From the top of a building 50m high, the angle of depression of a car is 55^0.

Find the distance of the car from the foot of the building.

Solution:



$$\tan 55^0 = \frac{50}{d}$$

$$d = \frac{50}{\tan 55^0} = \frac{50}{1.428}$$

$$= 35.01$$

$$= 35m$$

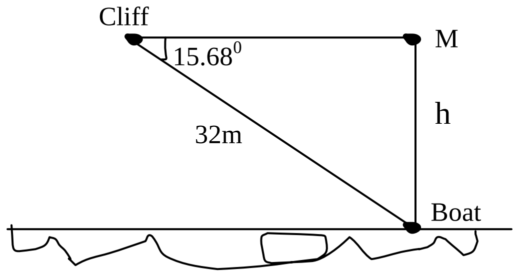
Period 3

Practice exercise on angles of depression

(1) The angle of depression of a boat as seen by a man on top of a cliff is 15.68^0. If the boat is 32m away from the man, what is the height of the cliff?

(2) Find the angle of depression from the top of a tower 50m high of an object on the ground distant 70m from the foot of the tower.

Solution

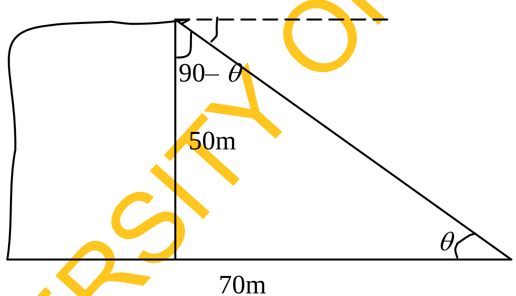


$$\frac{h}{32} = Sin\,15.68^0$$

$$\therefore h = 32\ Sin\,15.68$$

$$= 8.65m$$

(2)



$$\tan\theta = \frac{50}{70} = 0.7142$$

$$\theta = \tan^{-1} 0.7142$$

$$= 35.5^0$$

$$\cong 36^0$$

Assignment: From the top of a building 60m high, the angle of depression of a car is 45^0. Find the distance of the car from the foot of the building.

APPENDIX 2

EXPERIMENTAL GROUP 2

INSTRUCTIONAL GUIDE FOR EXPLICIT INSTRUCTIONAL STRATEGY (EIS)

(i) **General Information**

Subject: Mathematics

Topics: (i) Circles (ii) Volumes (iii) Angles of Elevation and Depression

Class: SS II

Sex: Male [ ] Female [ ]

(ii) **About the Procedure**

This guide was designed to teach the students in senior secondary school two the above topics. There are eighteen periods in the guide; the lessons were designed for 6 weeks of 3 periods each week.

(iii) **To the Teacher**

The guide was used by the teacher to teach the students. The teacher should follow the guide accordingly.

(iv) **General Objectives**

At the end of the lessons in this guide, students should be able to:

(i) define circle
(ii) name object with circular face
(iii) find the area and sector of a circle
(iv) work sums on chords and arcs of circles
(v) state some circle theorems
(vi) apply circle theorems when necessary
(vii) work sums on cyclic quadrilaterals
(viii) define prism with examples
(ix) find the volume of cube and cuboid

(x) identify triangular prism and find its volume

(xi) identify cylinder and find its volume

(xii) find the volume of pyramid

(xiii) differentiate between angles of elevation and depression.

(xiv) calculate angles of elevation and depression

(vi) **To the Students**

This learning guide is specially designed to teach you some topics within your school syllabus. Your total cooperation is very essential for your own benefit. The lesson will cover six weeks, if anything is not clear to you during the period of teaching, endeavour to ask questions.

Contents for each of the 6 Weeks

Week 1:

Topic: Circle Geometry

Period 1: Area and Arc of a Circle

2. Sectors and Chords of Circles

3. Circle Theorems

Week 2:

Topic: Circle Geometry

Period 1: Cyclic Quadrilaterals

2. Tangent to a Circle

3 Practice Exercises on Circle Geometry

Week 3:

Topic: Volume of Solids

Period 1: Volume of Solids (Prism)

2. Volume of Cube and Cuboids

3. Volume of Triangular Prism

Week 4:

Topic: Volume of Solids

Period 1: Volume of Cylinder

2. Volume and Frustum of Cone

3. Volume of Pyramid

Week 5:

Topics: Angles of Elevation and Depression

Period 1: Angles of Elevation

2. Angles of Elevation

3. Practice Exercises on Angles of Elevation

Week 6:

Topic: Angles of Elevation and Depression

Period 1: Angles of Depression

2. Angles of Depression

3. Practice Exercise on Angles of Depression

Experimental Group 2

Instructional Guide for Explicit Instructional Strategy (EIS)

Week 1

Topic: Circle Geometry

Period 1: Area and Arc of a circle

Review of previous knowledge: Ask the following questions, listen to their response and correct immediately.

1. What is a point?
2. What is a straight line?
3. Ask one student to turn round a point and then use chalk to join the student turning point. Then ask one student to measure any point from that turning arc to the center.

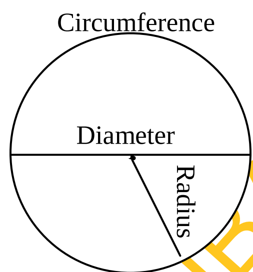
Presentation of the new topic: A circle is a closed curve in a plane. It is the locus of all points equidistant from a central point. The radius of a circle is the distance from the centre of the circle to the outside edge.

Step 1: Diameter of a circle

The diameter starts at one side of the circle, goes through the centre and ends on the other side. It is the longest distance across a circle, the diameter is twice the radius.

Diameter = 2 x radius

1. What is a circle?
2. Define radius of a circle?
3. Who can draw circle on the board?
4. Locate diameter from the circle on the board?



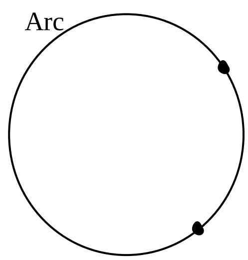
Step 2: Circumference is the distance around the edge of the circle. It is exactly π times the diameter.

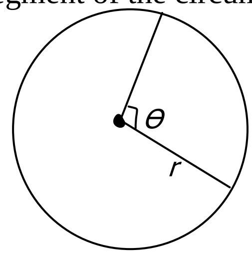
$$Circumference = \pi \text{ x diameter}$$
$$= \pi \text{ x 2 radii}$$
$$\pi = \frac{\text{Circumference}}{Diameter}$$

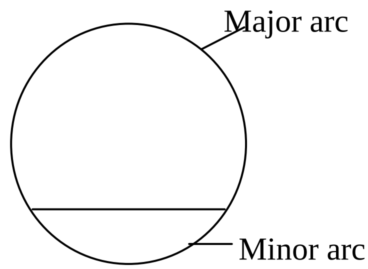
Area of circle**:** The area of a circle is π times the radius squared

$$A = \pi \text{ r}^2, \quad \text{where } \pi = \frac{22}{7} \text{ or } 3.142$$

An arc of a circle**:** An arc of a circle is a segment of the circumference of the circle.







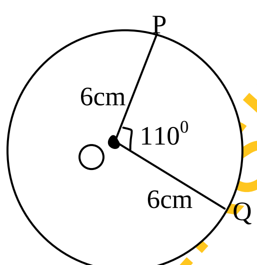
$$Arc\ \text{length} = \frac{\theta}{360^0} X\ 2\pi\ \text{r}$$

1. How do we find circumference of a circle?

2. What is the formula for finding the area of a circle?

Step 3: Example

1. An arc subtends an angle of 110^0 at the centre of a circle of radius 6cm. Find the length of the arc if $\pi = \frac{22}{7}$



Solution:

$$arc\ \text{length} = \frac{\theta}{360^0} X\ 2\pi\ \text{r}$$

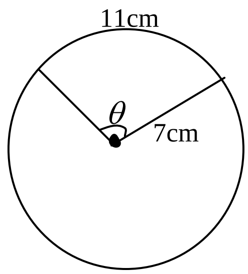
$$\therefore \text{arc PQ} = \frac{110^0}{360} X\, 2\, X\, \frac{22}{7} X\, \frac{6}{1}$$

$$= \frac{11X\,22}{3X\,7}$$

$$= \frac{242}{21}$$

$$= 11.5cm$$

2. Find the value of θ from the following diagram.



Solution:

$$arc\ \text{length} = \frac{\theta}{360^0} X\ 2\pi\ \text{r}$$

$$11 = \frac{\theta}{360} X\, 2\, X\ \pi\ X\ 7$$

$$\frac{11}{1} = \frac{\theta\ \text{X}\ 2\ \text{X}\ \pi\ \text{X}\ 7}{360^0}$$

$$\theta = \frac{11\ \text{X}\ 360}{2\ \text{X}\ \pi\ \text{X}\ 7}$$

$$= \frac{11X\,360^0\,X\,7}{2X\,22X\,7}$$

$$\theta = 90^0$$

3. Calculate the area of a circle of radius 7cm which subtends 108^0 at centre,

$$\pi = \frac{22}{7}$$

$$Area\ \text{of a circle} = \pi\ \text{r}^2$$

$$= \frac{22}{7} X\, 7\, X\, 7$$

$$= 154cm^2$$

Providing pictorial presentation: Show them the diagram of circle with diameter and radius of a circle.

Division of class into small groups: Divide the class into small groups.

Group activities: Ask the groups the following question, go through the work of each group and correct immediately.

Question: Calculate the arc length of a circle of radius 7cm which subtends 108^0 at centre $\left(\pi = \frac{22}{7}\right)$

Summary: A circle is a closed curve in a plane. Circumference of a circle can be calculated by using the following formula:

Circumference of a circle $= 2\pi$ r

Area of a circle $= \pi$ r^2

Arc length $= \frac{\theta}{360} X\, 2\pi$ r

Evaluation: Allow them to solve the following question in class individually.

Question**:** What is the length of an arc which subtends an angle of 60^0 at the centre of a circle of radius $\frac{1}{2}m$?

Assignment

1. In terms of π, what is the length of an arc which subtends an angle of 30^0 at the centre of a circle of radius $3^1/_2$cm?

Period 2

Topic: Sectors and chords of circles

Review of previous knowledge: Ask the following questions?

1. What is a circle?
2. Differentiate between radius and diameter of a circle.
3. How do we calculate arc length of a circle?
4. What is the formula for finding the area of a circle?

Presentation: Sectors and chords of circles

A sector of a circle is a portion enclosed by two radii and an arc of the circle.

A chord of a circle is a line segment that connects one point on the edge of the circle with another point on the circle. The diameter is the longest chord.

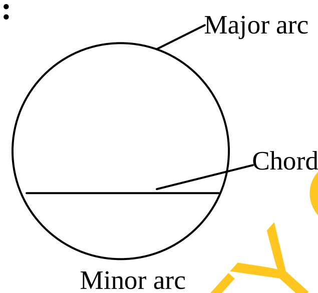
Step 1: The chord divides the circle into two segments of different sizes, a major and minor segment. Also a chord which is not a diameter divides the circumference into two arcs of different sizes a major arc and a minor arc.

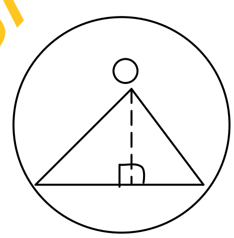
$$Area \text{ of a sector} = \frac{\theta}{360} \pi r^2$$

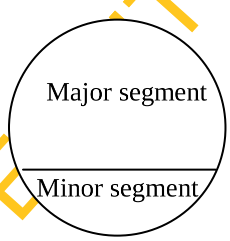
Questions

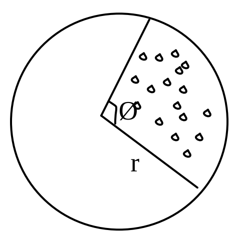
1. What is a sector?
2. What is a chord?

Step 2:









Question: What is the difference between sector and segment?

Step 3: Examples

1. Calculate the area of a sector of a circle which subtends an angle of 45^0 at the centre of the circle, radius 14cm if $\pi = \frac{22}{7}$

Solution:

$$Area \text{ of a sector} = \frac{\theta}{360} X \pi \, r^2$$

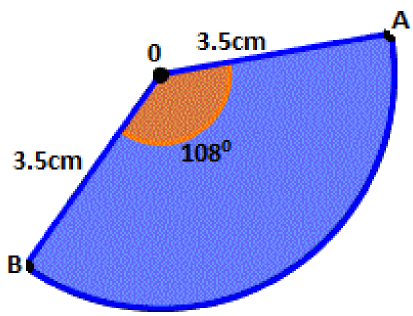
$$\theta = 45^0, \pi = \frac{22}{7}, \; r = 14$$

$$Area = \frac{45}{360} X \frac{22}{7} X \frac{14}{1} X 14$$

$$= \frac{45 X 22 X 7}{90}$$

$$= 77cm^2$$

2. Calculate the perimeter of the following diagram



Solution:

Perimeter is the distance round the shape

Perimeter of sector AOB = arc AB + A0 + B0

$$Arc \text{ AB} = \frac{\theta}{360} X \, 2\pi \, r$$

$$= \frac{108}{360} X 2 X \frac{22}{7} X 3.5$$

$$= \frac{33}{5}$$

$$Arc \text{ AB} = 6.6\text{cm}$$

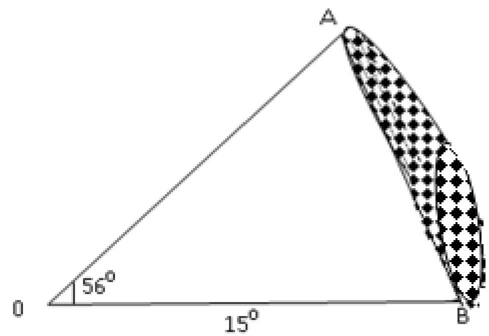
$$\overline{\text{AO}} = 3.5cm$$

$$\overline{BO} = 3.5cm$$

$$\therefore Perimeter = (6.6 + 3.5 + 3.5)cm$$

$$= 13.6cm$$

3. Calculate the area of the shaded segment of the following diagram, if $\pi = \frac{22}{7}$



Area of the shaded part = Area of sector AOB– Area of ΔAOB

$$Area\text{ of sector AOB} = \frac{\theta}{360} X \pi \text{ r}^2$$

$$= \frac{56}{360} X \frac{22}{7} X 15 X 15$$

$$= 110\text{cm}^2$$

$$Area\text{ of } \Delta\text{AOB} = \frac{1}{2} \overline{AO} X \overline{BO} \text{ Sin } 56^0$$

$$= \frac{1}{2} X 15 X 15 X 0.8290$$

$$= \frac{186.525}{2}$$

$$= 93.26\text{cm}^2$$

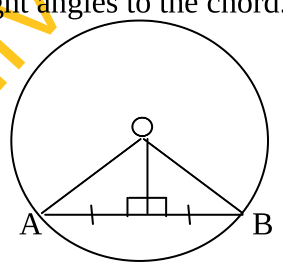
$\therefore$ *Area* of the shaded part

$$= (110 - 93.26) \text{ cm}^2$$

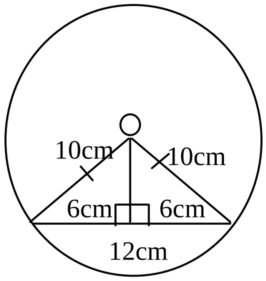
$$= 16.74 cm^2$$

Step 4: Chord of a circle

A straight line drawn from the centre of a circle to bisect a chord which is not a diameter is at right angles to the chord.



4. The radius of a circle is 10cm. The length of a chord is 12cm. Calculate the distance of the chord from the centre of the circle.



Solution:

$x^2 = 10^2 - 6^2$ (Pythagoras)

$= 100 - 36$

$= 64$

$x = \sqrt{64}$

$= 8cm$

Pictorial presentation: Show them the diagram of circle with sector and chord.

Group activities: Ask groups the following question, check their work and correct immediately.

Question: A chord of a circle is 9cm long. If its distance from the centre is 5cm, calculate the radius of the circle.

Summary: A chord is an internal segment of a circle that has both of its endpoints on the circumference of the circle. The diameter of a circle is the largest chord possible. A sector is enclosed by two radii and an arc in the circle

Area of sector $= \frac{\theta}{360} X \pi \, r^2$

Evaluation: Ask them the following questions.

1. What is a sector of a circle?
2. How do we find the area of a sector?
3. Define chord of a circle

Assignment

The radius of a circle is 10cm. The length of a chord is 16cm. Calculate the distance of the chord from the centre of the circle.

Period 3

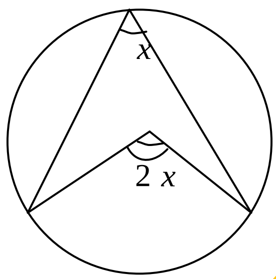
Topic: Circle Theorems

Review of previous knowledge: Ask them the following questions to know their level of knowledge on circle.

1. What do we call the largest possible chord?
2. How do we find area of a sector?

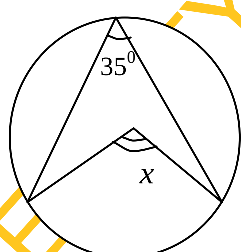
Presentation:

1. The angle that an arc of a circle subtends at the centre is twice that which it subtends at any point on the remaining part of the circumference.



Step 1: Example

Consider the following diagram; if the angle at the circumference is 35^0 find the angle at centre?



Solution:

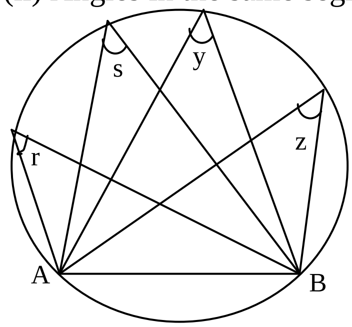
With the theorem

$x = 2 \times 35^0 = 70^0$

Question**:** Fill in the gap

The angle that an arc of a circle subtends at the centre is ______ that which it subtends at any point on the remaining part of the circumference?

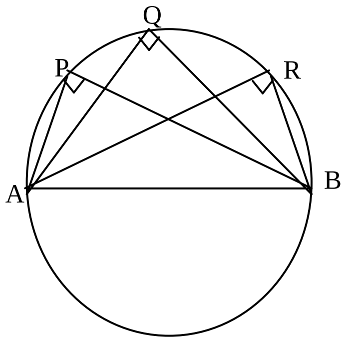
Step 2: (ii) Angles in the same segment of circle are equal.



$r = s = y = x$

If $r = 42^0 \Rightarrow S = 42^0$, $y = 42^0$ and $z = 42^0$

(ii) The angle in a semicircle is a right angle



$\Rightarrow P = Q = R = 90^0$

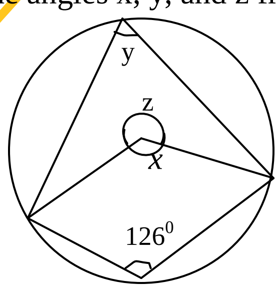
Question:

1. Angles in the same segment of a circle are _________

2. The angle in a semicircle is _________

Step 3: Examples

1. Find the angles x, y, and z from the following diagram



Solution:

z = 126 x 2 (angle at centre twice that at circumference)

$z = 252^0$

$x + z = 360^0$ (sum of angles at a point)

$x + 252^0 = 360^0$

$x = 360^0 - 252^0$

$= 108^0$

2y = x (angle at centre)

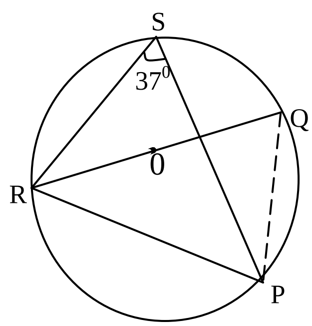
2y = 108

$$y = \frac{108}{2} = 54^0$$

$\therefore x = 108^0,\ y = 54^0,\ z = 252^0$

2. O is the centre of the following circle and QOR is the diameter.

PSR = 37^0, find PRQ



Join P and Q

RSP = RQP = 37^0 (angles in the same segment)

RPQ = 90^0 (angle subtended by diameter)

$\therefore$ PRQ = $180^0 - (90^0 + 37)$ (sum of angles of a triangle)

$= 180^0 - 127$

$= 53^0$

Summary: The angle that an arc of a circle subtends at the centre is twice that which it subtends at any point on the remaining part of the circumference. Angles in the same segment of a circle are equal.

The angle in a semicircle is a right angle.

Evaluation: Ask them the following questions:

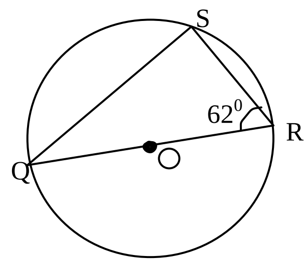
1. Angles in the same segment of a circle are ______ (equal/not equal)

2. The angle in a semicircle is a ______ (obtuse angle/right angle)

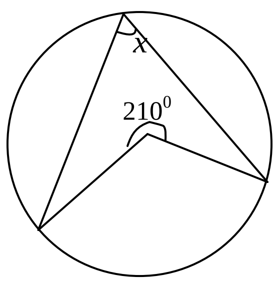
3. The angle that an arc of a circle subtends at the centre is ____ that which it subtends at any point on the remaining part of the circumference. (twice, three times)

Assignment: Solve the following questions

1. In the diagram below, 0 is the centre of the circle. If $Q\hat{R}S = 62^0$, find the value of SQR. (a) 14^0 (b) 28^0 (c) 31^0 (d) 90^0



2. If 0 is the centre of the following circle find x .



Week 2

Topic: Circle Geometry

Period 1: Cyclic quadrilateral

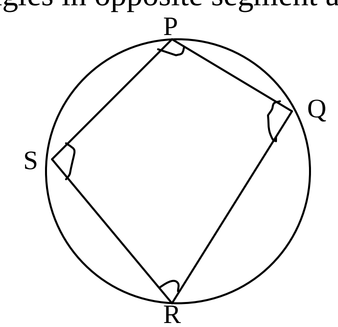
Review of previous knowledge:

1. What is a circle?

2. State two circle theorems

Presentation: Theorems

Step 1: Angles in opposite segment are supplementary.

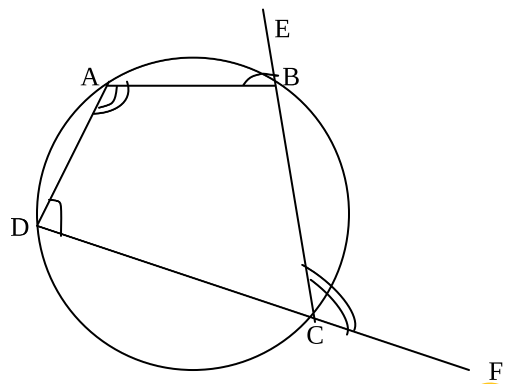


$P + R = 180^0$

$S + Q = 180^0$

Question**:** Angle in opposite segment are ______ (a) supplementary (b) complementary

Step 2: An exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.



$\Rightarrow BAD = BCF$

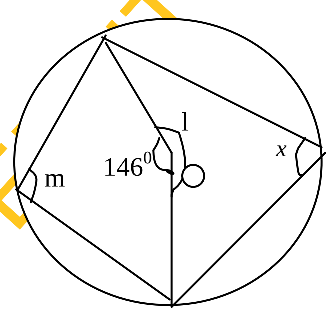
$\text{ADC} = \text{ABE}$

Question: What happen to exterior angle of a cyclic quadrilateral?

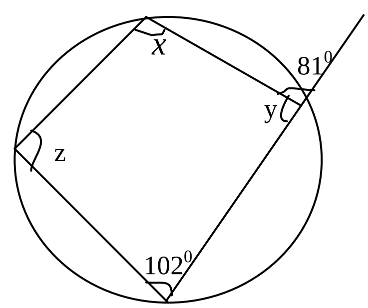
Step 3: Examples

Write down the sizes of the lettered angles in the following diagrams where 0 is given it is the centre of the circle.

(1)



(2)



Solution:

$$x = \frac{1}{2} X 146 \left(\text{angle at centre}\right)$$
$$= 73^0$$
$$l = 360^0 - 146^0 \text{ (angles at a point)}$$
$$= 214^0$$
$$m + x = 180^0 \text{ (opposite } \angle\text{s of a cyclic quadrilateral)}$$
$$\text{m} = 180^0 - x$$
$$= 180^0 - 73^0$$
$$= 107^0$$

(2) y + 81 = 180^0 (angles on a straight line)

y = $180^0 - 81^0$

= 99^0

$x + 102^0 = 180^0$ $\left(\textit{Opposite } \angle\text{s of a cyclic quadrilateral}\right)$

$x = 180^0 - 102^0$

= 78^0

z = 81^0 (exterior angle of a cyclic quadrilateral equal interior opp. $\angle$)

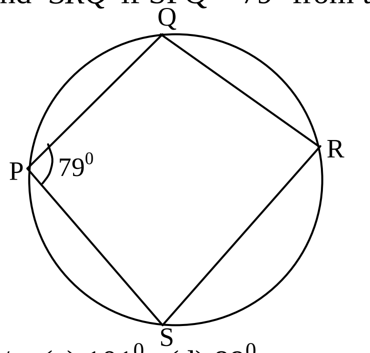
$\therefore$ $x = 78^0$, y = 99^0, z = 81^0

Summary: Opposite angles of a cyclic quadrilateral are added up to 180^0. Also the exterior angle of a cyclic quadrilateral is equal to interior opposite angle.

Evaluation: Ask the students the following questions.

1. Opposite angles of a cyclic quadrilateral are supplementary. True/False
2. What can you say about the exterior angle of a cyclic quadrilateral?

Assignment: Find $S\hat{R}Q$ if SPQ = 79^0 from the following diagram.



(a) 79^0 (b) $39^1/_2$ (c) 101^0 (d) 89^0

Period 2

Topic: Tangent to a circle

Review of previous knowledge: Ask them the following questions:

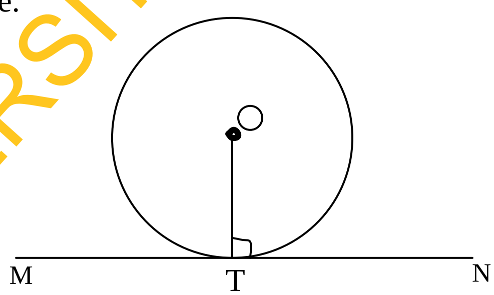
1. What is a chord of a circle?
2. What is diameter?
3. What is the difference between a diameter and radius

Presentation: Theorems

1. A tangent to a circle is perpendicular to the radius drawn to its point of contact.

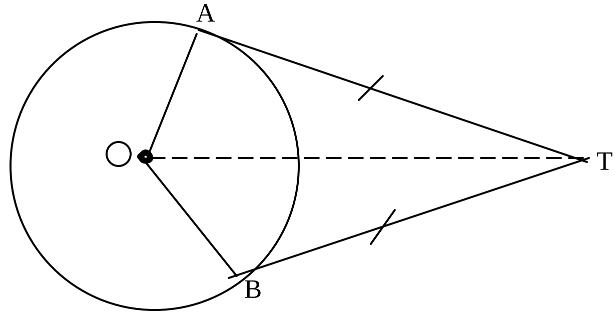
Step 1:

2. The perpendicular to a tangent at its point of contact passes through the centre of the circle.



Question**:** What happen to the perpendicular to a tangent?

Step 2: Tangents from an external point. The tangents to a circle from an external point are equal.

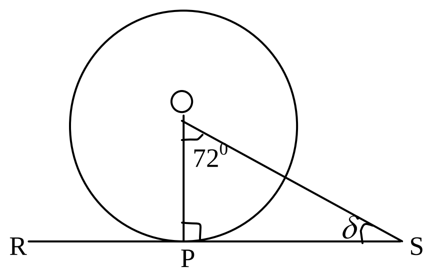


$\overline{TA} = \overline{TA}$

Question: The tangents to circle from an external point are ______

Step 3: Examples

1. Find the size of the angle δ from the following diagrams



Solution:

(1) $O\hat{P}S = 90^0$ (line from centre forms a $\perp$ line with the tanget)

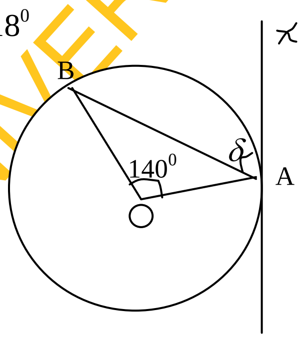
$\therefore 9 + 0^0 + 72^0 + \delta = 180^0$ (sum of angles of a triangle)

$162^0 + \delta = 180^0$

$\delta = 180^0 - 162^0$

$= 18^0$

(2)



Solution

$\overline{OA} = \overline{OB}$ (radii)

$O\hat{B}A = O\hat{A}B$ (base angles of isosceles)

$\therefore O\hat{B}A + O\hat{A}B + 140^0 = 180^0$ (sum of $\angle$s of a Δ)

$O\hat{B}A + O\hat{A}B = 180^0 - 140^0 = 40^0$

But $O\hat{B}A = O\hat{A}B$

$\Rightarrow 2\ O\hat{A}B = 40^0$

$$\text{OAB} = \frac{40^0}{2}$$

$$= 20^0$$

$OAB = 20^0$

$\Rightarrow OBA = 20^0$

OAB + BAX = 90^0 (line from centre)

$OAB + \delta = 90^0$

$20^0 + \delta = 90^0$

$\delta = 90^0 - 20^0$

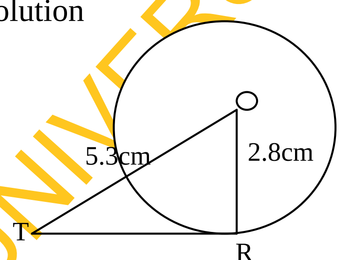
$= 70^0$

Pictorial presentation: Show them the diagram of the two theorems.

Group activities: Give the following question to all the groups, look at the result of their calculation and correct immediately.

Question**:** The tangent from a point T touches a circle at R. If the radius of the circle is 2.8cm, and T is 5.3cm from the centre. Calculate TR.

Solution



$(TR)^2 = (5.3)^2 - (2.8)^2$ (Pythagoras)

$(TR)^2 = 28.09 - 7.84$

$= 20.25$

$TR = \sqrt{20.25}$

$= 4.5cm$

Summary: The tangent does not cut the circle, it touches the circle. A tangent to a circle is perpendicular to the radius drawn to its point of contact. Also, the perpendicular to a tangent at its point of contact passes through the centre of the circle.

Evaluation: Ask the following questions:

1. A tangent to a circle _____ the circle (cut/touches)
2. The perpendicular to a tangent at its point of contact passes through _____ of the circle.

Assignment: Two circles have the same centre and their radii are 15cm and 17cm. A tangent to the inner circle at P cuts the outer circle at Q. Calculate $\overline{PQ}$.

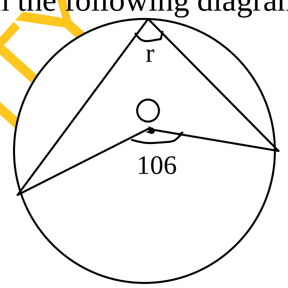
Period 3

Practice Exercises on Circle Geometry

Step 1: The following questions are given to students as class exercises.

Pick the correct answer from the following options.

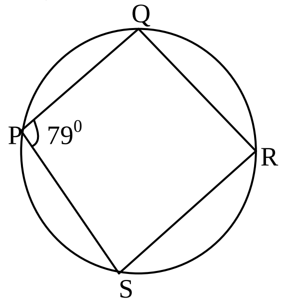
1. The radius of a circle is 10cm. The length of a chord of the circle is 16cm. Calculate the distance of the chord from the centre of the circle. (a) 5cm (b) 6cm (c) 7cm (d) 8cm
2. Calculate *r* from the following diagram?



(a) 53^0 (b) 63^0 (c) 35^0 (d) 36^0

3. A 216^0 sector of a sector of radius 5cm is bent to form a cone. Find the radius of the cone. (a) 3cm (b) 5cm (c) 6cm (d) 7cm

4. Find $S\hat{R}Q$ if $S\hat{P}Q = 79^0$ from the following diagram

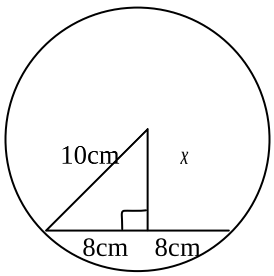


(a) 79^0 (b) $39^1/_2$ (c) 101^0 (d) 89^0

Step 2: Supervise their work and mark

Step 3: Solution to the questions

(1)



$x^2 = 10^2 - 8^2$ (Pythagoras)

$= 100 - 64$

$= 36$

$\therefore = \sqrt{36}$

$= 6cm$

(2) $2r = 106^0$ (angle at centre twice that at the circumference)

$r = \frac{106}{2}$

$= 53^0$

(3) Area of a sector $= \frac{\theta}{360} X \pi r^2$, r = 5cm

Curve surface of cone $= \pi$ rl, l = 5cm

$\Rightarrow$

$$\pi rl = \frac{\theta}{360} X \pi\ r^2$$

$$\pi rl = \frac{216 X \pi\ r^2}{360}$$

$$360^0 X \pi\ rl = 216\ \pi\ X\ 5\ X\ 5$$

$$r = \frac{216\pi\ X\ 5\ X\ 5}{\pi\ X\ 5\ X\ 360^0}$$

$$= \frac{216 X 5}{360}$$

$$= \frac{1080}{360^0}$$

$$= 3cm$$

(4) SRQ + SPQ = 180^0 (opposite angles of a cyclic quadrilateral)

SPQ = 79^0

SRQ + 79^0 = 180^0

$\therefore$ SRQ = $180^0 - 79^0$

= 101^0

Step 4: Allow them to ask questions

Week 3

Topic: Volume of Solids

Period: Prism–Volume of Cube

Review of previous knowledge: Ask them the following questions.

1. What is a point?
2. What is a straight line?
3. What is a square shape?
4. What is a rectangular shape?
5. A square shape has how many sides?
6. What can you say about the sides of a square?

Presentation: Volume of cube

Step 1: A Point is one of the basic terms in geometry. It is an entity that has only one characteristic, that is, it has position. A point has no length or width, it just specifies an exact location.

A•   B•

C•

Point A, B $ C

Straight line: A straight line is the shortest distance between two points. A straight line has one dimension which is length.

Prism is a solid which has a uniform cross-section. It is a solid that has two congruent parallel bases that are polygons. Examples of prism are cubes, cuboids, cylinder, and pyramid. All prisms are 3 dimensional shapes.

Step 2: Here is a line, it has got one dimension which is length (ID).

Length

The following is a rectangle, it has got two dimensions length and breadth, so it is 2D shape.

Breadth

Length

Examples of 2D shapes are squares, circles, triangles, parallelograms and all the polygons. 2D shapes have area but no depth. 3D shapes have length, breadth, and depth so all 3D shapes have volume. Examples of 3D shapes are cube, cuboids, triangular prism, cone, and pyramid.

Question**:** What are 2 dimensional and 3 dimensional shapes?

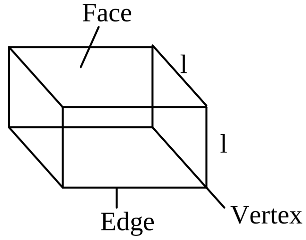
Step 3: The volume of a prism is given by the product of the area of its base and its height.

Volume of prism = area x height

Volume of cube = l x l x l

$= 1^3$

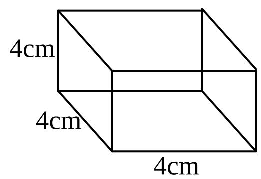
$\Rightarrow$ Height = length = breadth



Question**:** How do we find the volume of a prism?

Step 4: Examples

1. Find the volume of the following cube



Solution

Volume = (l x l x l) unit cube

= (4 x 4 x 4) cm^3

= $64cm^3$

2. Calculate the volume of a cube whose edges are 8cm each

Solution

l = 8cm, b = 8cm, h = 8cm

Volume = (8 x 8 x 8) cm^3

= $512cm^3$

Group activities: If the volume of a cube is $64cm^3$. Find the sides of the cube.

Summary: A prism is a solid with uniform cross section. A cube is a 3D shape. It has faces, edges and vertices.

Evaluation: Allow them to solve the following question in class. Find the volume of a cube with sides 70cm.

Assignment: Give the following questions as assignment.

1. Find the volume of a cube with sides 25cm
2. If the volume of a box in the shape of a cube is $2,197cm^3$, find the length of the edges.

Period 2

Topic: Volume of Cuboids

Review of previous knowledge: Ask the students the following questions.

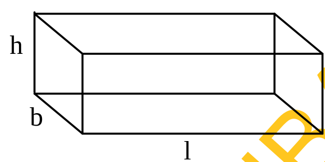
1. Differentiate between 2 dimensional and 3 dimensional shapes
2. What is a cube?
3. What can you say about the sizes of a cube?

Presentation: Volume of cuboid with rectangular faces. Example of material with cuboid shape is matches box. The sides of cuboid are not equal.

Question: (1) What is a cuboid?

(2) Differentiate between cube and cuboid

Step 2: Volume of cuboid = l x b x h where l = length, b = breadth and h = height



Volume of cuboid = (l x b x h) unit cube

$\Rightarrow$ Base area x h

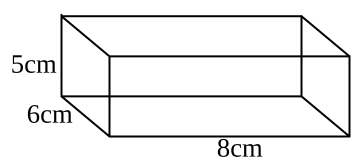
Where base area = l x b

Question: Differentiate between a cube and a cuboid

Step 3: Examples

1. Find the volume of a cuboid with the following dimensions, length 8cm, breadth 6cm and height 5cm

Solution



Volume of cuboid = (l x b x h)

= (8 x 6 x 5) cm^3

= $240cm^3$

2. Find the volume of a tank with length 30cm breadth 20cm and height 15cm

Solution:

Volume = l x b x h

$(30 \times 20 \times 15)\ cm^3$

$= 9000cm^3$

Group activities: Give the following question to them as group work.

A car petrol tank is 0.8m long, 25cm wide and 20cm deep. How many litres of petrol can it hold? ($1000cm^3$ = 1 litre)

Solution

Volume of tank = 0.8m x 25cm x 20cm

0.8m to cm

1m x 100cm

0.8m = 100 x 0.8 = 80cm

$\therefore$ Volume = $(80 \times 25 \times 20)\ cm^3$

$= 40{,}000cm^3$

$1{,}000cm^3$ = 1 litre

$$\therefore 40{,}000cm^3 = \frac{40{,}000}{1000}$$

= 40 litres

$\therefore$ Capacity of tank = 40 litres

Summary: Cuboid is also a 3 dimensional figure. It has faces, edges and vertices. It is an example of prism. It has 6 faces, 12 edges and 8 vertices.

Evaluation: Allow them to solve the following question on the board.

Find the volume of a box with length 15cm, breadth 6cm and height 4cm.

Assignment: The following questions were given as assignment.

1. Find the volume of a cuboid with height 12cm, breadth 6cm and length 8cm.
2. Find the volume of a cuboid with 3cm by 4cm by 6cm.

Period 3

Topic: Volume of Triangular Prism

Review of previous knowledge: Ask the following questions

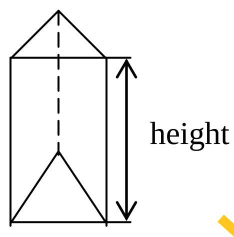
1. What is a prism?
2. Give three examples of prism?
3. Is cuboid a prism?
4. How do we find the volume of a prism?
5. What is the volume of a cuboid?

Presentation: Triangular prism

Step 1: A triangular based prism is a prism with a triangle-shaped base.

Question: What is a triangular prism?

Step 2:



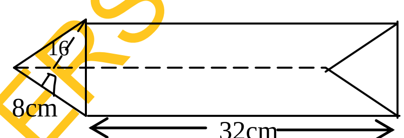
Vol. = area of base x height

Or area of base x length of the prism

Question**:** How do we find the volume of a triangular prism?

Step 3: Examples

1. Find the volume of a triangular prism below

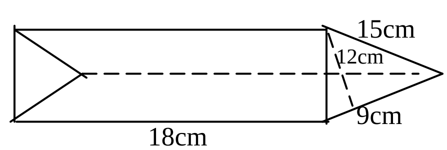


Solution

Vol. of prism = (Area of triangular base x height)

$$\frac{1}{2} X \frac{8}{1} X \frac{16}{1} X 32$$
$$= 2048 cm^3$$

2. Find the volume of the prism below



Solution:

$$Vol. = \frac{1}{2} X \frac{12}{1} X \frac{9}{1} X \frac{18}{1}$$
$$= 972cm^2$$

Group activities: Give the following question to them as group work.

Find the volume of a triangular prism with a triangle of base 9.3cm, height 8.1cm and the length 14.4cm

Solution

$$Vol.\text{ of prism} = \text{Base area x height}$$
$$= \frac{1}{2} \text{X}9.3\text{X}8.1\text{X}14.4$$
$$= 524.376\text{cm}^3$$

Summary: A prism is a solid of uniform cross section.

Volume of prism = area of cross section x height

A right triangular prism is a solid having a uniform triangular cross section.

Vol. = area of triangular base x length or height of the prism

Evaluation: Ask the following questions?

1. What is a prism?
2. How do we find the volume of a prism?
3. Find the volume of the following triangular prism with a right-angled triangular 3cm by 5cm, length 6cm.

Assignment: A right prism of length 10cm has as its cross-section equilateral triangle of side 6cm. Calculate its volume.

Week 4

Topic: Volume of Solids

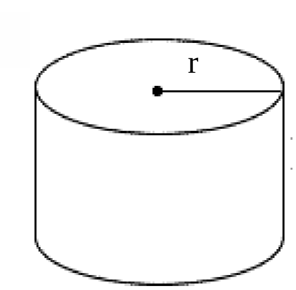
Period 1: Volume of Cylinder

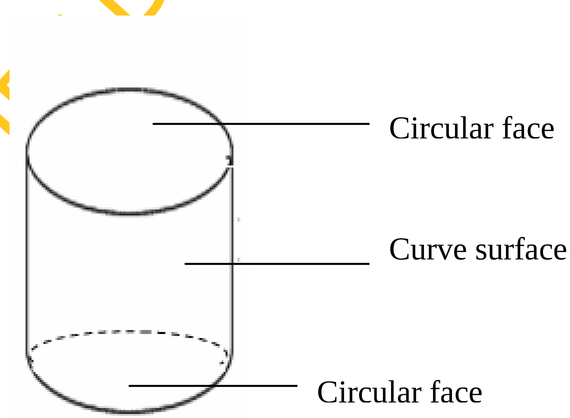
Review of previous knowledge: Ask them the following questions.

1. What is a rectangle?
2. Why is it called a 2-dimesional figure?
3. What is a circle?
4. Rectangle and circle, are they plane or solid shapes?
5. Differentiate between plane and solid shapes?
6. Give examples of solid and plane shapes.

Presentation: Volume of cylinder

Step 1: A cylinder is a solid with two congruent circles joined by a curved surface.





Question**:** What is a cylinder?

Step 2: The volume of the cylinder is the area of the base x height.

$\therefore$ Volume of cylinder $= \pi r^2 h$

Question**:** What is the volume of a cylinder?

Step 3: Examples

1. Find the volume of a cylinder of radius 43.4cm and length 550cm.

Solution

$$Vol.\text{ of cylinder} = \pi r^2 h$$
$$r = radius, \text{ h} = \text{height}$$
$$\text{r} = 43.4\text{cm, h} = 550\text{cm, } \pi = \frac{22}{7}$$
$$Vol. = \frac{22}{7} X \frac{(43.4)^2}{1} X \frac{550}{1}$$

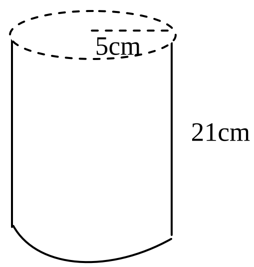
= 22 x 6.2 x 43.4 x 550

= (12100 x 269.08) cm^3

= $3255868cm^3$

= 3.26 x 10^6cm^3

2. Find the volume of the cylinder below



Solution:

$$Volume\text{ of cylinder} = \pi r^2 h$$
$$= \frac{22}{7} X \frac{5}{1} X \frac{5}{1} X \frac{21}{1}$$
$$= 22 \text{ x } 25 \text{ x } 3$$
$$= 1650\text{cm}^3$$

Group activities: A cylindrical container, closed at both ends, has a radius of 7cm and height 5cm. Find the volume of the container. $\left(\pi = \frac{22}{7}\right)$

Solution:

$$Volume\text{ of cylinder} = \pi r^2 h$$
$$\pi = \frac{22}{7}, \text{ r} = 7, \text{ h} = 5$$
$$\text{Vol. } \frac{22}{7} X \frac{7}{1} X \frac{7}{1} X 5$$
$$= 154 \text{ x } 5$$
$$= 770cm^3$$

Summary: A cylinder is a solid with two congruent circles joined by a curve surface. Volume of cylinder = $\pi r^2 h$

Evaluation: (1) A cylinder has how many circular faces.

(2) What is the formula for finding the volume of a cylinder?

Assignment: Find, correct to 1 decimal place, the volume of a cylinder of height 8cm and base radius 3cm $(Take\, \pi = 3.142)$.

Period 2

Topic: Volume and Frustum of Cone

Review of previous knowledge: Ask the following questions

1. What is a sector of a circle?
2. Can we use sector of a circle to form a cone?
3. By folding a sector to form a cone, the radius of the sector will form the height of the cone. True/false

Presentation: Volume and frustum of cone.

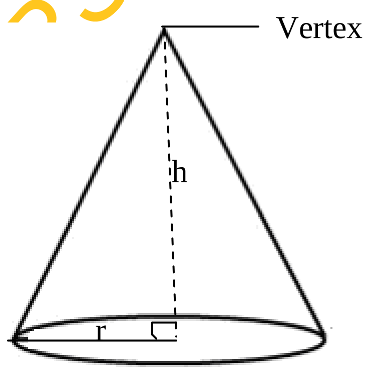
Step 1: A circular cone has a circular base, which is connected by a curved surface to its vertex. A cone is called a right circular cone, if the line from the vertex of the cone to the centre of its base is perpendicular to the base. A cone is also referred to as circular base pyramid.

Question**:** 1. What is a cone?

2. A cone can also be called circular based pyramid. True/false

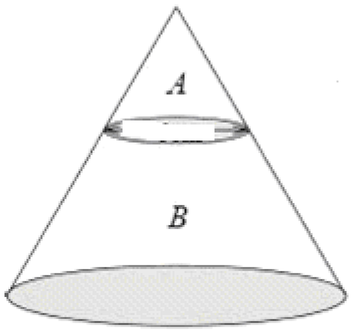
Step 2:

Cone

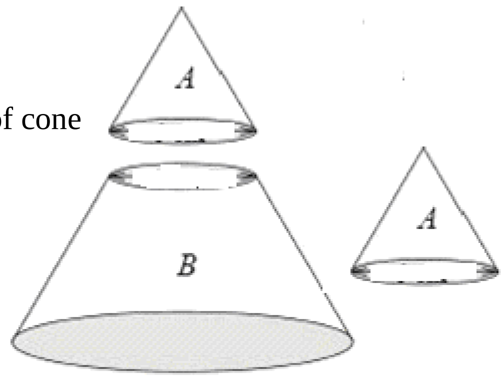


$$Volume \text{ of cone} = \frac{1}{3}\pi r^2 h$$

Frustum of cone



Frustum of cone



Volume of frustum of cone = Vol. of big cone – Vol. of small come

$$\frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h$$

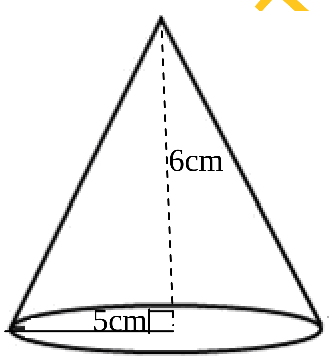
Or $Vol.\text{ of frustum of cone} = \frac{1}{3}\pi h\left(R^2 + r^2 + Rr\right)$

Question**:** How do we find volume of cone?

Step 3: Examples

1. Calculate the volume of cone with base radius 5cm and height 6cm

Solution



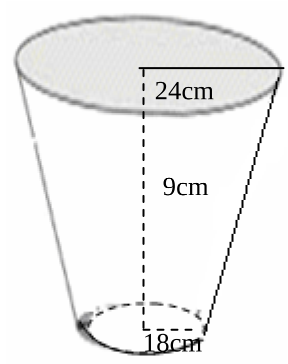
$$Vol.\text{ of cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} X \frac{22}{7} X \frac{5}{1} X \frac{5}{1} X \frac{6}{1}$$

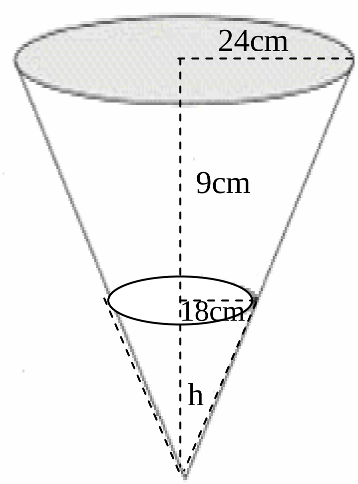
$$= \frac{1{,}100}{7}$$

$$= 157.14cm^3$$

2. A bucket inform of a frustum, has upper and lower radii as 24cm and 18cm respectively, if the height is 9cm find the volume of the bucket in terms of π.



Solution:



H = 9 + h
R = 24cm
r = 18cm
h = ?

$$\frac{H}{h} = \frac{R}{r}$$

$$\frac{9+h}{h} = \frac{24}{18} = \frac{4}{3}$$

$$4h = 3(9+h)$$

$$4h = 27 + 3h$$

$$4h - 3h = 27$$

$$h = 27$$

$$\therefore H = 9 + 27 = 36cm$$

$\therefore$ Vol. of bucket = Vol. of big – Vol. of small

$$= \frac{1}{3}\pi R^2H - \frac{1}{3}\pi r^2h$$

$$= \frac{1}{3}\pi \left(R^2H - r^2h\right)$$

$$= \frac{1}{3} x\, \pi \left(24^2\, X\, 36 - 18^2\, X\, 27\right)$$

$$= \frac{1}{3}\pi \left(20737 - 8748\right)$$

$$= \frac{1}{3}\pi \left(11988\right)$$

$$= 3996\pi cm^3$$

Group activities: Calculate the volume of a cone with base diameter 10cm and height $2^1/_2$cm.

Solution

$$Vol.\text{ of cone} = \frac{1}{3}\pi r^2 h \qquad r = \frac{10}{2} = 5cm$$

$$= \frac{1}{3} X \frac{22}{7} X\, 5\, X\, 5\, X\, 2\frac{1}{2}$$

$$= \frac{1}{3} X \frac{22}{7} X \frac{5}{1} X \frac{5}{1} X \frac{5}{2}$$

$$= \frac{1375}{21}$$

$$= 65.47cm^3$$

Summary: Volume of cone $\frac{1}{3}\pi r^2 h$ and Volume of frustum of cone =

Volume of big cone – Vol. of small cone

Or Volume of frustum of cone $\frac{1}{3}\pi h\left(R^2 + r^2 + Rr\right)$

Evaluation: 1. What is the formula for finding the volume of frustum of cone?

Assignment: Calculate the volume of cone with base radius 3.5cm, height 15cm.

Period 3

Topic: Volume of Pyramid

Review of previous knowledge: Ask the following questions.

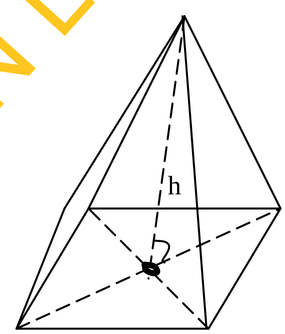
1. What is a cone?
2. Which type of pyramid is a cone?
3. How do we find the volume of a cone?

Presentation: Volume of pyramid

Step 1: A pyramid is a solid with a polygon base which meets at common point called the apex or vertex. Different types of pyramid are; cone (circle-based-pyramid), square-based pyramid, rectangular-based pyramid.

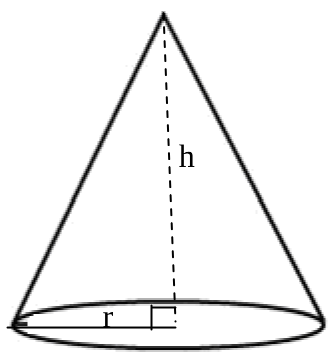
Question**:** Mention different types of pyramid.

Step 2:



Square-based

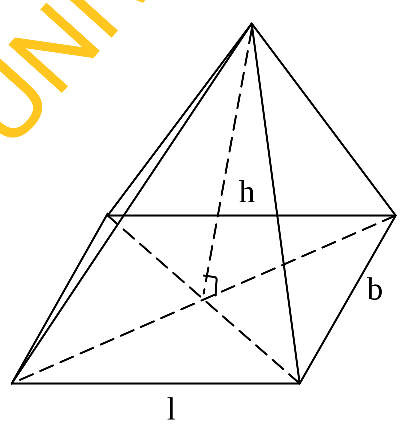
$Vol. \frac{1}{3} Ah = \frac{1}{3} l^2 h$



Circular-based pyramid

$Vol. \frac{1}{3} \pi r^2 h$

r = radius
A = area, l = length,
h = height



Rectangular base pyramid

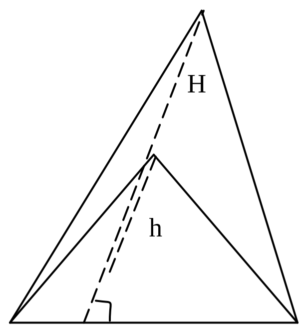
$Vol. \frac{1}{3} Ah$ where A = area

h = height

Or Vol. $\frac{1}{3}$ lbh where l = length, b = breadth

h = height

Triangular-based pyramid



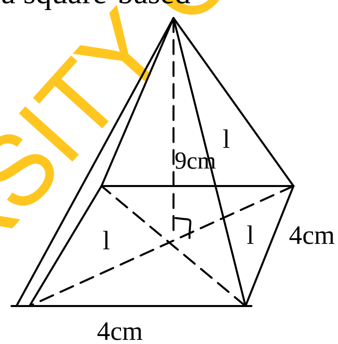
$$Vol. \ \frac{1}{3}\text{Ah}$$
$$= \frac{1}{3}\left(\frac{1}{2}bh\right)H$$
$$= \frac{1}{6}bhH$$

Question**:** How do we find the volume of a pyramid?

Step 3: Examples

1. A pyramid has a square base of side 4cm and height of 9cm. Find its volume

Volume of a square-based Pyramids

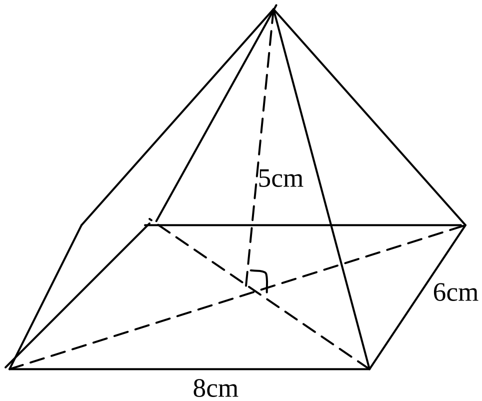


Solution

$$Volume = \frac{1}{3}Ah$$
$$= \frac{1}{3}\ l^2 h$$
$$= \frac{1}{3} X\, 4^2\, X\, 9$$
$$= 16 \text{ X } 3$$
$$= 48\text{cm}^3$$

Volume of rectangular-based pyramid

2. Find the volume of a rectangular-based pyramid whose base is 8cm by 6cm and height is 5cm.

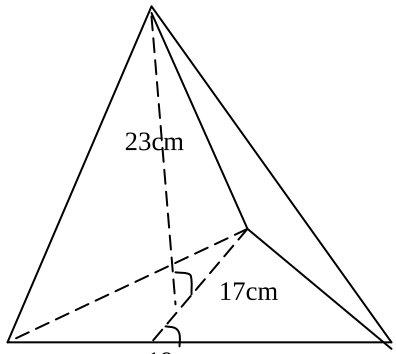


Solution

$$Volume = \frac{1}{3}Ah$$
$$= \frac{1}{3} \text{l b h}$$
$$= \frac{1}{3} X \frac{8}{1} X \frac{6}{1} X 5$$
$$= 80cm^3$$

3. Find the volume of the triangular-based pyramid with base 19cm, height of the triangle 17cm and height of the pyramid is 23cm. correct the answer to 2 decimal places.

Solution:



$$Volume = \frac{1}{3} AH$$

$$= \frac{1}{3}\left(\frac{1}{2} bh\right)H$$

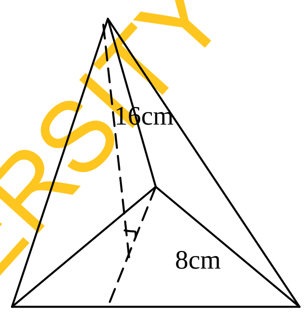
$$= \frac{1}{6} bhH$$

$$= \frac{1}{6} X \frac{19}{1} X \frac{17}{1} X \frac{23}{1}$$

$$= 1238.1667$$

$$= 1238.7 \; to \; 2 \text{ d.p.}$$

Group activities: Calculate the volume of the triangular based pyramid with base 12cm, height of the triangle 8cm and height of the pyramid 16cm.



Solution

$$Volume = \frac{1}{3}AH$$

$$= \frac{1}{3}\left(\frac{1}{2}bh\right)H$$

$$= \frac{1}{6}bhH$$

$$= \frac{1}{6} X \frac{12}{1} X \frac{8}{1} X \frac{16}{1}$$

$$= 256cm^3$$

Summary: Volume of pyramid = $\frac{1}{3}$ Ah

Different types of pyramid are: Circular-based pyramid (cone), Square-based pyramid, Rectangular-based pyramid, and Triangular-based pyramid.

Evaluation: (1) What is a pyramid?

(2) What is the formula for finding the volume of a pyramid?

Assignment: (1) A Pyramid has a square base of side 6cm and a height of 15cm. Find its volume

(2) Find the volume of a rectangular-based pyramid whose base is 10cm by 8cm and height 5cm.

Week 5

Topic: Angles of Elevation and Depression

Period 1: Angles of Elevation

Review of previous knowledge: Ask the following questions. Give explanation when necessary.

1. What is an angle?
2. How do we find the following?
   a. Sine of angles
   b. Cosine of angles
   c. Tangent of angles
3. What is the full meaning of SOHCAHTOA?

4. What is a right angle triangle?

5. In a right angle triangle, what do we call the side opposite right angle?

Presentation:

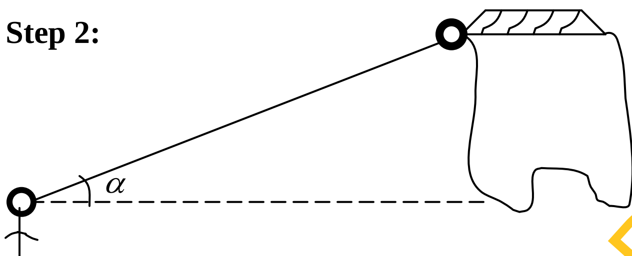
Step 1: The angle of elevation is the angle between a horizontal line from the observer and the line of sight to an object that is above the horizontal line.

Question:

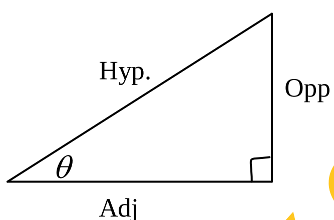
1. What is horizontal line?

2. What is angle of elevation?

Step 2:



θ is the angle of elevation.

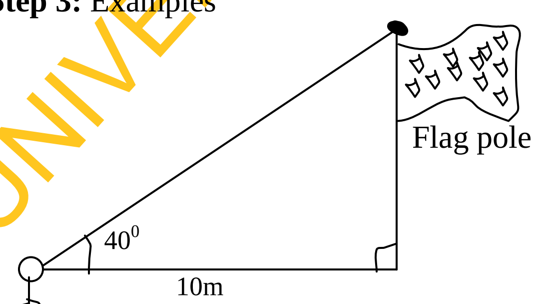


To find angle of elevation i.e. $\theta \Rightarrow$

$$Sin\ \theta = \frac{Opp}{hyp} \quad or \cdots \frac{Adj}{hyp}\ or\ \tan\theta = \frac{Opp}{Adj}$$ depends on the sides given.

Question**:** How do we find angle of elevation?

Step 3: Examples



Find the height of the flagpole

Solution:

$$\tan 40^0 = \frac{h}{10}$$

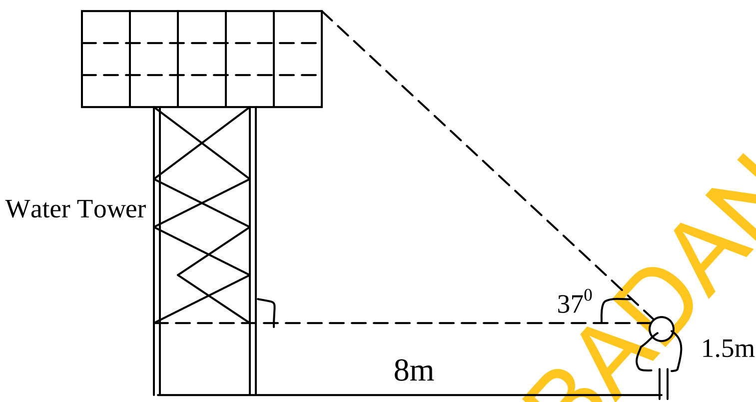
$h = 10 \tan 40^0$

$= 10 \text{ x } 0.839$

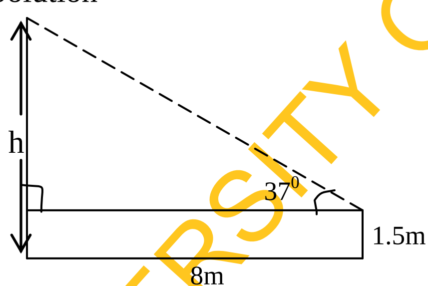
$= 8.39$

$= 8.4\text{m}$

2. Find the height of the water tower below to the nearest $^1/_2$m



Solution



$$\tan 37^0 = \frac{h - 1.5}{8}$$

$\Rightarrow h - 1.5 = 8 \tan 37^0$

$= 8 \text{ X } 0.7534$

$= 6.028$

$h - 1.5 = 6.028$

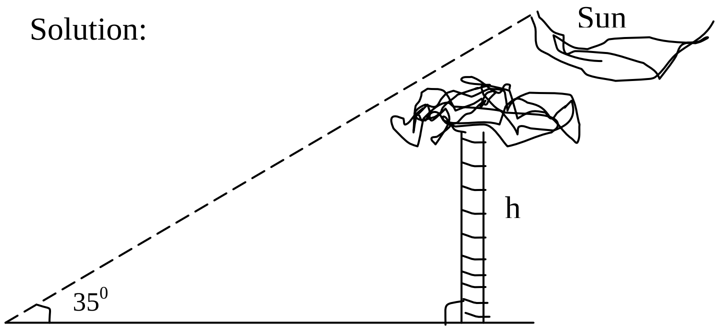
$h = 6.028 + 1.5$

$= 7.5284$

$= 7.5\text{m}$

Group activities: Allow each group to solve the following question and come out with their result. The angle of elevation of the sun is 45^0. A tree has a shadow 12m long. Find the height of the tree.

Solution:



$$\tan 35^0 = \frac{h}{12.8}$$

$$h = 12.8 \ \tan 35^0$$

$$= 12.8 \text{ X } 0.7002$$

$$= 8.96$$

$$= 9\text{m}$$

Summary: The angle of elevation of an object as seen by an observer is the angle between the horizontal and the line from the object to the observer's eye (the line of sight).

Evaluation: How do we find tangent of an angle?

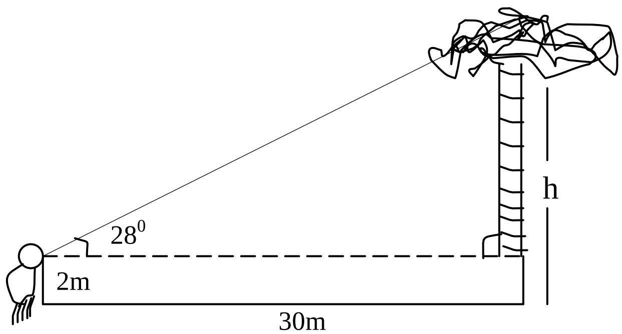
Assignment: The angle of elevation of the top of a tower as seen by a man 5cm away from the foot of the tower is 37^0. The angle of elevation of the top of the tower as seen by a second man standing on the opposite side of the tower is $49^{0.}$ How far are the men?

Period 2

Topic: More examples on angles of elevation

1. A man who is 2m tall stands on horizontal ground 30m from a tree. The angle of elevation of the top of the tree from his eyes is 28^0. Find the height of the tree.

Solution



$$\tan 28^0 = \frac{h-2}{30}$$

$$h-2 = 30\tan 28^0$$

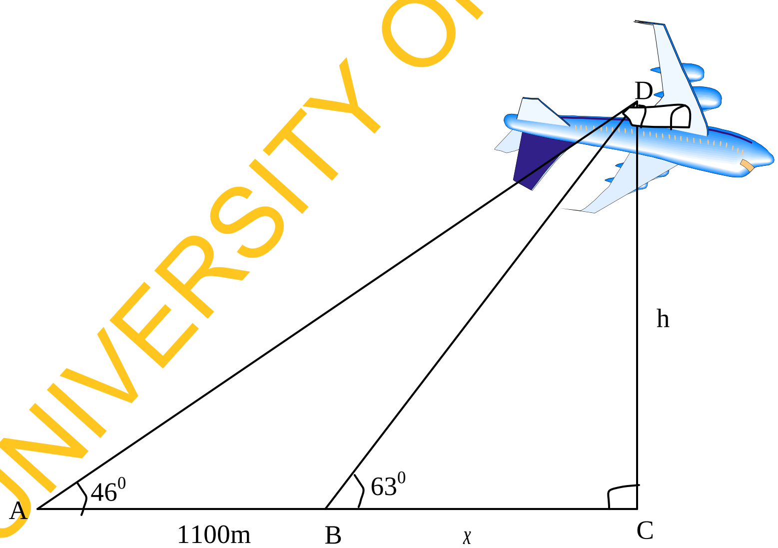
$$= 30 X 0.5317$$

$$h-2 = 15.95$$

$$h = 15.95 + 2$$

$$= 17.95m$$

2. The following figure shows the angle of elevation of an aircraft from two points 1100m apart. Find the height of the aircraft above the ground to the nearest 100m.



Let BC = x

$$\tan 46^0 = \frac{h}{1100 + x}$$

$$h = (1100 + x)\ \tan 46^0 \qquad ....(i)$$

$$\tan\ 63^0 = \frac{h}{x}$$

$$h = x \tan 63^0 \qquad ....(ii)$$

Since h = h

$\therefore$ Equate equations (i) and (ii)

$\Rightarrow$(1100 + x) tan 46^0 = x tan 63^0

1100 tan 46^0 + x tan 46^0 = x tan 63^0

1100 tan 46^0 = x tan 63^0 — x tan 46^0

1100 tan 46^0 = x (tan 63^0 — tan 46^0)

$$x = \frac{1100\ \tan 46^0}{\tan 63^0 \text{ - } \tan 46^0}$$

$$= \frac{1100 X 1.0355}{1.9626 - 1.0355}$$

$$= \frac{1139.08}{0.9271}$$

$$\therefore x = 1228.64$$

From equation (i)

h = (1100 + x) tan 46^0

= (1100 + 1228.64) 1.0355

=2328.64 x 1.0355

= 2411.306

To the nearest 100m

$\Rightarrow$ h= 2400m

Period 3

Practice exercise on angles of elevation

Allow them to practice the following questions in class.

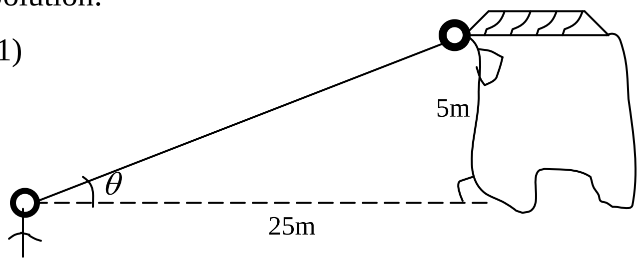
1. A man standing 25m away from a building observes the widow of the house 5m above the ground. Calculate the angle of elevation of the window from the man.

2. The angle of elevation of the top of a tower from a point 42cm away from its base on level ground is 36. Find the height of the tower.

3. The angle of elevation of the sun is 27^0. A man is 180cm tall. How long is his shadow? Give your answer to the nearest 10cm.

Solution:

(1)



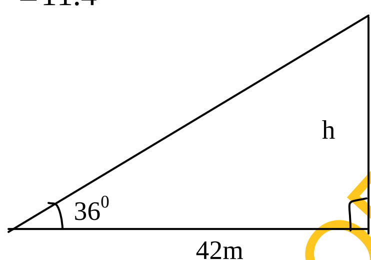
$$\tan\theta = \frac{5}{25}$$

$$\tan\theta = 0.2$$

$$\theta = \tan^{-1} 0.2$$

$$= 11.4^0$$

(2)



$$\tan 36^0 = \frac{h}{42}$$

$$h = 42 \ \tan 36^0$$

$$h = 42 \text{ X } 0.7265$$

$$= 30.5\text{m}$$

(3) $$\tan 27^0 = \frac{180}{x}$$

$$x = \frac{180}{\tan 27^0} = \frac{180}{0.509}$$

$$x = 353.6$$

to the nearest 10cm

= 350cm

Week 6

Topic: Angles of Elevation and Depression

Period 1: Angles of Depression

Review of previous knowledge:

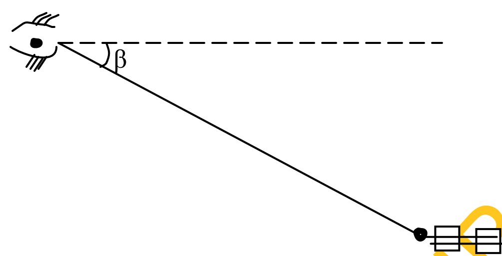
1. What is angle of elevation?

Presentation:

Step 1: The angle of depression is the angle between a horizontal line from the observer and the line of sight to an object that is below the horizontal line.

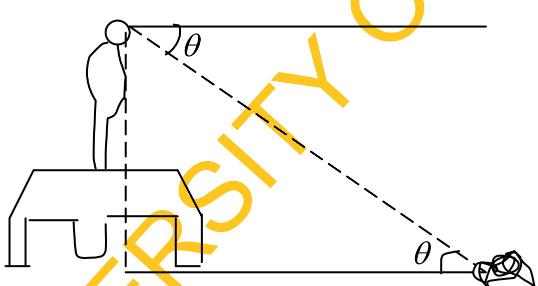
Question**:** What is angle of depression?

Step 2: Explain with the following diagram



β is the angle of depression.

$\Rightarrow$



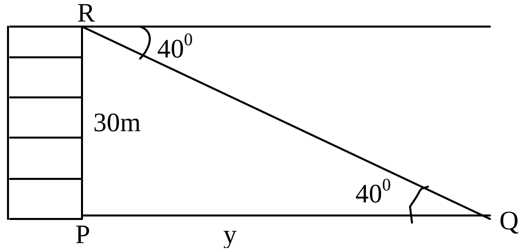
$\theta = \theta$ alternate angles

Question: If a man standing decides to look at an object on the floor the angle between his line of sight and object on the floor is called what?

Step 3: Examples

1. The angle of depression of a point Q from a vertical tower PR, 30m high is 40^0. If the foot P of the tower is on the same horizontal level as Q. Find correct to 2 decimal; places /PQ/.

Solution:



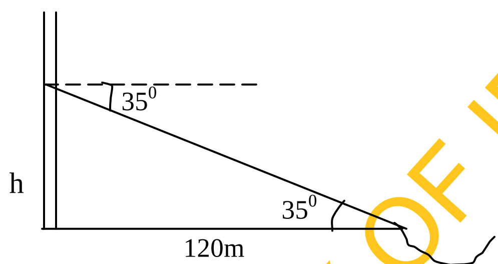
$\tan\ 40^0 = \frac{30}{y}$

$y = \frac{30}{\tan 40^0} = \frac{30}{0.8390}$

$= 35.75m$

2. The angle of depression of a boat from the mid-point of a vertical cliff is 35^0. If the boat is 120m from the foot of the cliff, calculate the height of the cliff.

Solution:



$\tan 35^0 = \frac{h}{120^0}$

h= 120^0 X tan 35^0

= 120^0 X 0.7002

= 84.02

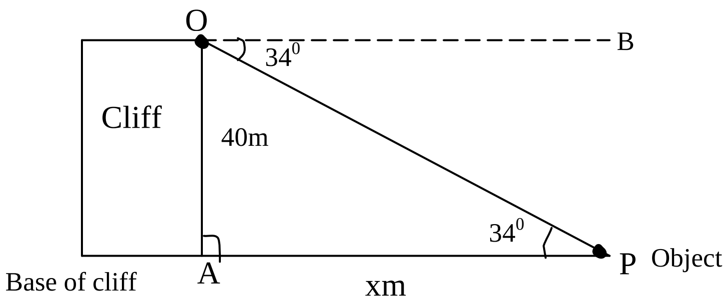
$\therefore$ 2h = 84.02 X 2

= 168m

Group activities: Allow each group to solve the following question and come out with their result.

Question**:** From the top of a vertical cliff 40m high, the angle of depression of an object that is level with the base of the cliff is 34^0. How far is the object from the base of the cliff?

Solution:



$A\hat{P}O = B\hat{O}P$ (*Alternate* angles)

$A\hat{P}O = 34^0$

$\tan 34^0 = \frac{40}{x}$

Multiply both sides by

$x \tan 34^0 = 40$

$0.6745x = 40$

$x = \frac{40}{0.6745}$

$= 59.30$

$\therefore$ The object is 59.3m from the base of the cliff.

Summary: If the object is below the level of the observer, then the angle between the horizontal and the observer's line of sight is called the angle of depression.

Evaluation: What is angle of depression?

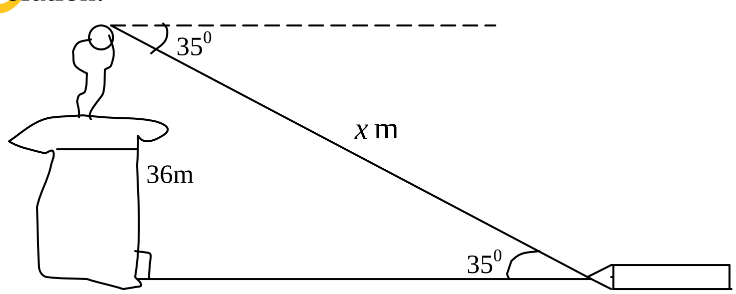
Assignment: From the top of a building 50m high, the angle of depression of a car is 55^0. Find the distance of the car from the foot of the building?

Period 2

Topic: More examples on angles of Depression.

1. A girl on top of a storey building 36m high dropped a pencil on the ground below. If the angle of depression of the pencil is 36^0, how far is the pencil away from the girl?

Solution:



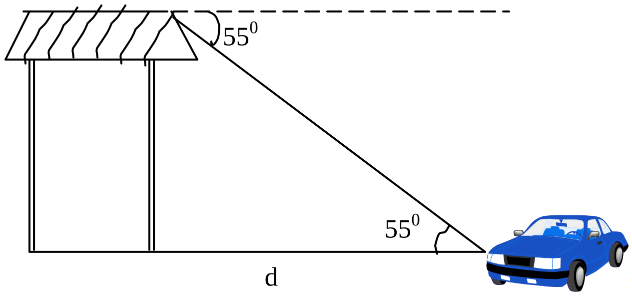
$$Sin \quad 35^0 = \frac{36}{x}$$

$$= \frac{36}{Sin\ 35^0} = \frac{36}{0.5735}$$

$$= 62.77m$$

2. From the top of a building 50m high, the angle of depression of a car is 55^0. Find the distance of the car from the foot of the building.

Solution:



$$\tan 55^0 = \frac{50}{d}$$

$$d = \frac{50}{\tan 55^0} = \frac{50}{1.428}$$

$$= 35.01$$

$$= 35m$$

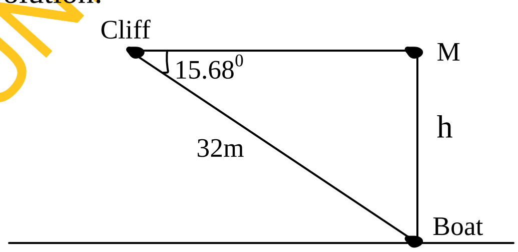
Period 3

Practice exercise on angles of depression

(1) The angle of depression of a boat as seen by a man on top of a cliff is 15.68^0. If the boat is 32m away from the man, what is the height of the cliff?

(2) Find the angle of depression from the top of a tower 50m high of an object on the ground distant 70m from the foot of the tower.

Solution:

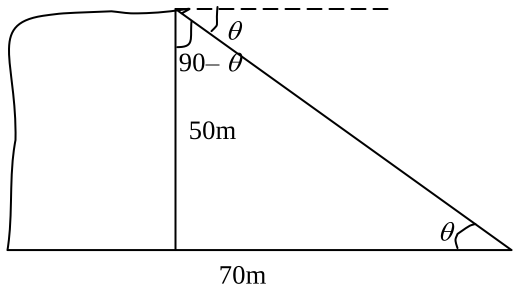


$$\frac{h}{32} = Sin\,15.68^0$$

$$\therefore h = 32\ Sin\,15.68$$

$$= 8.65m$$

(2)



$$\tan\theta = \frac{50}{70} = 0.7142$$

$$\theta = \tan^{-1} 0.7142$$

$$= 35.5^0$$

$$\cong 36^0$$

Assignment: From the top of a building 60m high, the angle of depression of a car is 45^0. Find the distance of the car from the foot of the building.

APPENDIX 3

CONTROL GROUP

INSTRUCTIONAL GUIDE FOR MODIFIED CONVENTIONAL TEACHING STRATEGY GROUP (MCTS)

General Information

Subject: Mathematics

Topics: (i) Circles (ii) Volumes (iii) Angles of Elevation and Depression

Class: SS II

Sex: Male [ ] Female [ ]

(ii) **About the Procedure**

The instructional guide was designed to teach students in SS II the following topics: Circle, Volume, and Angles of Elevation and Depression. The lessons were designed for 6 weeks of 3 periods each week.

(iii) **To the Teacher**

The guide was used by the teacher to teach the students. Use your normal way of teaching but by following the steps given in the guide accordingly.

(iv) **General Objectives**

At the end of the lessons in this guide, students should be able to:

(i) define circle
(ii) name object with circular face
(iii) find the area and sector of a circle
(iv) work sums on chords and arcs of circles
(v) state some circle theorems
(vi) apply circle theorems when necessary
(vii) work sums on cyclic quadrilaterals
(viii) define prism with examples
(ix) find the volume of cube and cuboid

(x) identify triangular prism and find its volume

(xi) identify cylinder and find its volume

(xii) find the volume of pyramid

(xiii) differentiate between angles of elevation and depression

(xiv) calculate angles of elevation and depression

Contents for each of the 6 Weeks

Week 1:

Topic: Circle Geometry

Period 1: Area and Arc of a Circle

2. Sectors and Chords of Circles

3. Circle Theorems

Week 2:

Topic: Circle Geometry

Period 1: Cyclic Quadrilaterals

2. Tangent to a Circle

3. Practice Exercises on Circle Geometry

Week 3:

Topic: Volume of Solids

Period 1: Volume of Solids (Prism)

2. Volume of Cube and Cuboids

3. Volume of Triangular Prism

Week 4:

Topic: Volume of Solids

Period 1: Volume of Cylinder

2. Volume and Frustum of Cone

3. Volume of Pyramid

Week 5:

Topic: Angles of Elevation and Depression

Period 1: Angles of Elevation

2. Angles of Elevation

3. Practice Exercises on Angles of Elevation

Week 6:

Topic: Angles of Elevation and Depression

Period 1: Angles of Depression

2. Angles of Depression

3. Practice Exercise on Angles of Depression

CONTROL GROUP

Instructional Guide for Modified Conventional Teaching Strategy (MCTS)

Week 1

Topic: Circle Geometry

Period 1: Area and Arc of a Circle

Presentation:

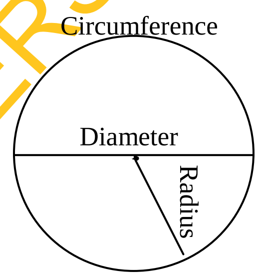
Step 1: Definition

A circle is a closed curve in a plane. It is the locus of all points equidistant from a central point.

Radius of a Circle: The radius of a circle is the distance from the centre of the circle to the outside edge.

Diameter of a Circle: The diameter starts at one side of the circle, goes through the centre and ends on the other side. It is the longest distance across a circle.

Step 2:



Circumference: The circumference of a circle is the distance around the edge of the circle. It is exactly π times the diameter.

$$Circumference = \pi \text{ x diameter}$$

$$But \text{ diameter} = 2r$$

$$\therefore \text{Circumference} = \pi \text{ X } 2 \text{ X r}$$

$$= 2\pi \text{ r}$$

$$\Rightarrow \pi = \frac{\text{Circumference}}{Diameter}$$

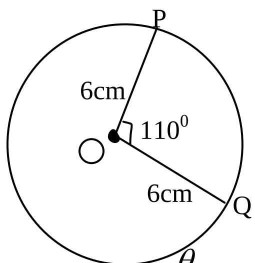
$$\text{Area of a circle} = \pi \text{ r}^2$$

$$Arc \text{ length} = \frac{\theta}{360^0} X \; 2\pi \text{ r}$$

Step 3: Examples

1. An arc subtends an angle of 110^0 at the centre of a circle of radius 6cm. Find the length of the arc if $\pi = \frac{22}{7}$

Solution:



$$arc \text{ length} = \frac{\theta}{360^0} X \; 2\pi \text{ r}$$

$$\therefore \text{arc PQ} = \frac{110^0}{360} X \, 2 \, X \frac{22}{7} X \frac{6}{1}$$

$$= \frac{11 X 22}{3 X 7}$$

$$= \frac{242}{21}$$

$$= 11.5cm$$

2. Calculate the area of a circle of radius 7cm which subtends 108^0 at centre, $\pi = \frac{22}{7}$

$$Area \text{ of a circle} = \pi \text{ r}^2$$

$$= \frac{22}{7} X \, 7 \, X \, 7$$

$$= 154cm^2$$

$$Arc\ \text{length} = \frac{\theta}{360^0} X\, 2\pi\, \text{r}$$

$$= \frac{108}{360^0} X\, 2 X \frac{22}{7} X \frac{7}{1}$$

$$= \frac{108\ \text{X}\, 11}{90}$$

$$\frac{1{,}188}{90}$$

$$13.2cm$$

Summary: A circle is a closed curve in a plane. Circumference of a circle can be calculated by using the following formula.

Circumference of a circle $= 2\pi\, \text{r}$

Area of a circle $= \pi\, \text{r}^2$

Arc length $= \dfrac{\theta}{360} X\, 2\pi\, \text{r}$

Evaluation: Allow them to solve the following question in class.

Question: What is the length of an arc which subtends an angle of 60^0 at the centre of a circle of radius $\dfrac{1}{2} m$?

Assignment

1. In terms of π, what is the length of an arc which subtends an angle of 30^0 at the centre of a circle of radius $3^1/_2$cm?

Period 2

Topic: Sectors and Chords of circles

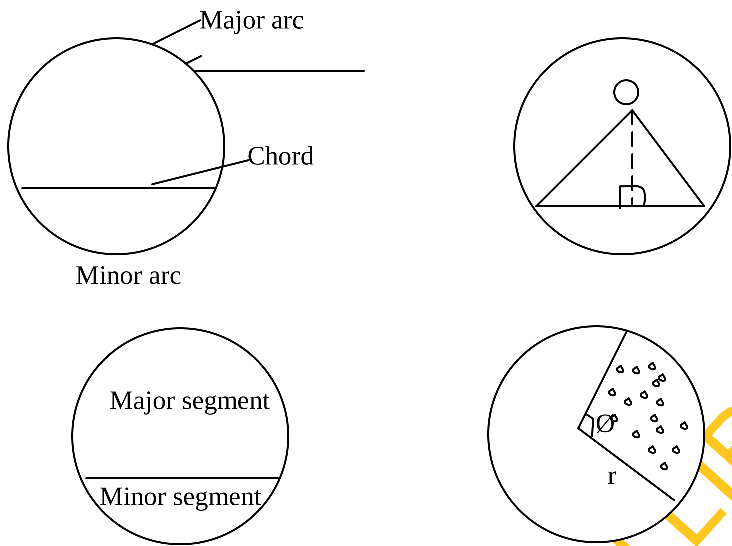
Presentation:

Step 1: Definition

A sector of a circle is a portion enclosed by two radii and an arc of the circle.

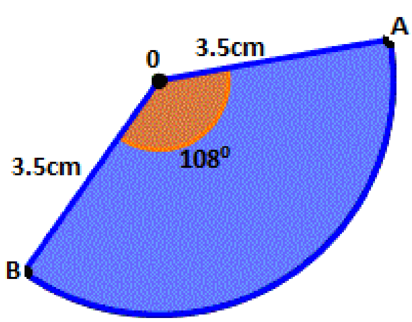
A chord of a circle is a line segment that connects one point on the edge of the circle with another point on the circle. The diameter is the longest chord.

Step 2:



Step 3: Examples

2. Calculate the perimeter of the following diagram



Solution:

Perimeter is the distance round the shape

Perimeter of sector AOB = arc AB + $\overline{AO} + \overline{BO}$

$$Arc\ \mathrm{AB} = \frac{\theta}{360} X\, 2\pi\ \mathrm{r}$$

$$= \frac{108}{360} X\, 2 X \frac{22}{7} X\, 3.5$$

$$= \frac{108}{360} X \frac{2}{1} X \frac{22}{7} X \frac{7}{2}$$

$$= \frac{33}{5}$$

$Arc\ \mathrm{AB} = 6.6\mathrm{cm}$

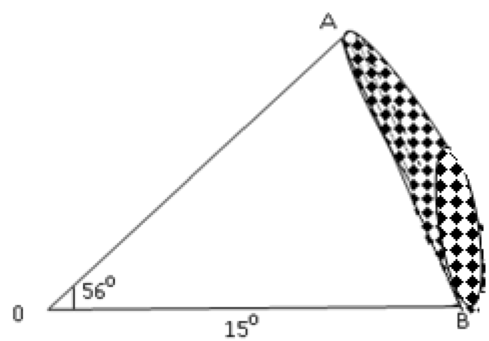
$\overline{\mathrm{AO}} = 3.5cm$

$\overline{BO} = 3.5cm$

$$\therefore Perimeter = (6.6 + 3.5 + 3.5)cm$$

$$= 13.6cm$$

2. Calculate the area of the shaded segment of the following diagram, if $\pi = \frac{22}{7}$



Area of the shaded part = Area of sector AOB– Area of ΔAOB

$$Area\ \mathrm{of\ sector\ AOB} = \frac{\theta}{360} X \pi\ \mathrm{r}^2$$

$$= \frac{56}{360} X \frac{22}{7} X \frac{15}{1} X \frac{15}{1}$$

$$= 110\mathrm{cm}^2$$

$$Area \text{ of } \Delta AOB = \frac{1}{2}\overline{AO} X \overline{BO} \text{ Sin } 56^0$$

$$= \frac{1}{2} X \frac{15}{1} X \frac{15}{1} X\, 0.8290$$

$$= \frac{186.525}{2}$$

$$= 93.2625 cm^2$$

$$\cong 93.3 cm^2$$

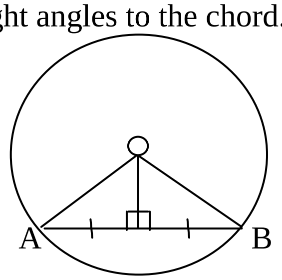
$\therefore Area$ of the shaded part =

$$(110 - 93.3) cm^2$$

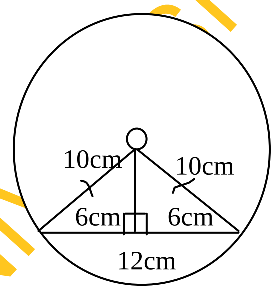
$$= 16.7 cm^2$$

Chord of a Circle

A straight line drawn from the centre of a circle to bisect a chord which is not a diameter is at right angles to the chord.



3. The radius of a circle is 10cm. The length of a chord is 12cm. Calculate the distance of the chord from the centre of the circle.



Solution:

$x^2 = 10^2 - 6^2$ (Pythagoras)

$= 100 - 36$

$= 64$

$x = \sqrt{64}$

$= 8cm$

Summary: A chord is an internal segment of a circle that has both of its endpoints on the circumference of the circle. The diameter of a circle is the largest chord possible. A sector is enclosed by two radii and an arc in the circle

Area of sector $= \frac{\theta}{360} X \pi \, r^2$

Evaluation: Ask them the following questions.

1. What is a sector of a circle?
2. How do we find the area of a sector?
3. Define chord of a circle
4. What do we call the largest possible chord?

Assignment

(1) The radius of a circle is 10cm. The length of a chord is 16cm. Calculate the distance of the chord from the centre of the circle.

(2) A sector with radius 14cm subtends an angle of 45^0 at the centre of a circle. What is the area of the sector?

Period 3

Topic: Circle Theorems

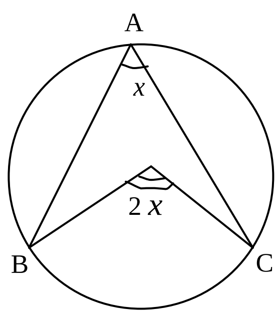
Presentation:

Step 1: Theorems

1. The angle that an arc of a circle subtends at the centre is twice that which it subtends at any point on the remaining part of the circumference.

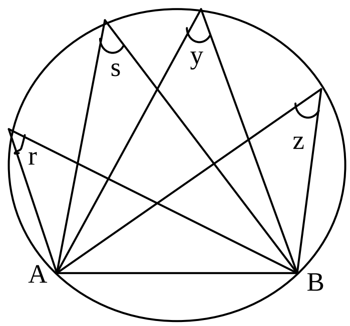
If BAC = x

Then BOC = 2 x



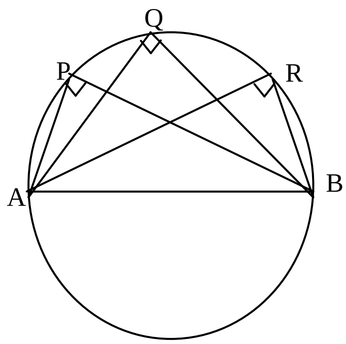
Step 2:

(ii) Angles in the same segment of circle are equal.



$\Rightarrow r = s = t = u$

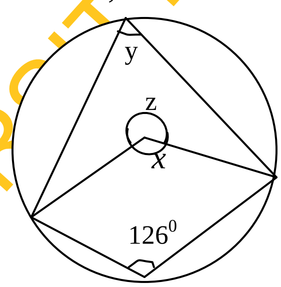
(ii) The angle in a semicircle is a right angle



$\Rightarrow$ P = Q = R = 90^0

Step 3: Examples

1. Find the angles x, y, and z from the following diagram



Solution:

z = 126 x 2 (angle at centre twice that at circumference)

z = 252^0

x + z = 360^0 (sum of angles at a point)

$x + 252^0 = 360^0$

$x = 360^0 - 252^0$

$= 108^0$

2y = x (angle at centre twice that at the circumference)

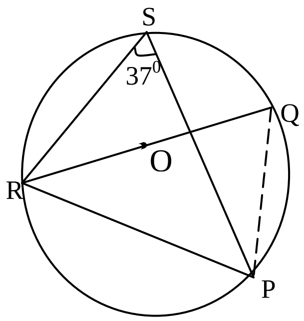
2y = 108^0

$$y = \frac{108}{2} = 54^0$$

$\therefore x = 108^0,\ y = 54^0,\ z = 252^0$

2. O is the centre of the following circle and QOR is the diameter.

PSR = 37^0. Find PRQ



Join P and Q

RSP = EQP = 37^0 (angles in the same segment)

RPQ = 90^0 (angle subtended by diameter)

$\therefore$ PRQ = $180^0 - (90^0 + 37)$ (sum of angles of a triangle)

= $180^0 - 127 = 53^0$

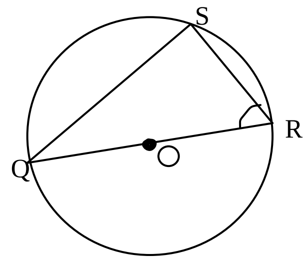
Summary: The angle that an arc of a circle subtends at the centre is twice that which it subtends at any point on the remaining part of the circumference. Angles in the same segment of a circle are equal. The angle in a semicircle is a right angle.

Evaluation: Ask them the following questions:

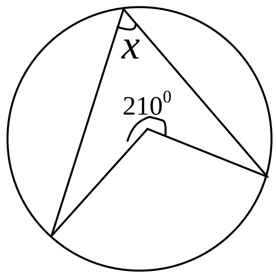
1. Angles in the same segment of a circle are ______ (equal/not equal)
2. The angle in a semicircle is a ______ (obtuse angle/right angle)
3. The angle that an arc of a circle subtends at the centre is ____ that which it subtends at any point on the remaining part of the circumference. (twice, three times)

Assignment: Solve the following questions

1. In the diagram below, 0 is the centre of the circle. If $Q\hat{R}S = 62^0$, find the value of SQR. (a) 14^0 (b) 28^0 (c) 31^0 (d) 90^0



2. If 0 is the centre of the following circle find x.



Week 2

Topic: Circle Geometry

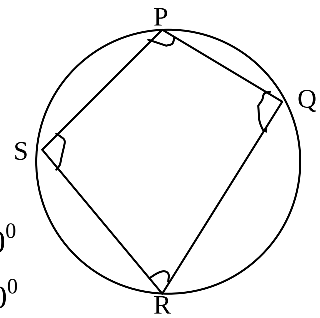
Period 1: Cyclic quadrilaterals

Presentation:

Step 1: Theorems

1. Angles in opposite segments are supplementary.
2. An exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

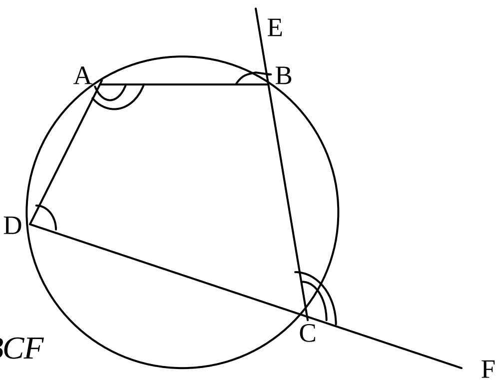
Step 2:



P + R = 180^0

S + Q = 180^0

(2)



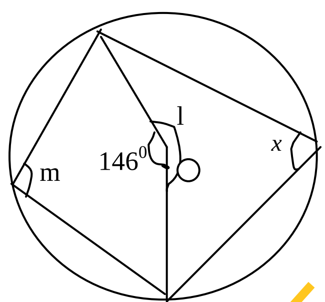
$\Rightarrow B\hat{A}D = BCF$

$ADC = ABE$

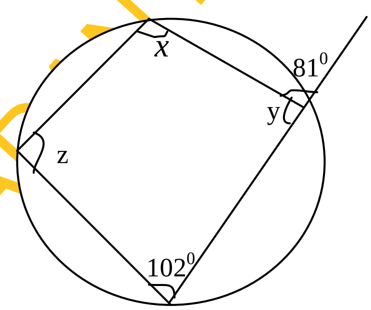
Step 3: Examples

Write down the sizes of the lettered angles in the following diagrams where 0 is given it is the centre of the circle.

(1)



(2)



Solution:

$x = \frac{1}{2} X 146^0$ (angle at centre)

$= 73^0$

$l = 360^0 - 146^0$ (angles at a point)

$= 214^0$

$m + x = 180^0$ (opposite∠s of a cyclic quadrilateral)

$m = 180^0 - x$

$= 180 - 73^0$

$= 107^0$

(2) $y + 81 = 180^0$ (angles on a straight line)

$y = 180^0 - 81^0$

$= 99^0$

$x + 102^0 = 180^0$ (*Opposite* ∠s of a cyclic quadrilateral)

$x = 180^0 - 102^0$

$= 78^0$

$z = 81^0$ (exterior angle of a cyclic quadrilateral equal interior opp. $\angle$)

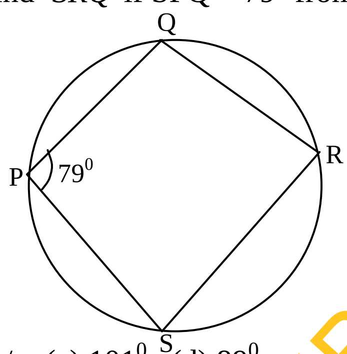
$\therefore\ x = 78^0$, $y = 99^0$, $z = 81^0$

Summary: Opposite angles of a cyclic quadrilateral are added up to 180^0. Also the exterior angle of a cyclic quadrilateral equal interior opposite angle.

Evaluation: Ask the students the following questions.

1. Opposite angles of a cyclic quadrilateral are supplementary. True/False
2. What can you say about the exterior angle of a cyclic quadrilateral?

Assignment: Find $S\hat{R}Q$ if SPQ = 79^0 from the following diagram.



(a) 79^0 (b) $39^1/_2$ (c) 101^0 (d) 89^0

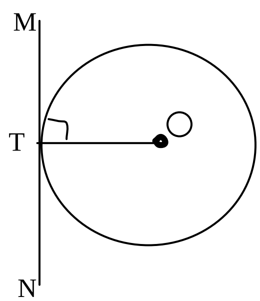
Period 2

Topic: Tangent to a Circle

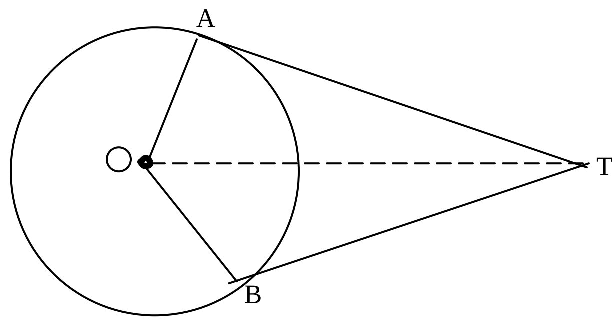
Step 1: Theorems

1. A tangent to a circle is perpendicular to the radius drawn to its point of contact.
2. The perpendicular to a tangent at its point of contact passes through the centre of the circle.

Step 2:



3. The tangents to a circle from an external point are equal.

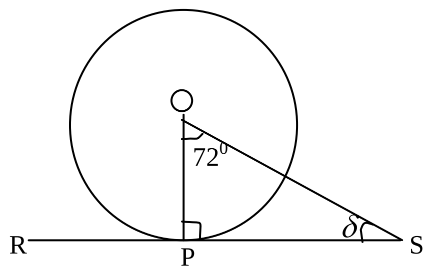


$\overline{TA} = \overline{TB}$

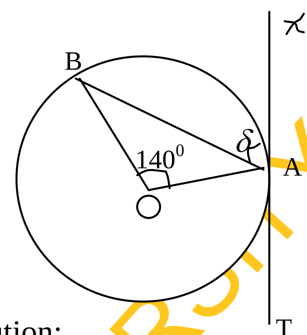
Step 3: Examples

Find the size of the angle δ from the following diagrams

(1)



(2)



Solution:

(1) $O\hat{P}S = 90^0$ (line from centre forms a $\perp$ line with the tanget)

$\therefore 9 + 0^0 + 72^0 + \delta = 180^0$ (sum of angles of a triangle)

$162^0 + \delta = 180^0$

$\delta = 180^0 - 162^0$

$= 18^0$

$(2)\quad \overline{OA} = \overline{OB}$ (radii)

$\hat{OBA} = \hat{OAB}$ (base angles of isosceles)

$\therefore \hat{OBA} + \hat{OAB} + 140^0 = 180^0$ (sum of angles of a triangle)

$\hat{OBA} + \hat{OAB} = 180^0 - 140^0 = 40^0$

$But\ \hat{OBA} = \hat{OAB}$

$\Rightarrow 2\ \hat{OAB} = 40^0$

$$OAB = \frac{40^0}{2}$$

$$= 20^0$$

$$O\hat{A}B = 20^0$$

$O\hat{B}A = 20^0$

OAB + BAX = 90^0 (line from centre)

$OAB + \delta = 90^0$

$20^0 + \delta = 90^0$

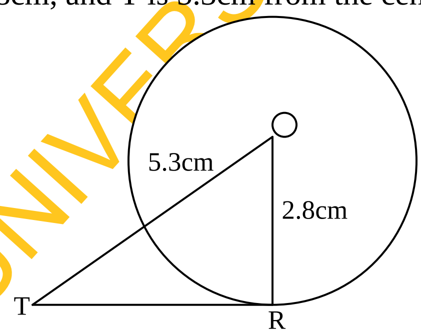
$\delta = 90^0 - 20^0$

$= 70^0$

Summary: The tangent does not cut the circle, it touches the circle. A tangent to a circle is perpendicular to the radius drawn to its point of contact. Also, the perpendicular to a tangent at its point of contact passes through the centre of the circle.

Evaluation: Allow them to solve the following question in class.

Question: The tangent from a point T touches a circle at R. If the radius of the circle is 2.8cm, and T is 5.3cm from the centre, calculate $\overline{TR}$.



$(TR)^2 = (5.3)^2 — (2.8)^2$ (Pythagoras)

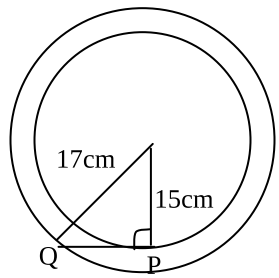
$(TR)^2 = 28.09 – 7.84$

$= 20.25$

$TR = \sqrt{20.25}$

$= 4.5cm$

Assignment: Two circles have the same centre and their radii are 15cm and 17cm. A tangent to the inner circle at P cuts the outer circle at Q. Calculate $\overline{PQ}$.



Period 3

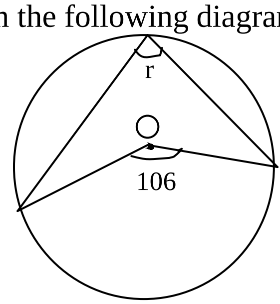
Topic: Practice Exercises on Circle Geometry

Step 1: The following questions are given to students as class exercises.

Pick the correct answer from the following options.

1. The radius of a circle is 10cm. The length of a chord of the circle is 16cm. Calculate the distance of the chord from the centre of the circle. (a) 5cm (b) 6cm (c) 7cm (d) 8cm

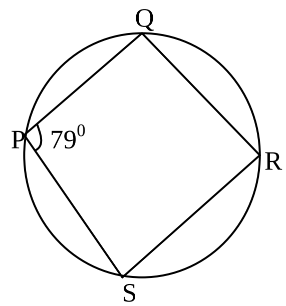
2. Calculate *r* from the following diagram?



(a) 53^0 (b) 63^0 (c) 35^0 (d) 36^0

3. A 216^0 sector of a circle of radius 5cm is bent to form a cone. Find the radius of the cone. (a) 3cm (b) 5cm (c) 6cm (d) 7cm

4. Find $S\hat{R}Q$ if $S\hat{P}Q = 79^0$ from the following diagram

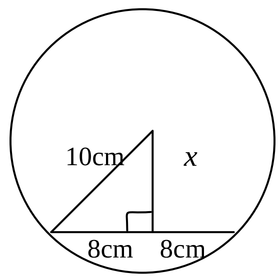


(a) 79^0 (b) $39^1/_2$ (c) 101^0 (d) 89^0

Step 2: Supervise their work and mark.

Step 3: Solution to the questions

(1)



$x^2 = 10^2 - 8^2$ (Pythagoras)

$= 100 - 64$

$= 36$

$\therefore = \sqrt{36}$

$= 6\text{cm}$

(2) $2r = 106^0$ (angle at centre twice that at the circumference)

$r = \dfrac{106}{2}$

$= 53^0$

(3) Area of a sector $= \dfrac{\theta}{360} X\pi\, r^2, \; r = 5\text{cm}$

Curve surface of cone $= \pi\, rl, \; l = 5\text{cm}$

$\Rightarrow$

$$\pi \mathrm{rl} = \frac{\theta}{360} X \pi \ \mathrm{r}^2$$

$$\pi \mathrm{rl} = \frac{216 X \pi \ \mathrm{r}^2}{360}$$

$$360^0 X \pi \ \mathrm{rl} = 216 \ \pi \ \mathrm{X}\ 5\ \mathrm{X}\ 5$$

$$\mathrm{r} = \frac{216\pi \ \mathrm{X}\ 5\ \mathrm{X}\ 5}{\pi \ \mathrm{X}\ 5\ \mathrm{X}\ 360^0}$$

$$= \frac{216 X 5}{360}$$

$$= \frac{1080}{360^0}$$

$$= 3cm$$

(4) SRQ + SPQ = 180^0 (opposite angles of a cyclic quadrilateral)

SPQ = 79^0

SRQ + 79^0 = 180^0

$\therefore$ SRQ = $180^0 - 79^0$

= 101^0

Step 4: Allow them to ask questions

Week 3

Topic: Volume of Solids

Period 1: Prism–Volume of Cube

Step 1: Definition

Point**:** A point is one of the basic terms in geometry. It is an entity that has only one characteristic, that is, it has position. A point has no length or width, it just specifies an exact location.

Straight line**:** A straight line is the shortest distance between two points. A straight line has one dimension which is length.

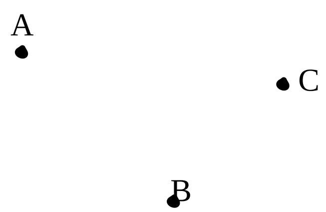
Rectangle**:** A rectangle has two dimensions length and breadth, it is 2 dimensional shapes. Examples of 2D shapes are squares, circles, triangles, parallelograms and all the polygons.

Prism**:** is a solid which has a uniform cross-section. It has two congruent parallel bases that are polygons.

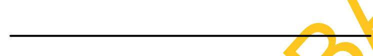
Examples of prism are cubes, cuboids, cylinder and pyramid. All prisms are 3 dimensional shapes

A cube is a 3 dimensional shape with square faces.

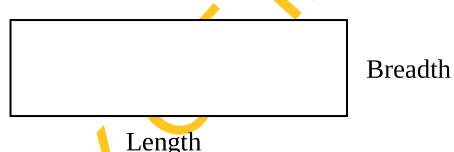
Step 2:



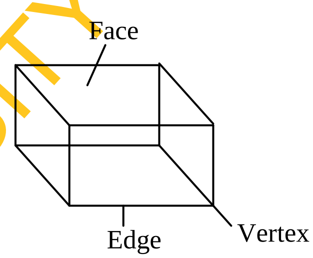
Points A, B and C



A line (ID)



A Cube



Volume of prism = Area of base x height

Volume of cube = length x length x length

$$= l \times l \times l$$

$$= l^3$$

$\Rightarrow$ Height = length = breadth

Area of base = l x l = l^2

$\therefore$ *Volume* of cube = Base area X height

$= l^2 \times l$

$= l^3$

Step 3: Examples

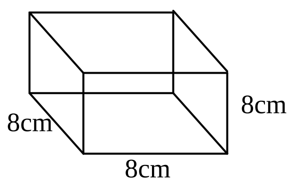
1. Calculate the volume of a cube whose edges are 8cm each.

Solution:

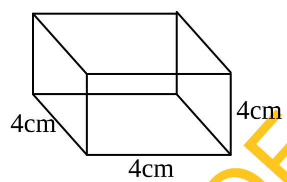
l = 8cm, b = 8cm, h= 8cm

Volume of cube = $(8 \times 8 \times 8)\ cm^3$

$= 512cm^3$



2. Find the volume of a cube below



Solution:

Volume $= (l \times l \times l)$ unit cube

$= (4 \times 4 \times 4)\ cm^3$

$= 64cm^3$

Summary: A prism is a solid with uniform cross section. A cube is a 3D shape. It has faces, edges and vertices.

Evaluation: Allow them to solve the following questions in class.

(1) Find the volume of a cube with sides 70cm.

(2) If the volume of a cube is 64cm find the sides of the cube?

Assignment: 1. Find the volume of a cube with sides 25cm

2. If the volume of a box in the shape of a cube is $2,197cm^3$, find the length of the edges.

Period 2

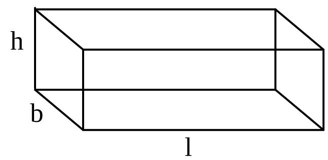
Topic: Volume of Cuboids

Step 1: A cuboid is a 3D shape with rectangular faces

Volume of cuboid = (l x b x h) unit cube

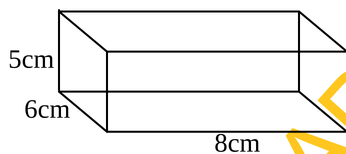
= Base area x h

Step 2:



Step 3: Examples

1. Find the volume of a cuboid with the following dimensions, length 8cm, breadth 6cm and height 5cm.



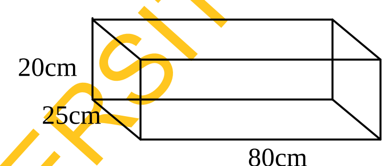
Solution:

Volume of cuboid = (l x b x h)

= (8 x 6 x 5) cm^3

= $240cm^3$

2. A car petrol tank is 0.8m long, 25cm wide and 20cm deep. How many litres of petrol can it hold?



Solution:

Volume of tank = 0.8m x 25cm x 20cm

$\Rightarrow 1m = 100cm$

$0.8m = \frac{100}{1} X 0.8$

$= 80\text{cm}$

Volume = 80cm x 25cm x 20cm

= $40{,}000cm^3$

$1000cm^3$ = 1 litre

Capacity of tank = $\frac{40,000}{1000} = 40\,litres$

Summary: Cuboid is also a 3 dimensional figure. It has faces, edges and vertices. It is an example of prism. It has 6 faces, 12 edges and 8 vertices.

Evaluation: Solve the following question.

Find the volume of a box with length 15cm, breadth 6cm and height 4cm.

Assignment: The following questions are given as assignment.

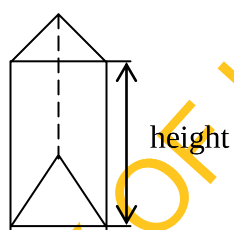
1. Find the volume of a cuboid with height 12cm, breadth 6cm and length 8cm.
2. Find the volume of a cuboid with 3cm by 4cm by 6cm.

Period 3

Topic: Volume of Triangular Prism

Step 1: A triangular based prism is a prism with a triangle-shaped base.

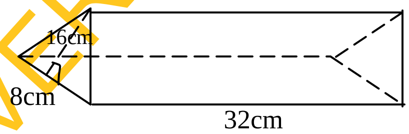
Step 2:



Vol. = area of base x height

Step 3: Examples

1. Find the volume of a triangular prism below



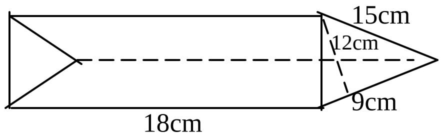
Solution:

Vol. of prism = (Area of triangular base x height)

$$\frac{1}{2} X \frac{8}{1} X \frac{16}{1} X \frac{32}{1}$$

$$= 2048cm^3$$

2. Find the volume of the following triangular prism



Solution:

$$Vol. = \frac{1}{2} X \frac{12}{1} X \frac{9}{1} X \frac{18}{1}$$
$$= 972cm^2$$

Summary: A prism is a solid of uniform cross section.

Volume of prism = area of cross section x height

A right triangular prism is a solid having a uniform triangular cross section.

Vol. = area of triangular base x length or height of the prism

Evaluation: Ask the following questions?

1. What is a prism?
2. How do we find the volume of a prism?
3. Find the volume of the following triangular prism with a right-angled triangular 3cm by 5cm, length 6cm.

Assignment: A right prism of length 10cm has as its cross-section equilateral triangle of side 6cm. Calculate its volume.

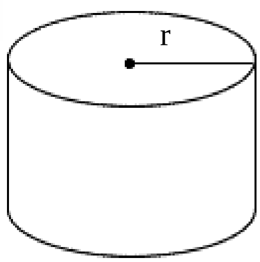
Week 4

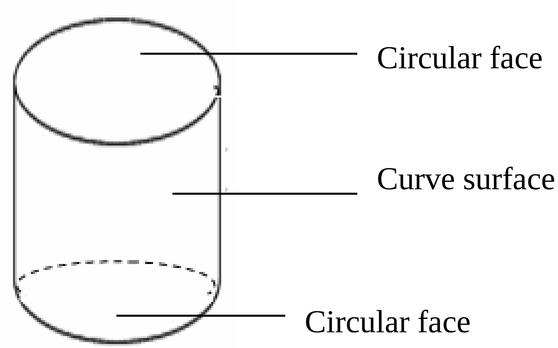
Topic: Volume of Solids

Period 1: Volume of Cylinder

Step 1: A cylinder is a solid with two congruent circles joined by a curved surface.

Step 2:





Volume of cylinder = $\pi r^2 h$

Step 3: Examples

1. Find the volume of a cylinder of radius 43.4cm and length 550cm.

Solution:

$$Vol.\text{ of cylinder} = \pi r^2 h$$

$$r = 43.4\text{cm},\ h = 550\text{cm},\ \pi = \frac{22}{7}$$

$$Vol. = \frac{22}{7} X \frac{(43.4)^2}{1} X\, 550$$

$$= \frac{22}{7} X\, 43.4 X\, 43.4 X\, 550$$

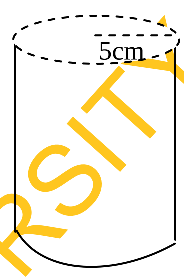
= (22 x 6.2 x 43.4 x 550) cm^3

= (12100 x 269.08) cm^3

= 3255868cm^3

= 3.26 x $10^6 cm^3$

2. Find the volume of the cylinder below



Solution:

$$Volume\text{ of cylinder} = \pi r^2 h$$

$$= \frac{22}{7} X \frac{5}{1} X \frac{5}{1} X \frac{21}{1}$$

$$= 22 \text{ x } 25 \text{ x } 3$$

$$= 1650\text{cm}^3$$

Summary: A cylinder is a solid with two congruent circles joined by a curve surface.

Volume of cylinder = $\pi r^2 h$

Evaluation: (1) A cylinder has how many circular faces.

(2) What is the formula for finding the volume of a cylinder?

Assignment: Find, correct to 1 decimal place, the volume of a cylinder of height 8cm and base radius 3cm $(Take\,\pi = 3.142)$.

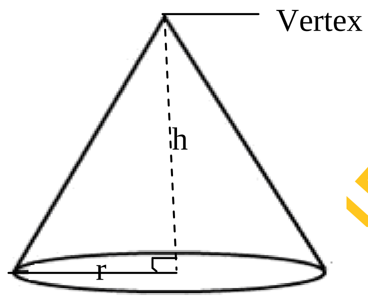
Period 2

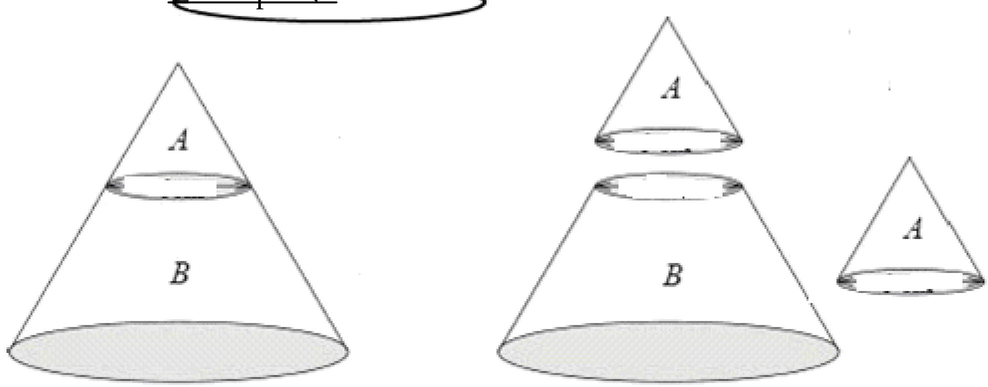
Topic: Volume and Frustum of Cone

Step 1: A circular cone has a circular base, which is connected by a curved surface to its vertex. A cone is called a right circular cone, if the line from the vertex of the cone to the centre of its base is perpendicular to the base. A cone is also referred to as circular base pyramid.

Step 2:

Cone





Frustum of cone

$$Volume\text{ of cone} = \frac{1}{3}\pi r^2 h$$

Volume of frustum of cone = Vol. of big cone – Vol. of small cone

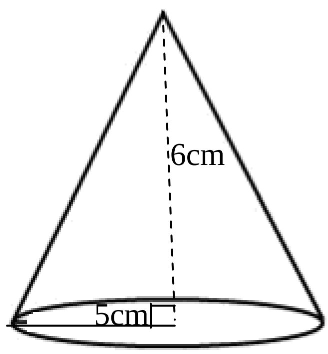
$$\frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h$$

Or Volume of frustum of cone = $\frac{1}{3}\pi h\left(R^2+r^2+Rr\right)$

Step 3: Examples

1. Calculate the volume of cone with base radius 5cm and height 6cm

Solution:



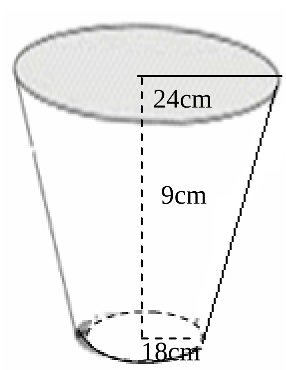
$$Vol.\text{ of cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} X \frac{22}{7} X \frac{5}{1} X \frac{5}{1} X \frac{6}{1}$$

$$= \frac{1{,}100}{7}$$

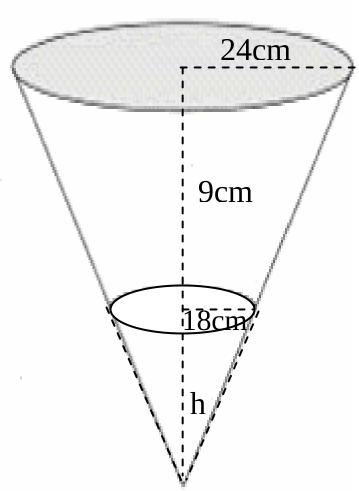
$$= 157.14cm^3$$

2. A bucket inform of a frustum, has upper and lower radii as 24cm and 18cm respectively, if the height is 9cm find the volume of the bucket in terms of π.



Solution:

H = 9 + h
R = 24cm
r = 18cm
h = ?



$$\frac{H}{h}=\frac{R}{r}$$

$$\frac{9+h}{h}=\frac{24}{18}=\frac{4}{3}$$

$$\frac{9+h}{h}X\frac{4}{3}$$

$$4h=3(9+h)$$

$$4h=27+3h$$

$$4h-3h=27$$

$$h=27$$

$$\therefore H=9+27=36cm$$

$\therefore$ Vol. of bucket = Vol. of big – Vol. of small

$$=\frac{1}{3}\pi R^2H-\frac{1}{3}\pi r^2h$$

$$=\frac{1}{3}\pi\left(R^2H-r^2h\right)$$

$$=\frac{1}{3}x\,\pi\left(24^2X36-18^2X27\right)$$

$$=\frac{1}{3}\pi\left(20737-8748\right)$$

$$=\frac{1}{3}\pi\left(11988\right)$$

$$=3996\pi cm^3$$

Summary: Volume of cone = $\frac{1}{3}\pi r^2h$ and Volume of frustum of cone =

Volume of big cone – Vol. of small cone

Or Volume of frustum of cone $\frac{1}{3}\pi h\left(R^2+r^2+Rr\right)$

Evaluation: 1. What is the formula for finding the volume of frustum of cone?

Assignment: Calculate the volume of cone with base radius 3.5cm, height 15cm. (2) A bucket inform of a frustum has upper and lower radii as 12cm and 8cm respectively, if the height is 10cm find the volume of the bucket in terms of π.

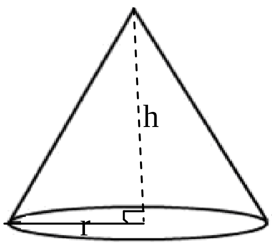
Period 3

Topic: Volume of Pyramid

Step 1: Volume of a pyramid

A pyramid is a solid with a polygon base which meets at common point called the apex or vertex. Different types of pyramid are; cone (circle-based-pyramid), square-based pyramid, rectangular-based pyramid.

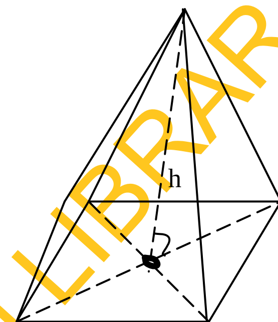
Step 2:



Circular-based pyramid

$Vol. \frac{1}{3}\pi r^2 h$

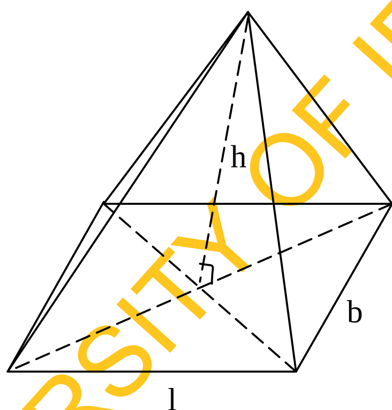
r = radius
h = height



Square-based

$Vol. \frac{1}{3}Ah = \frac{1}{3}l^2 h$

A = area, l = length,
h = height



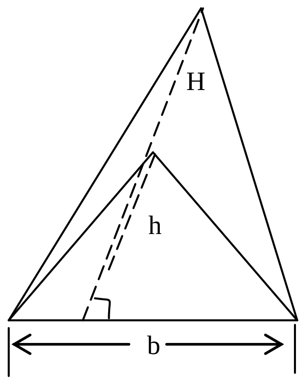
Rectangular base pyramid

$Vol. \frac{1}{3}Ah$ *where* A = area

h = height

Or Vol. $\frac{1}{3}$ lbh   where l = length, b = breadth

h = height



$$Vol. = \frac{1}{3}\text{ Ah}$$
$$= \frac{1}{3}\left(\frac{1}{2}bh\right)H$$
$$= \frac{1}{6}bhH$$

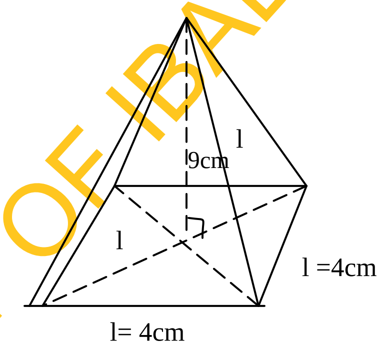
Triangular-based pyramid

Step 3: Examples

1. A pyramid has a square base of side 4cm and height of 9cm. Find its volume

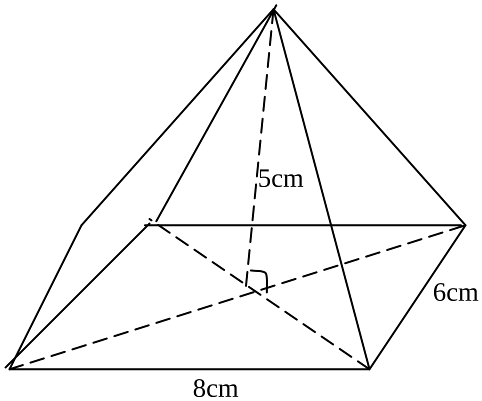
   Volume of a square-based Pyramid

Solution:



$$Volume = \frac{1}{3}Ah$$
$$= \frac{1}{3}\ l^2h$$
$$= \frac{1}{3}X\,4^2\,X\,9$$
$$= 16\text{ X }3$$
$$= 48\text{cm}^3$$

2. Find the volume of a rectangular-based pyramid whose base is 8cm by 6cm and height is 5cm.

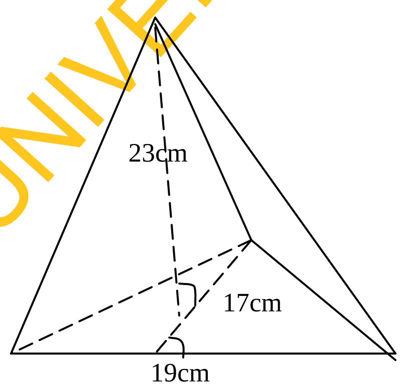


Solution:

$$Volume = \frac{1}{3}Ah$$
$$= \frac{1}{3} l\ b\ h$$
$$= \frac{1}{3} X \frac{8}{1} X \frac{6}{1} X \frac{5}{1}$$
$$= 80cm^3$$

3. Find the volume of the triangular-based pyramid with base 19cm, height of the triangle 17cm and height of the pyramid is 23cm, correct the answer to 2 decimal places.

Solution:



$$Volume = \frac{1}{3}AH$$
$$= \frac{1}{3}\left(\frac{1}{2}bh\right)H$$
$$= \frac{1}{6}bhH$$
$$= \frac{1}{6}X19X17X23$$
$$= 1238.1667$$
$$= 1238.17 \; to \; 2 \text{ d.p.}$$

Summary: Volume of pyramid = $\frac{1}{3}$Ah

Different types of pyramid are: Circular-based pyramid (cone), Square-based pyramid, Rectangular-based pyramid, and Triangular-based pyramid.

Evaluation: (1) What is a pyramid?

(2) What is the formula for finding the volume of a pyramid?

Assignment: (1) A pyramid has a square base of side 6cm and a height of 15cm. Find its volume

(2) Find the volume of a rectangular-based pyramid whose base is 10cm by 8cm and height 5cm.

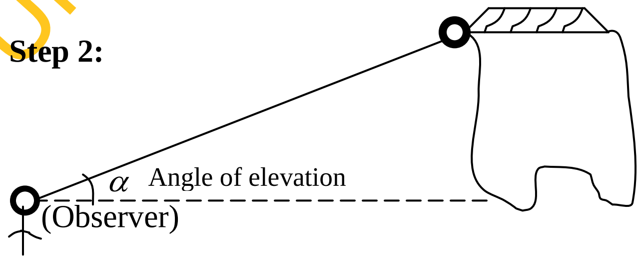
Week 5

Topic: Angles of Elevation and Depression

Period 1: Angles of Elevation

Step 1: The angle of elevation is the angle between a horizontal line from the observer and the line of sight to an object that is above the horizontal line.

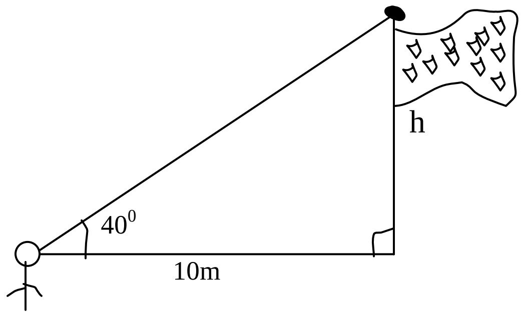
Step 2:



α is the angle of elevation.

Step 3: Examples

1. Find the height of the following flagpole



Solution:

$$\tan 40^0 = \frac{h}{10}$$

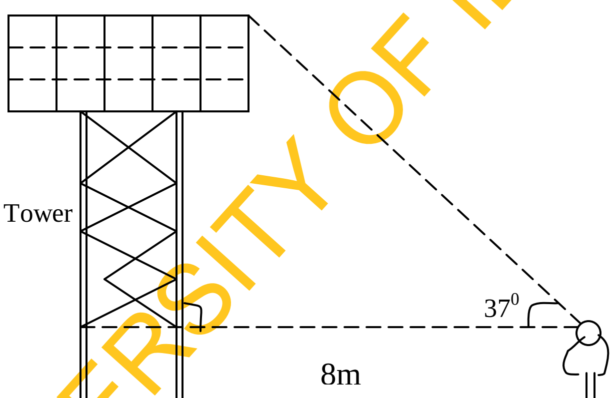
$h = 10 \tan 40^0$

$= 10 \times 0.839$

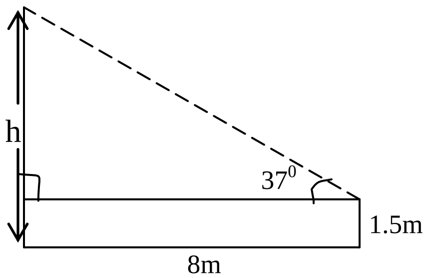
$= 8.39$

$= 8.4m$

2. Find the height of the water tower below to the nearest $^1/_2$m



Solution:



$$\tan 37^0 = \frac{h-1.5}{8}$$

$$\Rightarrow h - 1.5 = 8 \tan 37^0$$

$$= 8 \text{ X } 0.7534$$

$$= 6.028$$

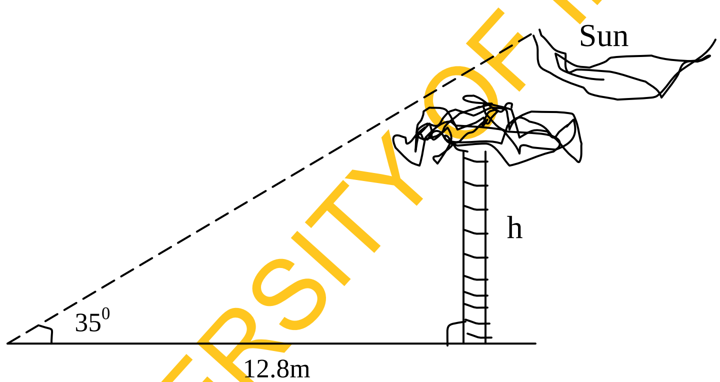
$$\text{h} - 1.5 = 6.028$$

$$\text{h} = 6.028 + 1.5$$

$$= 7.5284$$

$$= 7.5\text{m}$$

3. The angle of elevation of the sun is 45^0. A tree has a shadow 12m long. Find the height of the tree.



Solution:

$$\tan 35^0 = \frac{h}{12.8}$$

$$h = 12.8 \tan 35^0$$

$$= 12.8 \text{ X } 0.7002$$

$$= 8.96$$

$$= 9\text{m}$$

Summary: The angle of elevation of an object as seen by an observer is the angle between the horizontal and the line from the object to the observer's eye (the line of sight).

Evaluation: How do we find tangent of an angle?

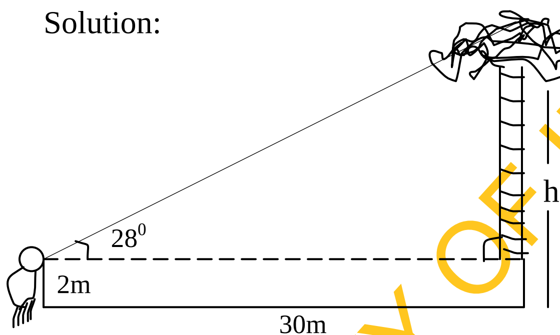
Assignment: The angle of elevation of the top of a tower as seen by a man 5cm away from the foot of the tower is 37^0. The angle of elevation of the top of the tower as seen by a second man standing on the opposite side of the tower is $49^{0.}$ How far are the men?

Period 2

Topic: More examples on angles of elevation

1. A man who is 2m tall stands on horizontal ground 30m from a tree. The angle of elevation of the top of the tree from his eyes is 28^0. Find the height of the tree.

Solution:



$$\tan 28^0 = \frac{h-2}{30}$$

$$h - 2 = 30 \tan 28^0$$

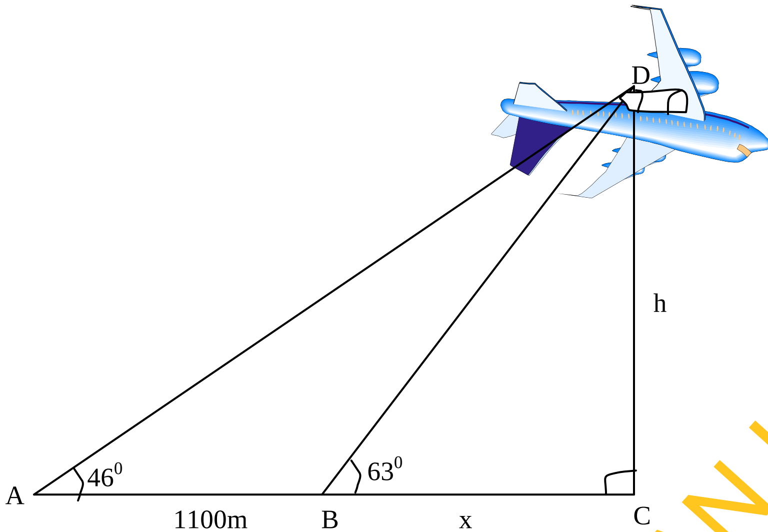
$$= 30 X 0.5317$$

$$h - 2 = 15.95$$

$$h = 15.95 + 2$$

$$= 17.95m$$

2. The following figure shows the angle of elevation of an aircraft from two points 1100m, apart. Find the height of the aircraft above the ground to the nearest 100m.



Let BC = x

$$\tan 46^0 = \frac{h}{1100 + x}$$

$$h = (1100 + x)\ \tan 46^0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(i)$$

$$\tan\ 63^0 = \frac{h}{x}$$

$$h = x \tan 63^0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

Since h = h

$\therefore$ Equate equations (i) and (ii)

$\Rightarrow$ (1100 + x) tan 46^0 = x tan 63^0

1100 tan 46^0 + x tan 46^0 = x tan 63^0

1100 tan 46^0 = x tan 63^0 — x tan 46^0

1100 tan 46^0 = x (tan 63^0 — tan 46^0)

$$x = \frac{1100\ \tan 46^0}{\tan 63^0\ \text{- } \tan 46^0}$$

$$= \frac{1100 X 1.0355}{1.9626 - 1.0355}$$

$$= \frac{1139.08}{0.9271}$$

$$\therefore x = 1228.64$$

From equation (i)

$h = (1100 + x) \tan 46^0$

$= (1100 + 1228.64)\ 1.0355$

$= 2328.64 \times 1.0355$

$= 2411.306$

To the nearest 100m

$\Rightarrow$ h= 2400m

Period 3

Practice exercise on angles of elevation

Allow them to practice the following questions in class.

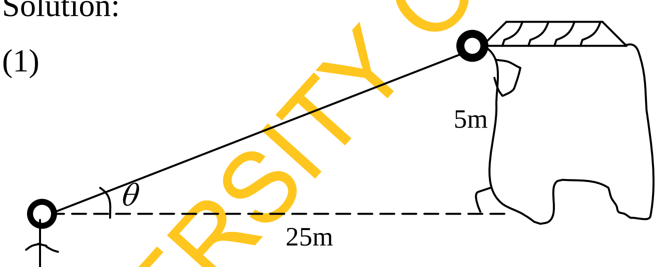
1. A man standing 25m away from a building observes the window of the house 5m above the ground. Calculate the angle of elevation of the window from the man.

2. The angle of elevation of the top of a tower from a point 42cm away from its base on level ground is 36. Find the height of the tower.

3. The angle of elevation of the sun is 27^0. A man is 180cm tall. How far is his shadow? Give your answer to the nearest 10cm.

Solution:

(1)



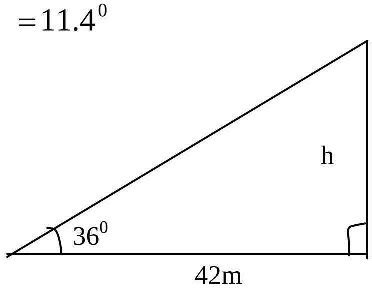
$\tan\theta = \frac{5}{25}$

$\tan\theta = 0.2$

$\theta = \tan^{-1} 0.2$

$= 11.4^0$

(2)



$$\tan 36^0 = \frac{h}{42}$$

$$h = 42 \tan 36^0$$

$$h = 42 \text{ X } 0.7265$$

$$= 30.5\text{m}$$

(3) $$\tan 27^0 = \frac{180}{x}$$

$$x = \frac{180}{\tan 27^0} = \frac{180}{0.509}$$

$$\text{X} = 353.6$$

to the nearest 10cm

= 350cm

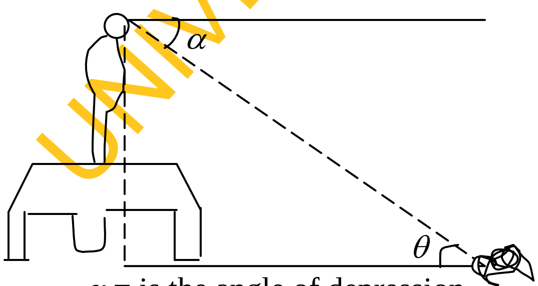
Week 6

Topic: Angles of Elevation and Depression

Period 1: Angles of Depression

Step 1: The angle of depression is the angle between a horizontal line from the observer and the line of sight to an object that is below the horizontal line.

Step 2:

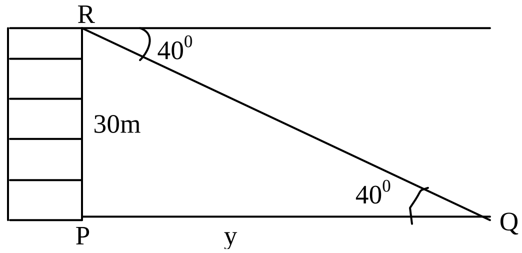


α = is the angle of depression.

Step 3: Examples

1. The angle of depression of a point Q from a vertical tower PR, 30m high is 40^0. If the foot P of the tower is on the same horizontal level as Q. Find correct to 2 decimal; places /PQ/.

Solution:

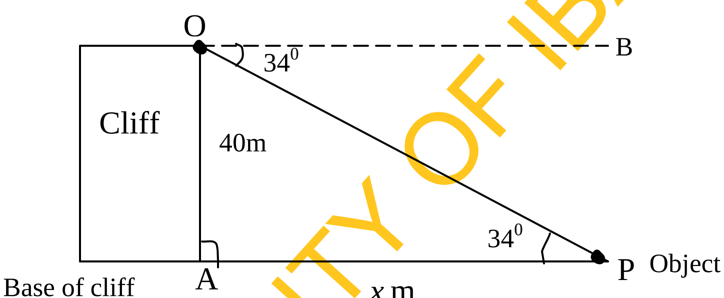


$$\tan\ 40^0 = \frac{30}{y}$$

$$y = \frac{30}{\tan 40^0} = \frac{30}{0.8390}$$
$$= 35.75m$$

2. From the top of a vertical cliff 40m high, the angle of depression of an object that is level with the base of the cliff is 34^0. How far is the object from the base of the cliff?

Solution



$A\hat{P}O = B\hat{O}P$ (*Alternate* angles)

$A\hat{P}O = 34^0$

$$\tan 34^0 = \frac{40}{x}$$

Multiply both sides by x

x tan 34^0 = 40

0.6745x = 40

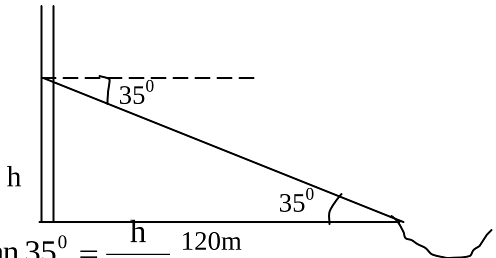
$$x = \frac{40}{0.6745}$$

= 59.30

$\therefore$ The object is 59.3m from the base of the cliff.

3. The angle of depression of a boat from the mid-point of a vertical cliff is 35^0. If the boat is 120m from the foot of the cliff, calculate the height of the cliff.

Solution:



$\tan 35^0 = \frac{h}{120^0}$

h= 120^0 X tan 35^0

= 120^0 X 0.7002

= 84.02

$\therefore$ 2h = 84.02 X 2

= 168m

Summary: If the object is below the level of the observer, then the angle between the horizontal and the observer's line of sight is called the angle of depression.

Evaluation: What is angle of depression?

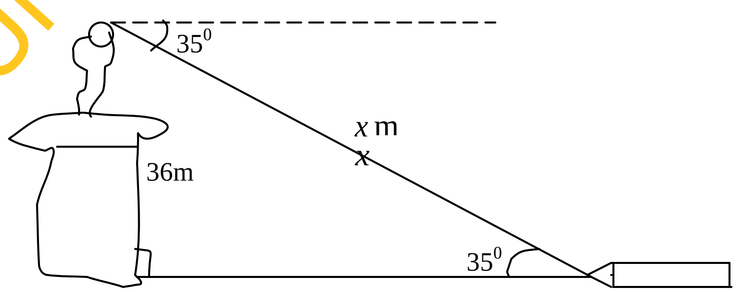
Assignment: From the top of a building 50m high, the angle of depression of a car is 55^0. Find the distance of the car from the foot of the building?

Period 2

More Examples on Angles of Depression.

1. A girl on top of a storey building 36m high dropped a pencil on the ground below. If the angle of depression of the pencil is 36^0, how far is the pencil away from the girl?

Solution



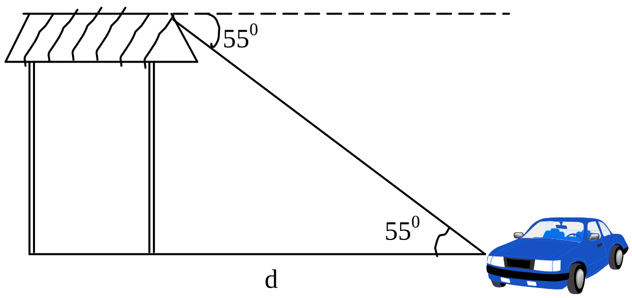
$$Sin \quad 35^0 = \frac{36}{x}$$

$$= \frac{36}{Sin\ 35^0} = \frac{36}{0.5735}$$

$$= 62.77m$$

2. From the top of a building 50m high, the angle of depression of a car is 55^0. Find the distance of the car from the foot of the building.

Solution:



$$\tan 55^0 = \frac{50}{d}$$

$$d = \frac{50}{\tan 55^0} = \frac{50}{1.428}$$

$$= 35.01$$

$$= 35m$$

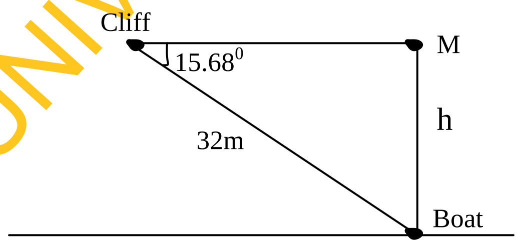
Period 3

Practice Exercise on Angles of Depression

(1) The angle of depression of a boat as seen by a man on top of a cliff is 15.68^0. If the boat is 32m away from the man, what is the height of the cliff?

(2) Find the angle of depression from the top of a tower 50m high of an object on the ground distant 70m from the foot of the tower.

Solution:

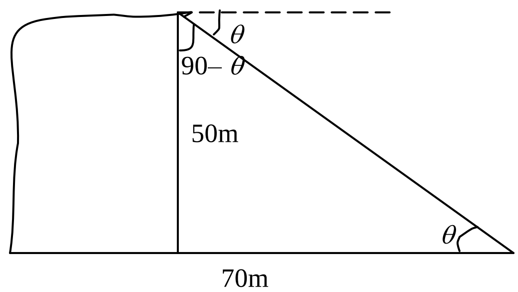


$$\frac{h}{32} = Sin\,15.68^0$$

$$\therefore h = 32\,Sin\,15.68$$

$$= 8.65m$$

(2)



$$\tan\theta = \frac{50}{70} = 0.7142$$

$$\theta = \tan^{-1} 0.7142$$

$$= 35.5^0$$

$$\cong 36^0$$

Assignment: From the top of a building 60m high, the angle of depression of a car is 45^0. Find the distance of the car from the foot of the building.

APPENDIX 4A

TEST ON STUDENTS FOR MATHEMATICS LEARNING DIFFICULTIES (TOSMALD)

Name of Student: ………………………………………

School: ………………………………………………… LGA:……………

Sex: ……………………………………………………..

Class: ……………………………………………………..

Age:…………………………………………………….

Part 1

Answer the following questions in your answer scripts, all workings must be shown. Then attach your question with your answer script.

(1) What must be added to 7,892 to make 10,101?

(2) A car covered a distance of 24km in 25minutes. Find its speed in km per hour.

(3) Express 216 as product of its prime factors using index notation.

(4) Simplify 2/3 x 4.

(5) Correct 4.002 to 1 decimal place.

(6) Share N18:00 in the ratio 2:7.

(7) Express 3/4 as percentage.

(8) Simplify $4^{c-1} = 64$.

(9) Express 0.0000824 in standard form.

(10) 9,936 drinking cups are imported in cartons holding 72 cups each; how many cartons are required?

APPENDIX 4B

TEST ON STUDENTS FOR MATHEMATICS LEARNING DIFFICULTIES (TOSMALD)

Name of Student: ………………………………………

School: ………………………………………………… LGA:……………

Sex: …………………………………………………..

Class: ………………………………………………..

Age:…………………………………………………….

Part 2

Answer the following questions by showing the workings. Then attach your answer script with your question paper.

(1) The sum of three numbers is 78,000; two of them are 25,520 and 37,672; find the third number.

(2) A man is paid N200 for 2½ days. Find his pay for 1 day?

(3) Find the prime factors of 30.

(4) Simplify 3/11 of 1 5/6.

(5) Correct 0.0052607 to 4 decimal places.

(6) Share 44 mangoes in the ratio 2:9.

(7) In a consignment of 125kg of bananas, 8% were found to be bad. What was the weight of good bananas?

(8) Solve the equation $4^{2y} = 1/32$.

(9) Express 0.00824 in standard form.

(10) A large Grammar school has library of 10,005 books arranged in shelves. If each shelf holds 145 books, how many shelves are there?

APPENDIX 4C

TEST ON STUDENTS FOR MATHEMATICS LEARNING DIFFICULTIES (TOSMALD)

Name of Student: ……………………………………

School: ……………………………………………… LGA:……………

Sex: ………………………………………………..

Class: ……………………………………………..

Age:……………………………………………….

Part 3

Work out the following questions**.** Attach your answer script with your question paper

(1). If the average amount of pocket money in a class of 28 children is 63 ½ kobo, find the total amount they received altogether.

2). How long would it take the engine of a train traveling at 48km per hour to pass through a station 80m long ?

3). Express 72 has product of its prime factors using the index notation.

4). Find the product of 2 ¾ and 5.

5). Correct 0.0052607 to 4 significant figure.

6). Share 40m in the ratio 3:7.

7). Express 90% as decimal fractions.

8). Solve the equation $32 = 8^3/2^x$.

9). Express two million in standard form.

10). Divide 559 by 13.

APPENDIX 5A

STUDENTS MATHEMATICS ACHIEVEMENT TEST (SMAT)

Supply the following information correctly.

Name of Student: …………………………………

School: ……………………………………………   **LGA:** ……………………

Age:…………………………………………………..

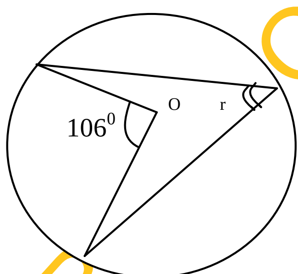
Sex: ……………………………………………….

Class: ……………………………………………...

Time: $1^1/_2$ hours

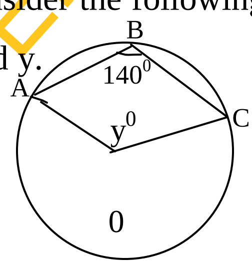
Instruction: This test consists of forty-five items. Each question is followed by four options lettered A, B, C, D. Choose the correct option from the alternatives provided. ***Answer the entire questions, avoid guessing.***

1. The radius of a circle is 10cm. The length of a chord of the circle is 16cm. Calculate the distance of the chord from the center of the circle. (a) 5cm (b) 6cm (c) 7cm (d) 8cm
2. Find the lettered angle from the following diagram, 0 is the centre of the circle.



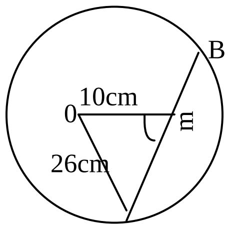
(a) 53^0 (b) 63^0 (c) 35^0 (d) 36^0

3. Consider the following figure, 0 is the centre of the circle and ABC = 140˚. Find y.



(a) 20^0 (b) 70^0 (c) 80^0 (d) 240^0

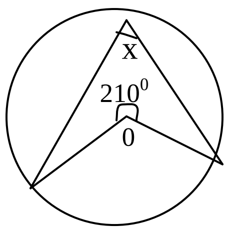
4. Calculate the length of a chord of a circle of radius 26cm if the chord is 10cm from the centre of the circle.



(a) 42cm (b) 45cm (c) 48cm (d) 24cm

5. A chord of length 24cm is 13cm from the center of the circle. Calculate the radius of the circle. (a) 17.69cm (b) 16.79cm (c) 19.76cm (d) 17.96cm

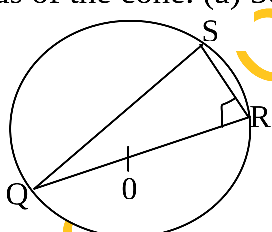
6. If O is the centre of the following circle find x.



(a) 70^0 (b) 75^0 (c) 105^0 (d) 150^0

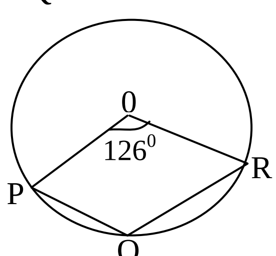
7. A 216^0 sector of a circle of radius 5cm is bent to form a cone. Find the radius of the cone. (a) 3cm (b) 5cm (c) 6cm (d) 7cm

8.



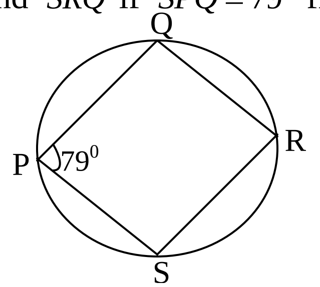
In the diagram above, 0 is the centre of the circle, If $Q\hat{R}S = 62^0$, find the value of $S\hat{Q}R$ (a) 14^0 (b) 28^0 (c) 31^0 (d) 90^0

9. Find PQR from the following diagram, 0 is the centre



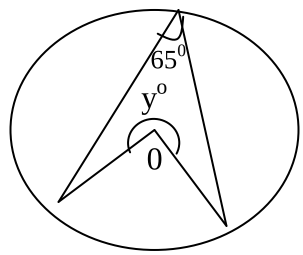
(a) 234^0 (b) 72^0 (c) 117^0 (d) 54^0

10. Find $S\hat{R}Q$ if $S\hat{P}Q = 79^0$ from the following diagram.



(a) 79^0 (b) $39^1/_2{}^0$ (c) 101^0 (d) 89^0

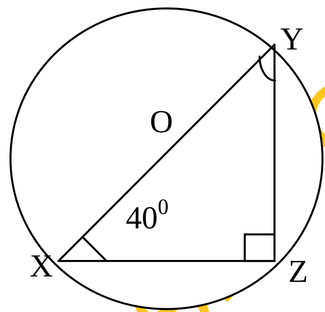
11.



In the diagram above, O is the centre of the circle. Find angle y.

(a) 230^0 (b) 60^0 (c) 65^0 (d) 130^0

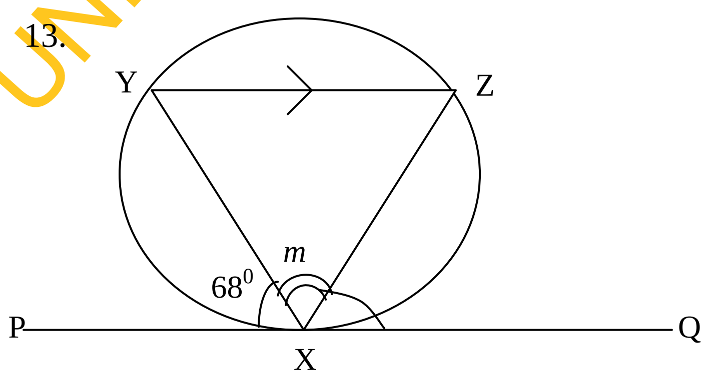
12.



In the diagram above, O is the centre of the circle. If angle YXZ = 40^0, find the value of ZYX.

(a) 140^0 (b) 130^0 (c) 90^0 (d) 50^0

13.



In the diagram above, X is the point of contact of PQ to the circle. YZ is a line joining two points on the circumference of the circle.

Find *m*, if PXY = 68^0.

(a) 136^0 (b) 112^0 (c) 68^0 (d) 44^0

14. A 120^0 sector of a circle of radius 21cm is bent to form a cone. What is the base radius of the cone? (a) 3½ cm (b)7cm (c) 10½ (d) 14cm

15. The angle of a sector of a circle is 108^0. If the radius of the circle is 3½cm, find the perimeter of the sector.

(a) $6\,^3/_5$ (b) $6^4/_5$cm (c) $7^1/_{10}$cm (d) $10^2/_5$cm

16. A pyramid 8cm high stands on a rectangular base 6cm by 4cm. Calculate the volume of the pyramid. (a) $46cm^3$ (b) $50cm^3$ (c) $64cm^3$ (d) $60cm^3$

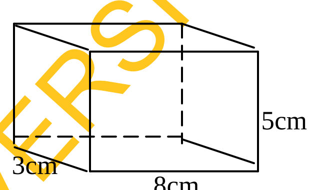
17. How many cylindrical glasses 6cm in diameter and 10cm deep can be filled from a cylindrical jug 10cm in diameter and 18cm deep?

(a) 4 (b) 5 (c) 6 (d) 7

18. A frustum of a cone has top and bottom diameters of 14cm and 10cm respectively and a depth of 6cm. Find the volume of the frustum in terms of π.

(a) 218 πcm^3 (b) 200 πcm^3 (c) 812 πcm^3 (d) $182\pi cm^3$

19. Calculate the volume of the following solid



(a) $210cm^3$ (b) $201cm^3$ (c) $102cm^3$ (d) $120cm^3$

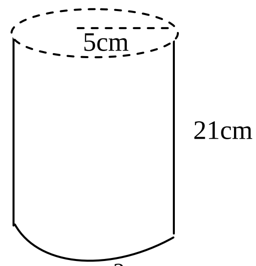
20. Calculate the volume of a cone of base diameter 14cm and height 5cm. $\left(Take\ \pi \text{ to be } \frac{22}{7}\right)$ your answer to 3.s.f. (a) $257cm^3$ (b) $250cm^3$ (c) $240cm^3$

(d) $620cm^3$

21. Calculate the volume of a cube with sides 4cm. (a) $12cm^3$ (b) $20cm^3$ (c) $64cm^3$ (d) $24cm^3$

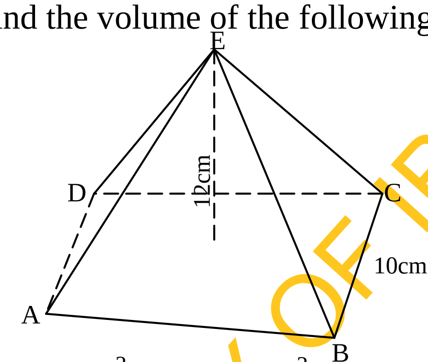
22. Find the volume of the rectangular solid with the dimensions 4cm length by 3cm breadth and 2cm height? (a) $24cm^3$ (b) $20cm^3$ (c) $42cm^3$ (d) $34cm^3$

23. Find the volume of the cylinder below



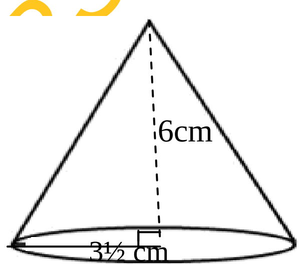
a) $16.50cm^3$ (B) $165.0cm^3$ (c) $1650cm^3$ (d) $1605cm^3$

24. Find the volume of the following square base pyramid



(a) $400cm^3$ (b) $400cm^2$ (c) $40cm^3$ (d) $4cm^3$

25. Calculate the volume of the following solid.



(a) $17cm^3$ (b) $42cm^3$ (c) $77cm^3$ (d) $70cm^3$

26. A water tank of height ½m has a square base of side 1½m. If it is filled with water from a water tanker holding 1500 litres, how many litres of water are left in the water tanker? [1000 litres = $1m^3$]

(a) 37.5 litres (b) 375 litres (c) 3750 litres (d) 37500 litres

A cylindrical container, closed at both ends, has a radius of 7cm and height 5cm. Use this information to answer questions 27 and 28. (Take $\pi = {}^{22}/_{7}$).

27. Find the total surface area of the container.

(a) $35cm^2$ (b) $154cm^2$ (c) $220cm^2$ (d) $528cm^2$

28. What is the volume of the container?

(a) $35cm^3$ (b) $154cm^3$ (c) $220cm^3$ (d) $770cm^2$

29. Find the total surface area of a solid circular cone with base radius 3cm and slant height 4cm. (Take $\pi = {}^{22}/_{7}$)

(a) $37^{5}/_{7}\,cm^2$ (b) $66cm^2$ (c) $75^{3}/_{7}$ (d) $78^{2}/_{7}cm^2$

30. Find, correct to 1 decimal place, the volume of a cylinder of height 8cm and base radius 3cm. (Take $\pi = 3.142$)

(a) $503.0cm^3$ (b) $300.0cm^3$ (c) $250.0cm^3$ (d) $226.2cm^3$

31. A man standing 25m away from a building observes the window of the house 5m above the ground. Calculate the angle of elevation of the window from the man. (a) 10^0 (b) 11.31^0 (c) 12^0 (d) 13^0

32. A man views the angle of elevation of the top of a tower to be $\theta^{0.}$ He is 40m from the foot of tower and at the same level. If the height of the tower is 23.09m, find θ. (a) 30^0 (b) 20^0 (c) 10^0 (d) 40^0

33. Find the angle of depression from the top of a tower 50m high of an object on the ground distant 70m from the foot of the tower. (a) 30^0 (b) 36^0 (c) 63^0 (d) 45^0

34. A boat is 125m away from the bottom of a cliff. If the cliff is 80m high, calculate the angle of depression of the boat from the top of the cliff.

(a) 32.6^0 (b) 35^0 (c) 40^0 (d) 50^0

35 The angle of elevation of the top of a building from a measuring instrument placed on the ground is 30^0. What is the angle of depression of the instrument from the top of the building? (a) 25^0 (b) 26^0 (c) 30^0 (d) 28^0

36. Find the angle of depression from the top window 10cm above the level ground of an object 21m from the foot of the building.
(a) 25.46^0 (b) 30^0 (c) 40^0 (d) 50^0

37. The angle of elevation of the top of a tower from a point 42m away from its base on level ground is 36^0. Find the height of the tower. (a) $28^1/_2$m (b) $30^1/_2$m (c) $20^1/_2$m (d) $40^1/_2$m

38. The angle of elevation of the sun is 45^0. A tree has a shadow 12m long. Find the height of the tree. (a) 10m (b) 11m (c) 12m (d) 13m

39. From the top of a building 50m high, the angle of depression of a car is 55^0. Find the distance of the car from the foot of the building. (a) 35m (b) 53m (c) 43m (d) 54m

40. The angle of elevation of the sun is 27^0. A man is 180cm tall. How long is his shadow? Give your answer to the nearest 10cm. (a) 350cm (b) 360cm (c) 340cm (d) 330cm

41. The angle of elevation of a point T on a tower from a point U on the horizontal ground is 30^0. If TU = 54m, how high is T above the horizontal ground? (a) 27m (b) 108m (c) 72m (d) 46.3m

42. The angle of depression of a point Q from a vertical tower PR, 30m high, is 40^0. If the foot P of the tower is on the same horizontal level as Q, find, correct to 2 decimal places, /PQ/. (a) 35.75m (b) 25.00m (c) 22.98m (d) 19.28m

43. The angle of elevation of the top of a tower from a point on the ground which is 36m away from the foot of the tower is 30^0. Calculate the height of the tower.
(a) 20.78m (b) 62.35m (c) 18.00m (d) 10.39m

44. The angle of depression of a boat from the mid-point of a vertical cliff is 35^0. If the boat is 120m from the foot of the cliff, calculate the height of the cliff.
(a) 168m (b) 170m (c) 185m (d) 187m

45. A girl on top of a storey building 36m high dropped a pencil on the ground below. If the angle of depression of the pencil is 35^0 how far is the pencil away from the girl?

(a) 62.76m (b) 44.49m (c) 49.55m (d) 20.64m

APPENDIX 5B

SOLUTION TO STUDENTS MATHEMATICS ACHIEVEMENT TEST (SMAT)

1.	B
2.	A
3.	C
4.	D
5.	A
6.	B
7	A
8.	B
9.	C
10.	C
11.	A
12.	D
13.	D
14.	B
15.	A
16.	C
17.	B
18.	A
19.	D
20.	A
21.	C
22.	A

23.	C
24.	A
25.	C
26.	B
27.	D
28.	D
29.	B
30.	D
31.	B
32.	A
33.	B
34.	A
35.	C
36.	A
37.	B
38.	C
39.	A
40.	A
41.	A
42.	A
43.	A
44.	A
45.	A

APPENDIX 6

STUDENTS MATHEMATICS ATTITUDES QUESTIONNAIRE (SMAQ)

Supply the following information correctly.
Name of Student: ……………………………………
School: ………………………………………………LGA:…………
Sex: ………………………………………………..
Class: …………………………………………

Instructions: Each of the statements expresses a feeling which a particular person has toward Mathematics. You are to express, on a four-point scale, the extent of agreement between the feeling expressed in each statement and your own feeling. The four points are: Strongly Agree (SA), Agree (A), Disagree (D), Strongly Disagree (SD). You are to tick the letter(s) which best indicates how closely you agree or disagree with the feeling expressed in each statement as it concerns you.

	Statements	**SA**	**A**	**D**	**SD**
1	I am sure that I can learn Mathematics				
2	Knowing Mathematics will help me earn a living				
3	I don't think I could do advanced Mathematics				
4	Mathematics will not be important to me in my life's work				
5	Males are not naturally better than females in Mathematics				
6	Mathematics is hard for me				
7	It's hard to believe a female could be a genius in Mathematics				
8	I'll need Mathematics for my future work				
9	When a woman has to solve a Mathematics problem, she should ask a man for help				
10	I am sure of myself when I do Mathematics				
11	I don't expect to use much teachers about a career that uses Mathematics				
12	Women can do just as well as men in Mathematics				
13	Mathematics is a worthwhile, necessary subject				
14	I would have more faith in the answer for a Mathematics problem solved by a man than a woman				
15	I'm not the type to do well in Mathematics				
16	Taking Mathematics is a waste of time				
17	Mathematics has been my worst subject				
18	Women who enjoy studying Mathematics are a little strange				
19	I think I could handle more difficult Mathematics				
20	I will use Mathematics in many ways as an adult				
21	Females are as good as males in geometry				
22	I see Mathematics as something I won't use very often when I get out of high school				
23	Women certainly are smart enough to do well in Mathematics				
24	Most subjects I can handle OK, but I just can't do a good job with Mathematics				

25	I can get good grades in Mathematics				
26	I'll need a good understanding of Mathematics for my future work				
27	I would expect a woman Mathematician to be a forceful type of person				
28	I know I can do well in Mathematics				
29	Studying Mathematics is just as good for women as for men				
30	Doing well in Mathematics is not important for my future				
31	I am sure I could do advanced work in Mathematics				
32	Mathematics is not important for my life				
33	I'm not good in Mathematics				
34	I study Mathematics because I know how useful it is				
35	I would trust a female just as much as I would trust a male to solve important Mathematics problems				