

# PERFORMANCES OF THE FULL INFORMATION ESTIMATORS IN A TWO-EQUATION STRUCTURAL MODEL WITH CORRELATED DISTURBANCES

A. A. ADEPOJU

(Received 19, November 2007; Revision Accepted 21, October 2008)

## ABSTRACT

The performances of two full information techniques, Three Stage Least Squares (3SLS) and Full Information Maximum Likelihood (FIML) of simultaneous equation models with correlated disturbance terms are compared with the Ordinary Least Squares (OLS) method in small samples. Comparative performance evaluation of the estimators was done using Average of Estimates, Total Absolute Bias (TAB) of Estimates, Root Mean Squared Error (RMSE) and Sum of Squared Residuals (RSS) of parameter estimates. The results of the Monte Carlo experiment showed that OLS is best with large negative or positive correlation, while 3SLS is best with feebly correlated error terms in the case of replication-based averages. The total absolute biases increase consistently as the sample size increases for OLS while FIML estimates reveal no distinct pattern. The magnitudes of the estimates yielded by two estimators, OLS and 3SLS, exhibited fairly consistent reaction to changes in magnitudes and direction of correlations of error terms.

**KEYWORDS:** Disturbance, Simultaneous Equation, Correlation, Structural Parameters, Bias.

## 1.0 INTRODUCTION

The single equation estimation methods lead to estimates that are consistent but, in general, not asymptotically efficient. The reason for lack of asymptotic efficiency is that single equation estimators do not take into account prior restrictions on the other equations in the model. This deficiency can be overcome by estimating all equations of the system simultaneously.

Many studies have revealed that full information methods such as 3SLS and FIML have an advantage over limited information methods like 2SLS and LIML in large samples. This result obtains because full information methods utilize the information concerning the contemporaneous disturbance,  $\varepsilon_t$ , and the over identifying restrictions arising from other equations, given that the simultaneous equation model is correctly specified. However, for incorrect specification (improper inclusion or exclusion of variables), it is not clear which estimator, limited information, full information or ordinary least squares, is to be preferred. The choice will depend on the form of the misspecification and which equations are involved. Summers<sup>11</sup> (1965) use Monte Carlo studies to study the performance of these estimators when specification error is present.

## The Model

The following two-equation simultaneous model with a mixture of exactly identified and over identified equations is assumed;

$$Y_{1t} = \beta_{12}Y_{2t} + \gamma_{11}X_{1t} + U_{1t}$$

$$Y_{2t} = \beta_{21}Y_{1t} + \gamma_{22}X_{2t} + \gamma_{23}X_{3t} + U_{2t}$$

where the Y's are the endogenous variables, X's are the predetermined variables and U's are the random disturbance terms,  $\beta$ 's and  $\gamma$ 's are the parameters.

The following levels of correlation between pairs of random deviates are assumed,

- (i) highly negatively correlated ( $r_{\varepsilon_1, \varepsilon_2} < -0.05$ )
- (ii) feebly negatively or positively correlated ( $-0.05 < r_{\varepsilon_1, \varepsilon_2} < +0.05$ )
- (iii) highly positively correlated ( $r_{\varepsilon_1, \varepsilon_2} > +0.05$ )

Other assumptions about the random error are,

$$E(U_t) = \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

$$E(U_{2N \times 1} U'_{1 \times 2N}) = E \left[ \begin{matrix} u_{11} \\ \vdots \\ u_{1N} \\ u_{21} \\ \vdots \\ u_{2N} \end{matrix} \begin{bmatrix} u_{11} & \dots & u_{1N} & u_{21} & \dots & u_{2N} \end{bmatrix} \right]$$

$$E(u_{it} \ u_{i't'}) = \begin{cases} \sigma_i^2, & i = i', \ t = t' \\ 0, & i = i', \ t \neq t' \\ \sigma_{ii'}, & i \neq i', \ t = t' \\ 0, & i \neq i', \ t \neq t' \end{cases}$$

$$= \begin{bmatrix} \sigma_{11} & 0 & 0 & \sigma_{12} & 0 & 0 \\ 0 & \ddots & 0 & 0 & \ddots & 0 \\ 0 & 0 & \sigma_{11} & 0 & 0 & \sigma_{12} \\ \sigma_{21} & 0 & 0 & \sigma_{22} & 0 & 0 \\ 0 & \ddots & 0 & 0 & \ddots & 0 \\ 0 & 0 & \sigma_{21} & 0 & 0 & \sigma_{22} \end{bmatrix}$$

$$= \Sigma = \Omega \otimes I_{2N}$$

where

$$\Omega = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} \text{ and}$$

$I_{2N}$  is the  $2N \times 2N$  identity matrix.

The reduced form equations are,

$$y_{1t} = \Pi_{11} X_{1t} + \Pi_{12} X_{2t} + \Pi_{13} X_{3t} + V_{1t} \tag{1.2}$$

$$y_{2t} = \Pi_{21} X_{1t} + \Pi_{22} X_{2t} + \Pi_{23} X_{3t} + V_{2t} \tag{1.3}$$

where,

$$\Pi = \begin{bmatrix} \gamma_{11} \beta^* & \beta_{21} \gamma_{22} \beta^* & \beta_{21} \gamma_{23} \beta^* \\ \beta_{12} \gamma_{11} \beta^* & \gamma_{22} \beta^* & \gamma_{23} \beta^* \end{bmatrix}$$

$$V_{1t} = (u_{1t} + \beta_{21} u_{2t}) \beta^*$$

$$V_{2t} = (\beta_{12} u_{1t} + u_{2t}) \beta^*$$

$$\beta^* = (1 - \beta_{12} \beta_{21})^{-1}$$

Equations (1.2) and (1.3) are subsequently used in deriving the values of stochastic endogenous variables,  $y_{1t}$ ,  $y_{2t}$  from selected values of  $X_1$ ,  $X_2$ ,  $X_3$ , assumed values of  $\beta_{12}$ ,  $\beta_{21}$ ,  $\gamma_{11}$ ,  $\gamma_{22}$ ,  $\gamma_{23}$  and assumed distribution of  $u_1$  and  $u_2$ .

The FIML estimator maximizes the likelihood function of an entire system's current endogenous variables, subject to the restrictions placed on the reduced form of

$$Y, [\beta \ \Gamma] \begin{bmatrix} \Pi \\ I_k \end{bmatrix} = 0, \text{ by the over identification of all}$$

equations.

The FIML estimator utilizes all information, hence the term "full", whereas the limited estimator uses only that information particular to the given equation. Naturally, the FIML estimator possesses all the properties of a maximum likelihood estimator, that is, consistency,

asymptotic normality and asymptotic efficiency in that the asymptotic covariance matrix of the FIML estimator achieves the asymptotic Cramer-Rao lower bound.

Where there exists no prior information on the variance-covariance matrix of the structural disturbances,  $\Sigma$  (for example, no covariance restrictions of the form  $\sigma_{ij} = 0$ ), the 3SLS and FIML estimators, though numerically distinct in small samples, have the same asymptotic distribution (Schmidt<sup>8</sup> (1976)). It follows that 3SLS, is asymptotically efficient in the presence of normally distributed errors. In contrast, however, when prior information concerning  $\Sigma$  is available, FIML estimator is asymptotically more efficient than 3SLS (Rothenberg and Leenders<sup>7</sup> (1964)). Finally, FIML estimator is defined only when all equations in the system are identified.

### 1.0 SIMULATION PROCEDURE

Monte Carlo methods constitute a fascinating, exacting and often indispensable craft with a range of applications that is already very wide yet far from being fully explored. The Monte Carlo method provides heuristic solutions to a variety of mathematical problems by performing statistical sampling experiments on a computer.

In econometrics, while asymptotic properties of estimators obtained by using various econometric methods are deductive in character, an approach which is often described as analytical, small sample properties of such estimators have always been studied from simulated data referred to as the Monte Carlo studies (an experimental approach) which is inductive in nature Nwabueze<sup>6</sup> (2005). The output of the analytical approach in the "finite-sample" area is very limited compared with what has been produced in the way of asymptotic results. The results of the analytical method in the finite-sample area are invariably very complex and exceedingly difficult to interpret which pose a major problem. In the Monte Carlo approach, findings are based on reasoning by inference. The use of this approach is due to the fact that real life observations on economic variables are in most cases plagued by one or all of the problems on multicollinearity, non-spherical disturbances and measurement errors.

The behavior of the system estimators are now available using Monte Carlo studies. The most important of these are Summers<sup>11</sup> (1965), Cragg<sup>2</sup> (1967), and Mosbaek and Wold<sup>5</sup> (1970). Interestingly, this method

has been excellently reviewed by many authors, especially Johnston<sup>4</sup> (1972), Smith<sup>9</sup> (1973), Intriligator<sup>3</sup> (1978) and Sowey<sup>10</sup> (1973). This approach is used in this work.

This approach may be described in broad terms as follows. The experimenter sets up an artificial system. Values are generated for the random disturbances for some specified sample size and using these values, values are calculated for the endogenous variables based on the assumptions of this artificial problem at each sample point. Pretending that the parameters are unknown and using only the values of the endogenous and predetermined variables at each sample point, several estimating techniques are applied in turn to obtain associated estimates of the parameters. The process of generating values for the disturbances, obtaining values for the endogenous variables, and calculating estimates of the parameters is repeated, or replicated, a large number of times. The set of estimates of each parameter by each estimator is then used to infer properties of the estimators for the given sample size and for the chosen values of the parameters.

The study uses three sample sizes  $N=15, 25$  and  $40$  each replicated  $50$  times. Each set of normal deviates with the different sample sizes and replications is then transformed using the upper ( $P_1$ ) triangular matrix. The procedure was repeated using the lower triangular matrix ( $P_1'$ ), such that in each case,  $\Omega = P_1 P_1'$ . Using the upper triangular matrix

$$\Omega = P_1 P_1' = \begin{bmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{bmatrix} \begin{bmatrix} S_{11} & 0 \\ S_{12} & S_{22} \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix}$$

to obtain a pair of random disturbances and for the lower triangular matrix,

$$\Omega = P_2 P_2'$$

$$\text{where } P_2 = \begin{bmatrix} S_{11} & 0 \\ S_{12} & S_{22} \end{bmatrix}$$

The behaviors of estimators are therefore examined across these triangular matrices. After estimating the parameters, the robustness of each estimator to the inadvertent correlation of the stochastic terms was examined using; average of estimates, absolute bias of estimates, root mean square error and sum of squared residuals of parameter estimates.

**Table 1: Summary of Estimators using Average R=50, P<sub>1</sub>**

Estimator	Level of correlation	EQ1					
		$\beta_{12}^{(1.5)}$			$\gamma_{11}^{(1.2)}$		
		N=15	N=25	N=40	N=15	N=25	N=40
OLS	r<-0.05	1.067074	1.070771	1.063577	-0.40953	-0.50417	-0.56386
	-0.05<r<0.05	1.033484	1.042028	1.046355	-0.49904	-0.53441	-0.50779
	r>0.05	1.013901	1.02698	1.030384	-0.60413	-0.57562	-0.58862
3SLS	r<-0.05	1.083419	1.296365	1.435163	-0.47362	0.029132	0.230474
	-0.05<r<0.05	1.042502	1.355854	1.43021	-0.53324	0.159156	0.302414
	r>0.05	0.989754	1.229802	1.271803	-0.40984	-0.19972	-0.12357
FIML	r<-0.05	-1.55879	-4.60894	-0.49344	-2.03921	-4.91175	-1.24092
	-0.05<r<0.05	-1.61711	-2.37039	-2.1021	-2.28008	-2.03702	-1.15926
	r>0.05	-0.8352	-1.06144	-0.91255	-0.41497	-0.80337	-0.73766

**Table 1: Summary of Estimators using Average R=50, P<sub>1</sub> (continued)**

Estimator	Level of correlation	EQ2								
		$\beta_{21}^{(1.8)}$			$\gamma_{22}^{(0.5)}$			$\gamma_{23}^{(2.0)}$		
		N=15	N=25	N=40	N=15	N=25	N=40	N=15	N=25	N=40
OLS	r<-0.05	0.89569 3	0.90118 3	0.89769 3	- 0.00314	0.08016 7	0.13679 7	0.50835 6	0.48881 6	0.40687 4
	-0.05<r<0.05	0.93639 3	0.93599 3	0.92698 3	0.03206 2	- 0.01435	- 0.00525	0.61272 2	0.62440 2	0.59301 7
	r>0.05	0.94834 3	0.96169 3	0.94834 3	0.05940 2	- 0.00299	0.26190 1	0.63842 7	0.753 7	0.41139 5
3SLS	r<-0.05	0.97595 7	1.13154 9	1.24462 9	0.00926 3	0.14684 2	0.49622 8	0.41031 8	1.06033 8	1.08499 2
	-0.05<r<0.05	0.99489 5	0.94269 5	1.12331 5	- 0.23727	0.34551 2	0.10778 4	1.04167 2	0.41698 2	1.01786 2
	r>0.05	1.30056 6	1.03855 6	1.21632 1	- 0.07144	- 0.00088	0.62785 5	1.78297 5	0.87216 7	0.81249 9
FIML	r<-0.05	- 0.96486	- 2.77071	- 0.16686	- 0.39955	- 1.64419	- 0.26099	- 0.68349	- 1.76555	- 0.02052
	-0.05<r<0.05	1.28313 6	1.50835 6	1.46402 1	0.85273 5	0.67438 5	0.22465 5	1.28932 5	0.70913 7	0.81512 9
	r>0.05	- 0.64733	- 0.72131	- 0.61169	- 0.01846	- 0.09378	0.03053 5	- 0.18251	- 0.11286	- 0.29474

**Table 2: Summary of Estimators using Average R=50, P<sub>2</sub>**

Estimator	Level of correlation	EQ1					
		$\beta_{12}^{(1.5)}$			$\gamma_{11}^{(1.2)}$		
		N=15	N=25	N=40	N=15	N=25	N=40
OLS	r<-0.05	0.992898	1.007177	1.005596	-0.58899	-0.65394	-0.61268
	-0.05<r<0.05	1.03872	1.043855	1.046446	-0.50066	-0.53163	-0.57261
	r>0.05	1.0644	1.071153	1.077282	-0.45167	-0.50358	-0.47483
3SLS	r<-0.05	1.138964	1.257422	1.382585	-0.39606	-0.12574	0.196764
	-0.05<r<0.05	1.091362	1.272043	1.405338	-0.42857	-0.08312	0.210843
	r>0.05	1.248948	1.481571	1.433504	-0.04207	0.460571	0.307667
FIML	r<-0.05	-2.24675	-1.56517	-2.07313	-2.35591	-3.29957	-3.00509
	-0.05<r<0.05	-3.76624	-6.44124	-1.57687	-3.62188	-5.85855	-2.09379
	r>0.05	-3.80075	-1.10505	-1.05307	-3.17179	-1.12594	-0.34167

**Table 2: Summary of Estimators using Average R = 50, P<sub>2</sub> (Continued)**

Estimator	Level of correlation	EQ2								
		$\beta_{21}^{(1.8)}$			$\gamma_{22}^{(0.5)}$			$\gamma_{23}^{(2.0)}$		
		N=15	N=25	N=40	N=15	N=25	N=40	N=15	N=25	N=40
OLS	r<-0.05	0.96792	0.96679	0.919973	0.07633	0.071831	0.137499	0.693704	0.701117	0.499387
	- 0.05<r<0.05	0.936266	0.938178	0.923779	0.062311	0.011683	0.131808	0.606942	0.67296	0.478739
	r>0.05	0.907646	0.91977	0.913413	-0.02132	-0.04718	0.10778	0.571083	0.663349	0.461893
3SLS	r<-0.05	1.173429	1.558212	1.068858	-0.4057	0.78732	0.233205	1.658431	1.876514	0.689285
	-0.05<r<0.05	1.668581	1.513603	1.206698	2.308587	0.685876	0.491424	0.617603	1.772122	0.955078

FIML	r>0.05	1.127006	0.950839	0.884521	0.016592	0.067383	-0.07385	1.183495	0.596628	0.502641
	r<-0.05	-1.76366	-1.49297	-0.87906	-0.96703	-0.49748	-0.37876	-1.73279	-1.03067	-0.83246
	-	-2.99361	-5.18181	-0.74722	-1.71173	-2.08676	-0.30778	-3.02344	-5.33491	-0.32156
	0.05<r<0.05	-2.98186	-0.64979	-0.79036	-1.02985	-0.34432	-0.13676	-2.84699	-0.03626	-0.19601

**Table 3:** Summary of Total Absolute Bias R=50, P<sub>1</sub>

Level of correlation	OLS			3SLS			FIML		
	N=15	N=25	N=40	N=15	N=25	N=40	N=15	N=25	N=40
r<-0.05	4.941557	4.961139	5.058924	4.994664	3.335791	2.508514	12.645898	22.70113	9.182728
-0.05<r<0.05	4.88493	4.946335	4.946682	6.691437	3.779803	3.018422	14.322373	14.299279	12.765143
r>0.05	4.944047	4.836939	4.936597	3.407983	4.060074	3.450798	9.098455	9.792754	9.526034

**Table 4:** Summary of Total Absolute Bias R=50, P<sub>2</sub>

Level of correlation	OLS			3SLS			FIML		
	N=15	N=25	N=40	N=15	N=25	N=40	N=15	N=25	N=40
r<-0.05	4.858138	4.907023	5.050227	3.830927	2.220908	3.429303	16.066133	14.885862	14.168504
-0.05<r<0.05	4.856425	4.864958	4.991839	5.359608	2.210336	2.73062	22.617801	31.903266	12.047216
r>0.05	4.929852	4.896483	4.914466	3.466025	3.443008	3.94552	20.831248	10.261373	9.517872

**Table 5:** Summary of Estimators using Root Mean Square Error R=50, P<sub>1</sub>

Estimator	Level of correlation	EQ1					
		$\beta_{12} (1.5)$			$\gamma_{11} (1.2)$		
		N=15	N=25	N=40	N=15	N=25	N=40
OLS	r<-0.05	0.436156	0.431532	0.437016	1.62725	1.717513	1.766868
	-0.05<r<0.05	0.467685	0.458839	0.454077	1.712024	1.742782	1.710205
	r>0.05	0.491549	0.47391	0.469965	1.8375	1.78329	1.791164
3SLS	r<-0.05	0.809737	0.342553	0.546158	2.432383	1.397053	1.632861
	-0.05<r<0.05	0.708987	0.584134	0.478117	2.012958	1.873058	1.422934
	r>0.05	0.953527	0.609297	0.471323	2.031971	2.068709	1.641591
FIML	r<-0.05	4.610025	9.772582	2.398144	6.264157	11.52345	3.128115
	-0.05<r<0.05	7.99254	6.476522	4.782457	10.47733	5.782321	3.21884
	r>0.05	2.486464	3.111921	2.918378	1.851861	2.693205	2.598102

**Table 5:** Summary of Estimators using Root Mean Square Error R=50, P<sub>1</sub> (continued)

Estimator	Level of correlation	EQ2								
		$\beta_{21} (1.8)$			$\gamma_{22} (0.5)$			$\gamma_{23} (2.0)$		
		N=15	N=25	N=40	N=15	N=25	N=40	N=15	N=25	N=40
OLS	r<-0.05	0.90552 1	0.89962	0.90252 9	0.70126 9	0.54722 3	0.43359 1	1.57697 9	1.54299 3	1.61150 7
	-	0.86375 9	0.86411 6	0.87312 7	0.62253 5	1.60600 7	0.56012 6	1.46225 6	1.43659 1	1.42830 1
	r>0.05	0.85711 2	0.83852 9	0.85181	0.58444 7	0.57740 7	0.32696 5	1.48406 8	1.27960 6	1.60844
3SLS	r<-0.05	1.28677 5	1.04272	1.51538 2	1.60038 2	1.52198 4	2.29374 9	3.56752 9	2.64613 1	3.06893 5
	-	1.17223 7	1.8286	1.02171 6	1.85341 5	2.02085 2	0.98687 5	3.99424 3	4.55724 1	2.72383 2
	r>0.05	2.00650 4	1.06009 3	1.02140 5	1.99529	1.69107 4	2.03809 6	6.52847 3	2.36978 9	2.18759
FIML	r<-0.05	3.41604 7	7.18922 6	2.22517	1.60380 7	4.61801 8	1.21946 3	3.47206 1	5.62756 7	2.11689 8
	-	7.10255 5	5.45487 1	4.13702 6	4.16893 6	3.57313 4	1.13353 1	9.37822 1	4.66970 1	3.76264 7
	r>0.05	2.57813 1	2.99622	2.7794	0.59975 9	0.77844 7	0.53376 2	2.50586 5	2.65070 2	2.73452 5

**Table 6:** Summary of Estimators using Root Mean Square Error R=50, P<sub>2</sub>

Estimator	Level of correlation	EQ1					
		$\beta_{12} (1.5)$			$\gamma_{11} (1.2)$		
		N=15	N=25	N=40	N=15	N=25	N=40
	r<-0.05	0.510122	0.494626	0.501262	1.81861	1.862027	1.81824
	-0.05<r<0.05	0.46242	0.456944	0.454163	1.714671	1.738476	1.776118

	r>0.05	0.440366	0.430319	0.423331	1.667882	1.710873	1.678407
3SLS	r<-0.05	0.605028	0.675491	0.748272	2.121374	2.186761	2.136393
	-0.05<r<0.05	1.218544	0.61113	0.473233	3.623711	2.062664	1.468751
	r>0.05	0.57849	0.518955	0.375886	1.814418	1.600259	1.265556
FIML	r<-0.05	6.109593	6.743089	5.832326	5.124391	7.467952	6.575807
	-0.05<r<0.05	10.61125	16.28581	4.148964	11.802	17.38281	4.989155
	r>0.05	9.730886	3.049391	2.95474	8.309207	3.412404	1.872224

**Table 6:** Summary of Estimators using Root Mean Square Error R=50, P<sub>2</sub> (continued)

Estimator	Level of correlation	EQ2								
		$\beta_{21}$ (1.8)			$\gamma_{22}$ (0.5)			$\gamma_{23}$ (2.0)		
		N=15	N=25	N=40	N=15	N=25	N=40	N=15	N=25	N=40
OLS	R<-0.05	0.832829	0.833769	0.899442	0.637005	0.57612	0.459278	1.373695	1.354464	1.533501
	-0.05<r<0.05	0.86398	0.861902	0.876361	0.57803	0.571873	0.418968	1.442819	1.360709	1.537516
	r>0.05	0.894405	0.880426	0.886701	0.642272	0.606852	0.435069	1.514568	1.362601	1.555075
3SLS	R<-0.05	1.694989	2.379137	1.127651	3.0698	2.197369	1.178233	4.036605	6.737785	2.776149
	-0.05<r<0.05	5.632721	1.295517	1.110134	11.1051	3.852231	1.503287	13.75275	4.833901	2.78772
	r>0.05	1.225078	1.148319	1.083567	2.633028	1.437763	1.226975	3.262547	2.661511	1.996823
FIML	r<-0.05	5.638482	5.638683	4.562609	3.606066	3.417611	2333067	7.162911	5.380956	4.560933
	-0.05<r<0.05	9.115358	14.04307	3.39546	6.555847	7.772889	1.629755	10.22504	16.02082	2.778681
	r>0.05	8.230567	2.807013	2.877896	3.422276	2.124536	1.187865	9.410859	2.50089	2.403147

**Table 7:** Summary of Sum of Squared Residuals for Three Correlation Levels R=50, P<sub>1</sub>

Estimator	Level of correlation	EQ1			EQ2		
		N=15	N=25	N=40	N=15	N=25	N=40
		OLS	r<-0.05	8.519252	14.19056	22.9875	5.652294
-0.05<r<0.05	7.872591		13.48918	22.01066	5.293851	9.039257	15.69381
r>0.05	7.976691		11.86255	19.50978	5.506655	7.679018	14.58404
3SLS	r<-0.05	52.23134	31.7618	129.0425	100.7806	133.8592	700.6628
	-0.05<r<0.05	28.26443	89.056	134.4355	75.10572	547.6233	216.7182
	r>0.05	37.30773	81.22743	96.78534	545.47	128.758	270.4573
FIML	r<-0.05	1648.747	15698.51	1369.229	878.4006	9469.272	874.2625
	-0.05<r<0.05	7087.641	8074.355	7036.368	6076.905	5769.992	4840.792
	r>0.05	685.7844	1859.201	2800.426	555.1461	1465.665	2029.094

**Table 8:** Summary of Sum of Squared Residuals for Three Correlation Levels R=50, P<sub>2</sub>

Estimator	Level of correlation	EQ1			EQ2		
		N=15	N=25	N=40	N=15	N=25	N=40
		OLS	r<-0.05	8.138315	14.0019	21.92372	5.932354
-0.05<r<0.05	7.629978		13.73168	21.76534	5.294499	9.180186	16.11784
r>0.05	8.45091		11.76171	19.15493	5.629083	7.617836	13.30717
3SLS	r<-0.05	31.65482	83.26091	203.0776	266.5712	983.7893	233.0846
	-0.05<r<0.05	146.3053	79.31666	124.7349	3638.535	353.7829	345.611
	r>0.05	46.27148	100.0351	111.6931	150.4811	144.4524	138.6418
FIML	r<-0.05	4018.641	8884.87	10138.66	3108.333	5480.683	6191.815
	-0.05<r<0.05	13077.01	51373.8	5114.238	10195.22	41057.14	3128.517
	r>0.05	12450.62	1785.939	2811.317	9531.924	1248.937	2149.904

**4.0 RESULTS/DISCUSSION**

Tables 1-8 are used to summarize the relative performances of the three estimators using, average of estimates, total absolute bias (TAB), root mean square error (RMSE) and sum of squared residuals (RSS). The three levels of correlation coefficients used are given in the tables using three different sample sizes. This experiment is performed using the upper and lower

triangular matrices (P<sub>1</sub> and P<sub>2</sub>). The results of the experiment reveal that on the basis of TAB criterion, the 3SLS method shows asymptotic behavior (the total absolute biases decrease with increased sample size) under P<sub>1</sub> while the other two estimators, OLS and FIML exhibit no such pattern. For both equations and irrespective of whether P<sub>1</sub> or P<sub>2</sub> is used, OLS RSS estimates increased consistently as sample size increased making it inferior to the other estimators.



Judging the performance of the estimators by RMSE shows that as the correlation coefficient changes through the three critical levels, OLS RMSE estimates increase consistently for equation 1 and decrease consistently for equation 2 under  $P_1$ , the reverse is the case for  $P_2$ . However, under  $P_1$ , FIML is remarkably best in the open-ended intervals and remarkably poor at the closed interval

## 2.0 CONCLUSION

The magnitudes of the estimates yielded by two estimators, OLS and 3SLS exhibited fairly consistent reaction to changes in magnitudes and direction of correlations of error terms. The results of this study show that the performances of the estimators vary with the correlation interval with OLS ranking best especially in the open-ended intervals. The results also reveal that choice of triangular matrix upper ( $P_1$ ) or lower ( $P_2$ ) as well as the identifiability status of the equations are factors to be reckoned with. Consequently, the interaction of these factors: correlation levels of error term, identifiability status of the equations and the use of  $P_1$  or  $P_2$  have not facilitated a conclusive ranking of the estimators using our own definition of 'best' estimators.

## REFERENCES

- Adejumobi, A. A., 2006. Robustness of Simultaneous Estimation Techniques To over-identification and Correlated Random Deviates. PhD Thesis, Unpublished, University of Ibadan.
- Cragg, J., 1967. On the Relative Small-sample Properties of Several Structural-Equation Estimators. *Econometrica*, 35: 89-110.

- Intriligator, M. D., 1978. *Econometric models, Techniques and Applications.* Prentice-Hall, Englewood Cliffs, N.J., 189.
- Johnston, J., 1972. *Econometric Methods.* 2nd ed. New York: McGraw Hill.
- Mosbaek, E. and Wold, H., 1970. *Interdependent Systems: Structure and Estimation.* North-Holland, Amsterdam.
- Nwabueze, J. C., 2005. Performances of Estimators of Linear Models with Auto correlated Error Terms when the Independent Variable is Normal. *Journal of the Nigerian Association of Mathematical Physics.* 9: 379 – 384.
- Rothenberg, T. and Leenders, C., 1964. Efficient Estimation of Simultaneous Equation Systems. *Econometrica*, 406-425.
- Schmidt, P., 1976. *Econometrics.* New York: Marcel Dekker.ss
- Smith, V., 1973. *Monte Carlo Methods.* D. C. Heath, Lexington Mass.
- Sowey, E., 1973. A Classified Bibliography of Monte Carlo Studies in Econometrics. *Journal of Econometrics*, 1: 377- 395.
- Summers, R., 1965. A Capital Intensive Approach to the Small Sample Properties of Various simultaneous Equation sEstimators. *Econometrica*, 33: 1-41.