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Trends in Petroleum Engineering

Research Article

Pressure Drop Determination for Multiphase Flow in a Vertical Well Tubing

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Abstract

Several studies have been carried out, by researchers to predict multiphase flow pressure drop in the oil and gas industry, but yet there seems to be one being generally acceptable for accurate prediction of pressure drop. This is as a result of some constraints in each of these models, which makes the pressure drop predicted by the model far from accurate when compared to measured data from the field. This study is aimed at developing a multiphase fluid flow model in a vertical tubing using the Duns and Ros flow model. Data from six wells were utilized in this study and results obtained from the Modified model compared with that of Duns and Ros model along other models. From the result, it was observed that the newly developed model (Modified Duns and Ros Model) gives more accurate result with a R-squared value of 0.9936 over the other models. The Modified model however, is limited by the choice of correlations used in the computation of fluid properties.

Keywords: Duns and Ros model, Modified duns and Ros flow model multiphase flow, Multiphase pressure drops

Introduction

Multiphase flow is a common occurrence in the petroleum, chemical and process, space, geothermal energy plant, air-conditioning system and nuclear reactor industries. The need to study multiphase flow in these industries arises for a number of reasons which include; proper design and safety purposes. The polyphasic flow of Newtonian fluids in vertical pipes has been investigated both theoretically and experimentally by several researchers.[1-3] While others dealt with the problem of predicting pressure gradients in an oil wells, where the flowing fluid may be gas, oil or water mixture with interphase mass transfer. The complexity of fluids encountered, the large diameters, long lengths of pipe and often times in hostile environments, make multiphase flow unique in the petroleum industry[4].

The essence of multiphase flow studies in the petroleum industry is to determine the pressure drop in pipes (vertical, horizontal or inclined). Several approaches have been used to achieve the best method to obtain accurate prediction of pressure drops in pipes. The methods used to predict pressure gradient can be classified as empirical correlations and mechanistic models (Table 1). These correlations are based on experiments performed mostly in the laboratory.

The large variation in pressure and temperature along a wellbore suggests that different flow patterns would exist at various depths[5] Crude oil usually enters the wellbore as a single phase, but as the fluid moves upward, pressure decreases gradually and gas will evolve from liquid and bubble flow starts. Flow pattern is the various configurations that exist as interface between different phases in a multiphase fluid flow in the wellbore or pipeline. These flow patterns have been described by different investigators as internal structures or interface existing between the phases present in a multiphase flow. They are usually obtained from flow pattern maps (empirically or mechanistic model). In some cases, these flow

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Received: 11 November, 2021 **Published:** 15 December, 2021

Citation: Akintola SA, Olakunle F. Pressure Drop Determination for Multiphase Flow in a Vertical Well Tubing. *Trends Petro Eng*. 2021;1(2):1–7. DOI: 10.53902/TPE.2021.01.000510

patterns are subjective based on the visual observation of the investigator[6] from the dimensional or dimensionless parameters deduced from experiments.

The four major flow patterns for vertical flow in pipes: bubble flow (bubbles of gas of small sizes present in liquid), slug/plug flow (large gas mass formed from the merged individual gas bubbles), churn flow (the slugs begin to disintegrate due to changes in phase flow rates) and annular flow (the gas flows in the middle of the pipe, while the liquid phase occupies the space adjacent to the pipe wall)[7] were described by as presented in the Figure 1.

Table 1: Empirical Correlations.

Category		
A	B	C
Poettmann and Carpenter	Hagedorn and Brown	Duns and Ros
Baxendell and Thomas	Gray	Orkiszewski
Francher and Brown	Asheim	Aziz[9]
		Chierici
		Beggs[10]
		Mukherjee and Brill

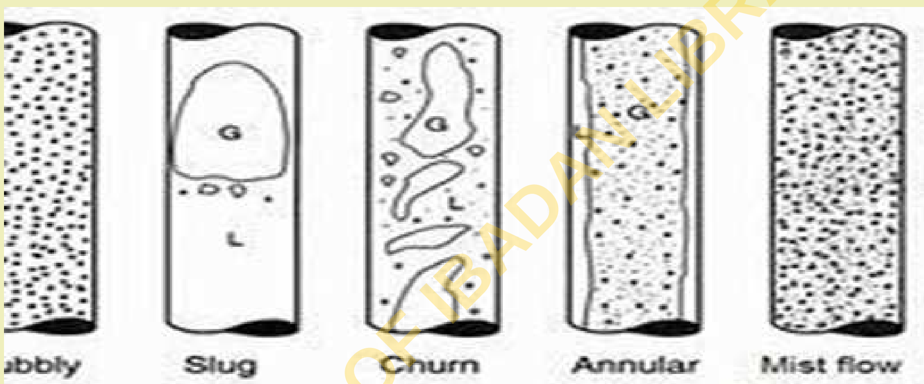

Figure 1: Sketches of multiphase flow regimes in vertical pipe.[6]

The sand-grain[7] experiments formed the basis for friction factor data for rough pipes wall is given as equation 1.0.

$$\frac{1}{\sqrt{f}} = 1.74 - 2\log\left(\frac{2\varepsilon}{d}\right) \quad \ldots\ldots 1.0$$

An empirical equation 2.0 to describe the variation of f in the transition region was proposed[8]. It has become the basis for modern friction factor charts

$$\frac{1}{\sqrt{f}} = 1.74 - 2\log\left(\frac{2\varepsilon}{d} + \frac{18.7}{N_{Re}\sqrt{f}}\right) \quad \ldots\ldots 2.0$$

Solving equation 1.0 for f requires a trial and error process. Equation 3.0 can be expressed as

$$f_c = \left[1.74 - 2\log\left[2\left(\frac{\varepsilon}{d}\right) + \frac{18.7}{(N_{Re})_n\sqrt{fest}}\right]\right]^{-2} \quad \ldots\ldots 3.0$$

Values of f are estimated, $fest$, and then calculated, f_c, until they agree to within an acceptable tolerance. The pressure gradient equation for single phase flow can be modified for multiphase flow by considering the fluid to be a homogeneous mixture, hence equation 4.0

$$\left(\frac{dp}{dL}\right)t = f_{tp}\rho V^2 / 2gD + \frac{g}{g_c}\rho_m \sin\theta + \frac{\rho V_m}{g_c}\frac{DV_m}{dL} \quad \ldots\ldots 4.0$$

Where the definition for ρ and V_m can vary with different investigators. For vertical flow, sin θ =1, $dL = dz$ and the equation can be written as shown in the equation 5.0

$$\left(\frac{dp}{dz}\right)t = \left(\frac{dp}{dz}\right)el + \left(\frac{dp}{dz}\right)f + \left(\frac{dp}{dz}\right)acc \quad \ldots\ldots 5.0$$

Except for conditions of high velocity, most of the pressure drop in vertical flow is caused by this component. The pressure drop caused by acceleration is normally negligible and is considered only for cases of high flow velocities. Many methods have been developed to predict two-phase flowing pressure gradients. This study presents a modified Duns and Ros Model for predicting pressure drop in a gas, oil and water 3-phase flow in vertical pipes. These models will combine three empirical correlations; Duns and Ros[2]. model for selecting the flow regimes, Aziz, et al.[9] model to account for the Liquid hold up and the Beggs and Brills model[10] method of calculating the two-phase friction factor.

The input volume fractions for liquid and gas in –situ volume fraction are defined as expressed in the equations 6.0a and 16.0b, respectively.

$$\lambda_L = \frac{q_l}{q_l+q_g} \quad \text{......} \quad 6.0a$$

and

$$\lambda_g = \frac{q_g}{q_l+q_g} \quad \text{......} \quad 6.0b$$

Where,

λ_L and λ_g =In-situ liquid and gas volume fraction , respectively

q_l and q_g = Liquid and gas flow rate at prevailing pressure and Temperature (bbl/d, respectively

The input volume fractions, λ_L and λ_g are also referred to as the 'no slip holdups'.

In-situ Volume fraction (Liquid Holdup Effect): With the in-situ volume fraction of the denser and lighter liquid phase are expressed in the equations 7.0a and 7.0b

$$H_L = \frac{V_L}{V} \quad \text{......} \quad 7.0a$$

while the in-situ volume fraction of the lighter gas phase is defined as

$$H_g = \frac{V_g}{V} \quad \text{......} \quad 7.0b$$

Where,

H_L and H_g =In-situ liquid and gas volume fraction, respectively.

V_L and V_g ==Volume of liquid and gas phase in pipe segment (cubic feet), respectively

V=Volume of the pipe segment (cubic feet)

If the gas is completely occupied by the phases equation 8.0 applies,

$$H_g = 1 - H_L \quad \text{......} \quad 8.0$$

The in-situ volume fraction or Liquid holdup is often estimated from the multiphase (Empirical or mechanistic) correlations.

The Slip Velocity is expressed mathematically as presented in the equation 9.0

$$v_s = \bar{v}_g - \bar{v}_l \quad \text{......} \quad 9.0$$

where,

v_s= Slip velocity(ft/s)

$\bar{v}_L$ and $\bar{v}_g$ =Average in-situ velocity of liquid and gas (ft/s), respectively

The superficial velocity of each phase is expressed mathematically as shown in the equations. 10.0 and 11.0

For the liquid phase,

$$v_{sL} = \frac{4q_l}{\pi D^2} \quad \text{......} \quad 10.0$$

For the gas phase,

$$v_{sg} = \frac{4q_g}{\pi D^2} \quad \text{......} \quad 11.0$$

Where,

v_{sL} and v_{sg} =Superficial liquid and gas velocity (ft/s), respectively

q_l and q_g = Liquid flow rate at prevailing pressure and Temperature (bbl/D), q_g

D=Pipe Diameter (ft)

The average in-situ velocities, $\bar{v}_g$ and $\bar{v}_L$ are related to the superficial velocities and the in-situ fractions by the following equations 12.0 and 13.0.

$$\overline{v_g} = \frac{v_{sg}}{H_g} \quad \text{......} \quad 12.0$$

$$\overline{v_L} = \frac{v_{sL}}{H_L} \quad \text{......} \quad 13.0$$

Therefore, the slip velocity is presented in the equation 14.0y,

$$v_s = 4/\pi D^2 \left[\frac{q_g}{1-H_L} - \frac{q_L}{H_L}\right] \quad \text{......} \quad 14.0$$

The input volume fraction is related to the superficial velocity by the following equation 15.0 and 16.0,

$$\lambda_L = \frac{v_{sL}}{v_m} \quad \text{......} \quad 15.0$$

$$\lambda_L = \frac{v_{sg}}{v_m} \quad \text{......} \quad 16.0$$

Where;

v_m=Mixture velocity(ft/s)

The Mixture Density is expressed in the equations 17.0

$$\rho_m = \rho_L H_L + \rho_g(1- H_L) \quad \text{......} \quad 17.0$$

Where,

ρ_m= Mixture density (lbf/cu.ft)

ρ_L and ρ_g = Liquid and Gas density (lbf/cu.ft), respectively

H_L=In-situ liquid volume fraction

The mixture velocity is given by equation 19.0

$$vm = vsL + vsg \quad \text{......} \quad 18.0$$

Where,

v_m=mixture velocity (ft/s)

v_{sL} and v_{sg} =Superficial liquid and gas velocity (ft/s), respectively

The mixture viscosity is expressed as showned in the equation 19.0

$$\mu_m = \mu_L H_L + \mu_g(1 - H_L) \quad \text{......} \quad 19.0$$

Where,

μ_m =mixture viscosity (cp)

μ_L and μ_g=Liquid viscosity (cp)

H_L=In-situ gas volume fraction

The no-slip density is defined in the equation 20.0 as follows

$$\rho_n = \rho_L \lambda_l + \rho_g \lambda_g \quad \text{......} \; 20.0$$

Where,

ρ_n=no-slip density (lb/cu.ft)

ρ_L and ρ_g =Liquid and gas density (lb/cu.ft)

λ_L and λ_g =In-situ liquid and gas volume fraction

The no-slip viscosity calculated with the equation 21.0

$$\mu_n = \mu_L \lambda_l + \mu_g \lambda_g \quad \text{......} \; 21.0$$

Where,

μ_n=No slip viscosity (cp)

μ_L and μ_g=Liquid and Gas viscosity (cp)

$\lambda_L$and λ_g =In-situ liquid and gas volume fraction

Methodology

The general energy equation (equation 22.0), is the theoretical basis for most fluid flow equations, and most mechanistic models and multiphase correlations which can be used for estimating pressure gradient in a producing well are derived from it.

$$U_1 + P_1V_1 + \frac{mv_1^2}{2g_c} + \frac{mgz_1}{g_c} + Q + W_s = U_2 + P_2V_2 + \frac{mv_2^2}{2g_c} + \frac{mgz_2}{g_c} \quad \text{......} \; 22.0$$

Where;

U = Internal energy of the fluid lbf-ft

P = pressure, psia

V = Volume, cuft

m = mass of fluid, lbm

v = average fluid velocity, ft/sec

z = distance in the vertical direction, ft

g = acceleration due to gravity, ft/sec^2

g_c = conversion factor = 32.17lbm ft/lbf-sec^2

PV = Pressure volume (Energy of compression or Expansion), lbf-ft

$\frac{mv^2}{2g_c}$ = Kinetic energy of the fluid, lbf-ft

$\frac{mgz}{g_c}$ = Potential energy of the fluid, lbf-f

Q = Energy added or removed from the fluid, lbf-ft

W_s =Shaft work done on the fluid by the surroundings (shaft work done by the fluid to the surroundings is negative), lbf-ft

The equation 23.0 gives an expression for the total pressure gradient for a vertical pipeline or tubing.

$$\frac{\partial P}{\partial z} = -\left(\frac{\rho v \partial v}{g_c \partial z} + \frac{\rho g}{g_c} + \frac{2f\rho V^2}{g_c D}\right) \quad \text{......} \; 23.0$$

Where,

$\frac{\partial P}{\partial z}$= Total pressure gradient, lbf/ft^3

$\frac{\rho g}{g_c}$= Elevation or static pressure gradient, lbf/ft^3

$\frac{\rho v \partial v}{g_c \partial z}$= Acceleration or Kinetic pressure gradient, lbf/ft^3

since $\frac{g}{g_c} \approx 1$, characterizing pressure drop associated with fluid flow in a vertical tubing, leads to the equation 24.0

$$\frac{\partial P}{\partial z} = \frac{1}{144}\left(\frac{\rho v \partial v}{g_c \partial z} + \rho + \frac{2f\rho V^2}{g_c D}\right) \quad \text{......} \; 24.0$$

The introduction of the two phase friction factor, f_{tp}, gas liquid mixture density, ρ_m and the gas-liquid mixture velocity (total fluid velocity), V_m, results in the equation 25.0

$$\frac{\partial P}{\partial z} = \frac{1}{144}\left(\frac{\rho_m v_m \partial v_m}{g_c \partial z} + \rho + \frac{2f_{tp}\rho_m V_m^2}{g_c D}\right) \quad \text{...} \; 25.0$$

The Model Development

This model is aimed at the modification of Duns and Ros Model for modelling two phase flow in vertical pipes. This method combines the multiphase flow pattern criteria with the physical models for pressure drop and liquid holdup calculations for each of the flow patterns being considered. The Duns and Ros Model is modified with Aziz, et al. model used to determine the liquid hold up for bubble and slug flow regimes (Taylor bubble effect) with the liquid holdup calculation for mist flow calculated using Duns and Ros method. The Beggs and Brills method was used for calculating and estimating the friction factor

The following assumptions were made:

The flow is steady, isothermal, Newtonian, turbulent- N_{Re} is more than 2100 and no work is done.

The acceleration/kinetic energy pressure gradient term in the energy equation and end effects is negligible

The fluid behaves as a continuum and pipe is a smooth pipe.

With the assumptions made above, the mechanical Energy Balance Equation can be in psi/ft as expressed in the equation 26.0

$$\frac{\partial P}{\partial z} = \frac{1}{144}\left(\rho_m + \frac{2f_{tp}\rho_m V_m^2}{g_c D}\right) \quad \text{......} \; 26.0$$

Flow Regime Determination

The Duns and Ros model developed a map for flow pattern prediction, by identifying separate regions and dimensionless groups developed in their correlation to predict the flow patter ns

(Equations 27.0 a. 27.0b and 27.0c).

$$NLv = 1.938V_{sl}\left(\frac{\rho_l}{\sigma}\right)^{0.25} \qquad ....27.0a.$$

$$Ngv = 1.938V_{sg}\left(\frac{\rho_l}{\sigma}\right)^{0.25} \qquad ....27.0b$$

$$Nd = 120.872D\left(\frac{\rho_l}{\sigma}\right)^{0.5} \qquad ....27.0c$$

Where,

ρ_l is the liquid mixture density in lbm/cu-ft

σ is the liquid mixture surface tension in dynes/cm

D is the flow diameter of the tubing in ft

V_{sl} and V_{sg} are the liquid superficial and gas superficial velocities respectively.

In this study, the flow patterns considered are bubble, slug and mist (equations 28.0 a., 28.0b and 28.0c) type of flow

Bubble Flow

$0 \le Ngv \le (L1 + L2)\ NLv$ .... 28.0a.

Slug Flow

$(L_1 + L_2)\ N_{Lv} < N_{gv} < (50 + 36\ N_{Lv})$ 28.0b

Mist Flow

$N_{gv} > (75 + 84\ N_{gv}^{0.75})$ 28.0c

Pressure Gradient Determination

The liquid holdup factor for bubble and slug types of flow was estimated using the equation 29.0 and the rise velocity of small gas bubbles in a flowing liquid calculated using the equation 30.0

$$H_L = 1 - \frac{V_{sg}}{V_{bf}} \qquad ....29.0$$

V_{bf} is the rise velocity of small gas bubbles in a flowing liquid and can be calculated as follows

$$V_{bf} = 1.2V_m + V_{bs} \qquad ....30.0$$

Where,

V_{bs}, the rise velocity of a continuous swarm of small bubbles in a static liquid column

The mixture density is a function of liquid hold up H_L, given by equation 31.0

$$\rho m = \rho L\ HL + \rho G(1 - HL) \qquad ....31.0$$

Substituting equation 29.0 into 31.0 to obtain equation 32.0

$$\rho_m = \rho_g + \left(\frac{V_{bf} - V_{sg}}{V_{bf}}\right)(\rho_L - \rho_g) \qquad ....32.0$$

The equation 33.0 is the modified model for estimating pressure gradient for bubble and slug types of flow in psi/ft

$$\frac{\partial P}{\partial z} = \frac{1}{144}\left(\rho_g + \left(\frac{V_{bf} - V_{sg}}{V_{bf}}\right)(\rho_L - \rho_g)\right)\left(1 + {2f_{tp}V_m}^2/g_cD\right) \qquad ....33.0$$

For bubble flow: V_{bs}, the rise velocity of a continuous bubbles in a static liquid column in equation 34.0 is given by

$$V_{bs} = 1.41\left[\frac{\sigma g(\rho_L - \rho_g)}{{\rho_L}^2}\right]^{0.25} \qquad ....34.0$$

And for Slug flow, the rise velocity is presented in the equation 35.0

$$V_{bs} = C\sqrt{\left[\frac{gD(\rho_L - \rho_g)}{\rho_L}\right]} \qquad ....35.0$$

Where C was given by Wallis as shown in the equation 36.0

$$C = 0.345\left[1 - e^{(-0.029N_v}\right]\left[1 - e^{\left(\frac{3.37 - N_E}{m}\right.}\right] \qquad ......36.0$$

And

$$N_v = \sqrt{\frac{D^3 g\rho_L(\rho_L - \rho_g)}{\mu_L}} \qquad ....37.0$$

With m determined from the expression in the equation 38.0

N_v	m
≥ 250	10
$250 > N_v > 18$	$69{N_v}^{-0.35}$
≤ 18	25

........38.0

MIST FLOW

Assume there is no slippage in the flow. i.e. S=0, the liquid-hold up is equation 39.0

$$H_L = \frac{1}{1 + \frac{V_{sg}}{V_{sl}}} \qquad ....39.0$$

Two-Phase Friction Factor Determination

To account for the no slip friction factor f_n. Payne, et al. (1979)[11] modifications to Beggs and Brills model was employed. A normalizing friction factor obtained from the Colebrook equation (40.0) was used with an iterative procedure used to obtain f_c

$$f_c = \left[1.74 - 2\log\left[2\left(\frac{\varepsilon}{d}\right) + \frac{18.7}{(N_{Re})_n\sqrt{fest}}\right]\right]^{-2} \qquad .....40.0$$

Where,

$\frac{\varepsilon}{d}$ = Relative pipe roughness

D = Pipe diameter in ft

N_{Re} = no slip Reynolds number

For No slip Reynold's number, is obtained from the equation 41.0

$$(N_{Re})_n = \frac{G_m D}{\mu_m} \quad \text{.......41.0}$$

And the friction factor ratio calculated with the equation 42.0

The friction factor ratio, $\frac{f_{tp}}{f_n} = e^S$ 42.0

Then the slip factor,S is obtained calculated with the equation 43.0

$$S = \frac{[\ln y]}{\{-0.0523+3.182\,[\ln y]-0.8725\,[\ln y]^2+0.01853\,[\ln y]^4\}} \quad \text{.......43.0}$$

The equation 43.0 applies for $y \geq 1.2$

Where $y = \frac{\lambda_l}{[H_L]^2}$ 44.0

H_L is the liquid hold up obtained for each flow regime

S becomes unbounded at a point in the interval $1 > y < 1.2$; for this interval the function S is calculated from the equation 45.0:

$$S = \ln(2.2y - 1.2) \quad \text{.......45.0}$$

Hence the two phase friction factor, equation 46.0

$$f_{tp} = f_n\left(\frac{f_{tp}}{f_n}\right) \quad \text{.......46.0}$$

The Python program included in the Appendix A (along the additional correlation information used Appendix B) was developed to first select the flow regime with the given fluid, pipe and well properties using the boundaries previously stated then to evaluate the pressure drop at any depth using the Modelled pressure gradient formula given by the equation 33.0

Results and Discussion

With the Modified Model in place, a Python program was generated to carry out the computations. The program developed on Duns and Ros method to select the flow regimes that might occur in a vertical oil and Gas well at different rates of flow of fluid in the wellbore and also to evaluate the pressure drop at a given depth within that region. Fluid production data from six different wells (Table 2) were used in testing the model. The predictions were made based on the Modified model and Duns and Ros model and the data obtained were compared with measured data and then compared with some of the previous models Aziz, Ansari, Hagedorn and Brown, and Beggs and Brill as presented in the Table 3. The Relative Mean Square Error (RMSE), Mean Absolute Error (MAE) and the R-squared were estimated the for all the Models and presented in the Table 4.[12]

Table 2: The Data Employed for the Six Wells Being Examined.

Well	Pipe ID (Inches)	Well Depth, (ft)	WHP (psia)	Avg Temp. (°f)	Q_o (bbl/day)	Q_w (bbl/day)	GOR (scf/ bbl)	SG_o	Density of Gas
A	3.83	11373	2191	150	9922	0	1375	0.85	4.23
B	2.441	7150	369	90	2201	0	78	0.93	0.65
C	2.992	3890	670	110	1850	0	575	0.89	2.5043
D	1.995	10184	820	90	2000	0	500	0.9	2.4258
E	2.441	8010	210	90	800	200	160	0.9	0.59
F	2.992	5151	505	90	1140	0	450	0.9	2.19

Table 3: Test and Predicted Pressure Drop Results.

Well	WHP (psia)	ΔP (Measured)	ΔP (Modified)	ΔP (Duns and Ros)	ΔP (Aziz)	ΔP (Ansari)	ΔP Hagedorn and Brown	ΔP (Beggs & Brill)
A	505	1600	1656	1556.64	1738	1463	1529	1530.76
B	210	2310	2364	2179.92	2280	2220	2530	2119.6
C	820	3620	3587	3420.69	3560	3660	3830	3451.5
D	670	760	747	787.24	750	640	950	771.95
E	369	2594	2643	2677.09	2688	2510	2174	2784.1
F	2191	3961	4039	4197.48	4341	3629	4203	4071.92

Table 4: The Relative Mean Square Error, Mean Absolute Error and the R-Squared Values.

MODELS	RMSE	MAE	R-SQUARED
Modified Model	124	10.6	0.9936
Duns and Ros	142	11	0.9916
Aziz	171	10.9	0.9878
Ansari	163	11.6	0.9889
Beggs and Brills	140	11.1	0.9918

Conclusion

With the helpful of python program, the pressure drop estimation process presented in this paper, is relatively easy and requires little computing time and absolutely cost effective when compared to the use of pressure gauges for pressure measurement in the field.

A distinction has to be made between the different flow regimes that might occur in a tubing/pipe, as this greatly affects the pressure gradient. Also, an in-depth knowledge of the multiphase flow variables, tubing properties and how they individually and collectively affect flow of fluids in pipes is necessary to ensure proper interpretations of the model.

The following observations were made during this study:

- The modified Model gave a mean absolute error of 10.6% while 11% MAE was obtained from the original work of Duns and Ros. Hence, there is a reduction margin of error between the predicted pressure drop values using this model and measured values.
- It was observed that the model gives the most reasonable results for Pressure drop, when compared with the results obtained from other existing models available.

Recommendation

- More well data should be used in the analysis in order to validate the result of the model.
- Plots of different test data and variations in the dependent variables to establish empirical relationships between well head pressures and flowing bottom-hole pressures as estimated by the model.

Generally, an improved result is obtained with the use of the Modified Model in prediction when compared to the result obtained from the original work of Duns and Ros.

Acknowledgments

None.

Funding

None.

Conflicts of Interest

Author declares that there is no conflict of interest.

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